Road inclinations in the design of LPV-based adaptive cruise control *

Balázs Németh and Péter Gáspár

Systems and Control Laboratory, Computer and Automation Research Institute, Hungarian Academy of Sciences, Hungary
Kende u. 13-17, H-1111 Budapest, Hungary;
Fax: +36-1-4667503; Phone: +36-1-2796171; E-mail: gaspar@sztaki.hu

Abstract: The paper presents the design of a new adaptive cruise control method which takes into consideration the knowledge of the inclinations of the road and the velocity regulations along the route of the vehicle. By choosing the velocities of the vehicle that are the most suitable for the given road conditions, the number of unnecessary accelerations and brakings and their durations can be reduced. First a control-oriented model which contains the longitudinal control forces and disturbances is formalized. In the model the predicted road inclinations and velocity regulations are considered together with the safety requirements. Then the control system is designed by using the Linear Parameter Varying (LPV) control theory. The operation of the longitudinal controlled system and the influence of different designed parameters are also analyzed.

Keywords: vehicle control, adaptive cruise control, look-ahead control, LPV control, robust stability and performance, optimization.

1. INTRODUCTION AND MOTIVATION

The paper presents a method to reduce energy and fuel consumption when the external road and traffic information is taken into consideration during the journey. The cruise control systems automatically maintain a steady speed of a vehicle as set by the driver by setting the longitudinal control forces. The adaptive cruise control (ACC) systems are able to switch from speed control to spacing control automatically and follow the preceding vehicle at a safe distance using automated throttle control. The current ACC systems are able to take into consideration only instantaneous effects of road conditions, since they do not have information about the oncoming road sections. In the paper predicted road inclinations are taken into consideration in the design of the longitudinal control force. The aim in this calculation is to achieve a control force which is similar to the driver’s requirement.

The driver is usually able to perceive the road conditions. For example before the downhill slope the driver can see the change in the curve of the road. Here the velocity of the vehicle increases, thus the control force of the vehicle before the slope can be reduced. As a result at the beginning of the slope the velocity of the vehicle decreases, thus it will increase from a lower value. Consequently, the brake system can be activated later or it is not necessary to activate it at all. If the velocity in the next road section changes it is possible to set the adequate control force.

Using the idea of slope and velocity regulation fuel consumption and the energy required by the actuators can be reduced. Moreover, the unnecessarily frequent activation of the brake is undesirable because of the wear of the brake pad/disc and the loss in kinetic energy. The control of longitudinal dynamics requires the integration of these vehicle components, see Kiencke and Nielsen (2000); Trachtler (2004).

Several methods in which the road conditions are taken into consideration have already been proposed, see Ivarsson et al. (2009); Nouveliere et al. (2008); Németh and Gáspár (2010). The look-ahead control methods assume that information about the future disturbances to the controlled system is available. To find a compromise solution between fuel consumption and travelling time leads to an optimization problem. The optimization was handled by using a receding (sliding) horizon control approach in Hellström et al. (2010); Passenberg et al. (2009). In another approach the terrain and traffic flow were modeled stochastically using a Markov chain model in Kolmanovsky and Filev (2009). In Hellström et al. (2009) the approach was evaluated in real experiments.

In the proposed method besides road inclinations the speed regulations and traffic environment are also taken into consideration. The aim of the method is to calculate the longitudinal forces by using an optimization method. The optimal solution is built into a closed-loop interconnection structure in which a robust controller is designed by using an LPV method. In the LPV method uncertainties, disturbances and nonlinear properties of the system are also handled. In the paper the conditions on the oncoming road sections are assumed to be known. Several methods have been proposed for slope estimation. Cameras, differential
GPS or a GPS/INS systems have been used, see e.g. Hahn et al. (2004).

The paper is organized as follows: Section 2 formalizes the road characteristics, speed regulations and traffic environment in a control-oriented model and establishes the concept of weighting strategy. Section 3 presents the optimization of vehicle cruise control by the appropriate choice of prediction weights. Section 4 develops the control scheme of velocity tracking according to the optimal prediction weights and performs the robust control design. Section 5 shows the operation of the controlled system on a transportational route with real data.

2. CONSIDERATION OF ROAD CONDITIONS

The principle of the consideration of the predicted road conditions is introduced in this section. The road ahead of the vehicle is divided into equidistant sections. The rates of the inclinations of the road and those of the speed limits are assumed to be known at the endpoints of each section. It is assumed that the vehicle travels in a segment from the initial point to the first division point. The velocity at the initial point is predefined and it is called original reference velocity. The dynamics of the vehicle is described between the initial and the first division points. The journey is carried out with constant longitudinal force in a segment. An important question is how velocity should be selected at the initial point (it is called modified reference velocity) by which the reference velocity of the first point can be reached using a constant longitudinal force between the first two division points. The thought can be extended to the next segments and division points. In case of n number of segments n equations are formalized between the first and the end points.

![Fig. 1. Simplified vehicle model](image)

The simplified model of the longitudinal dynamics of the vehicle is shown in Figure 1. The longitudinal movement of the vehicle is influenced by the traction force $F_l$ as the control signal and disturbances $F_d$ (such as rolling resistance $F_r$ and aerodynamic forces $F_{aer}$). The acceleration of the vehicle is $\ddot{s} = (F_l - F_d)/m$ with the mass of the vehicle $m$ and the acceleration of the vehicle $\ddot{s}$. The predicted course of the vehicle can be divided into equidistant sections using $n+1$ number of points as Figure 2 shows. The acceleration of the vehicle is considered to be constant between these points. Using simple kinematic equations the movement of the vehicle is:

$$s = \frac{1}{2}a(t_2 - t_1)^2 + \left(\frac{a(t_1 + t_2)}{2}\right)(t_2 - t_1) + s_0.$$

The velocity in the first section point should reach a predefined reference velocity $v_{ref,1}$. It might be a maximum authorized velocity due to the limitations. Therefore the next assumption must be fulfilled: $\ddot{s}_1^2 \rightarrow v_{ref,1}^2$.

![Fig. 2. Division of predicted road](image)

Similarly, the velocity of the vehicle can be formalized in the next $n$ section points. It is important to emphasize that the longitudinal force $F_{l1}$ affects only the first section, and it does not affect the next sections. The purpose of considering road conditions is to determine a control force by which the vehicle can travel along its way. Therefore at the calculation of the control force it is assumed that additional longitudinal forces will not act on the vehicle, i.e., the longitudinal force $F_{l2}$ will not affect the second section. The velocities of vehicle are described at each section point of the road by using similar expressions. The velocity of the $n^{th}$ section point is the following:

$$\ddot{s}_n^2 = \ddot{s}_0^2 + 2s \left(\sum_{i=1}^{n} F_{l,i} - \sum_{i=1}^{n} F_{d,i}\right) \rightarrow v_{ref,n}^2$$

(2)

It is also an important goal to track the momentary value of the velocity. It can also be considered in the following equation:

$$\ddot{s}_0^2 \rightarrow v_{ref,0}^2$$

(3)

The vehicle travels in traffic and it may happen that the vehicle is overtaken. Because of the risk of collision it is necessary to consider the velocity in front of us (leader vehicle):

$$\ddot{s}_0^2 \rightarrow v_{lead}^2$$

(4)

Since equations (3) and (4) might be in contradiction a trade-off must be created between them. The $F_{di}$ disturbance force can be divided in two parts: the first part is the force resistance from road slope $F_{di,r}$, while the second part $F_{di,o}$ contains all of the other resistances. We assume that $F_{di,o}$ is known while $F_{di,r}$ is unknown. $F_{di,r} = mg\sin \alpha_i$ depends on the mass of the vehicle and the angle of the slope $\alpha_i$. The consequence of this assumption is that the model does not contain all the information about the predicted road disturbances, therefore it is necessary to design a robust speed controller. This controller can ignore the undesirable effects. Consequently, the equations of the vehicle at the section points are calculated in the following way:

$$\ddot{s}_0^2 + 2s \frac{F_{l1}}{m} - 2s \frac{F_{di,o}}{m} \rightarrow v_{ref,j}^2 + \frac{2s}{m} \sum_{i=1}^{n} F_{di,r}$$

(5)

In the next step a weight $Q$ is applied to the momentary (initial) velocity (3), (4) and weights $\gamma_1, \gamma_2, ..., \gamma_n$ are applied to the predicted reference velocities (5). While the weights $\gamma_i$ represent the rate of the road conditions, weight $Q$ determines the tracking requirement of the momentary reference velocity $v_{ref,0}$. An additional weight $W$ is applied in (3). $W$ represents the tracking of velocity of the leader.
vehicle \(v_{\text{lead}}\) in order to avoid a collision. The weights should sum up to one, i.e., \(\gamma_1 + \gamma_2 + \ldots + \gamma_n + Q + \mathcal{W} = 1\). By increasing \(\mathcal{W}\) the predicted road conditions and the momentary reference velocity become less important. By summarizing the equations (3)…(5) and taking the prediction weights into consideration the following formula is yielded:

\[
\dot{\xi}_0^2 + \frac{2s}{m}(1 - Q - \mathcal{W})F_{11} - \frac{2s}{m}(1 - Q - \mathcal{W})F_{d1,o} = \vartheta \tag{6}
\]

where the value \(\vartheta\) depends on the predicted road slopes, the reference velocities and the prediction weights

\[
\vartheta = \mathcal{W}v_{\text{lead}}^2 + Qv_{\text{ref},0}^2 + \sum_{i=1}^{n}\gamma_iv_{\text{ref},i}^2 + \frac{2s}{m}\sum_{i=1}^{n}F_{dc,r}\sum_{j=1}^{n}\gamma_j. \tag{7}
\]

The interpretation of the control problem is the following. In order to take the predicted road conditions into consideration in the control design equation (6) is applied as a performance. Note that weights have an important role in control design. By making an appropriate selection of the weights \(Q\) and \(\gamma_i\) the importance of the predicted road condition is considered. For example when \(Q = 1\) and \(\mathcal{W} = \gamma_i = 0\), \(i \in [1, n]\) the control exercise is simplified to a cruise control problem without any predicted road conditions. When \(W = 1\) and \(Q = \gamma_i = 0\), \(i \in [1, n]\) only the tracking of leader vehicle is realized. The optimal determination of weights has an important role, i.e. to achieve a balance between the momentary velocity and the effect of the road slope, i.e., a balance between velocity and the economy parameters.

In addition to the consideration of predicted road conditions it is also important to consider the traffic environment. It means that the leader vehicle must be considered in the reference velocity design because of the risk of collision. Thus, all of the kinetic energy of the vehicle is dissipated by friction. This estimation of the safe stopping distance could be conservative in a normal traffic situation, where the leader vehicle also brakes, therefore the distance between the vehicles can be reduced.

In this paper the safe stopping distance between the vehicles is determined according to the 91/422/EEC, 71/320/EEC UN and EU directives (in case of M1 vehicle category, the velocity in \(\text{km/h})\): \(d_s = 0.1\dot{\xi}_0 + \xi_0^2/150\). It is also necessary to consider that without a leader vehicle the consideration of safe stopping distance is not possible and not necessary. The consideration of the leader vehicle is determined by \(\mathcal{W}\), therefore this weight is determined according to Figure 3.

**Fig. 3. Adhesion coefficient dependence**

In the final step a control-oriented vehicle model in which reference velocities and prediction weights are taken into consideration is constructed. The dynamical equation of the vehicle is the following: \(m\ddot{\xi}_0 + F_{d1,r} = F_{11} - F_{d1,o}\). Then equation (6) is rearranged in order to determine the modified reference velocity:

\[
\dot{\xi}_0 = \lambda \tag{8}
\]

where the parameter \(\lambda\) is calculated in the following way:

\[
\lambda = \sqrt{\vartheta - 2s(1 - Q)(\xi_0 + \xi_{\text{ref}})} \tag{9}
\]

Consequently, the predicted road conditions can be considered by velocity tracking. The momentary velocity of the vehicle \(\dot{\xi}_0\) should be equal to parameter \(\lambda\), which contains the predicted road information. The calculation of \(\lambda\) requires the measurement of the longitudinal acceleration \(\ddot{\xi}_0\). Note that in the control design it is assumed that the reference velocity is defined by using the safety conditions, such as adhesion conditions and vehicle parameters.

### 3. Optimization of the Cruise Control

Equation (6) shows that the modified reference velocity \(\dot{\xi}_0\) depends on prediction weights \((Q\) and \(\gamma_i)\). In this section the task is to find an optimal selection of the prediction weights in such a way that both the minimization of control force and the traveling time are taken into consideration.

The unmeasured disturbances such as rolling resistance and aerodynamic force can be expressed by the quadratic form of the velocity. The rolling resistance is modeled by an empiric form \(F_r = F_{r0}(1 + 5\kappa^2)\), where \(F_{r0}\) is the vertical load of the wheel, \(F_{r0}\) and \(\kappa\) are empirical parameters depending on tyre and road conditions and \(\xi\) is the velocity of the vehicle, see Pacejka (2004). The aerodynamic force is formulated as \(F_{\text{aux}} = 0.5C_w\rho A_0\xi_{\text{rel}}^2\), where \(C_w\) is the drag coefficient, \(\rho\) is the density of air, \(A_0\) is the reference area, \(\xi_{\text{rel}}\) is the velocity of vehicle relative to the air.

The sum of these forces is written in the following form:

\[
F_{d1,o} = F_r + F_{\text{aux}} = F_{r0}(1 + 5\kappa^2) + 0.5C_w\rho A_0\xi_{\text{rel}}^2 = A + T\xi_{\text{rel}}^2\tag{10}
\]

where \(A\) and \(T\) are assumed to have been calculated in advance. By substituting the expression in (6) the following equation is obtained:

\[
\xi_0^2 + \frac{2s(1 - Q - \mathcal{W})F_{11} - 2s(1 - Q - \mathcal{W})(A + T\xi_{\text{rel}}^2)}{m} = \vartheta \tag{9}
\]

From (9) the longitudinal force \((F_{11})\) can be expressed as the linear function of prediction weights:

\[
F_{11} = \beta_0(Q) + \beta_1(Q)\gamma_1 + \beta_2(Q)\gamma_2 + \ldots + \beta_n(Q)\gamma_n \tag{10}
\]

where \(\beta_i\) are the coefficients of \(\gamma_i\), and they depend on the prediction weight \(Q\). The vehicle cruise control problem can be divided into two optimization problems in the following forms:

- **Optimization 1**: The longitudinal control force must be minimized, i.e., \(|F_{11}| \rightarrow \text{Min}!\). In practice, however, the \(F_{11}^2 \rightarrow \text{Min}!\) optimization is used because of the simpler numerical computation. This performance is met by the transformation of the quadratic form with the following constrains:

\[
F_{11}^2(Q, \gamma_i) = (\beta_0(Q) + \beta_1(Q)\gamma_1 + \ldots + \beta_n(Q)\gamma_n)^2 \leq Q, \gamma_i \leq 1 \text{ and } Q + \sum \gamma_i = 1 - \mathcal{W} \tag{11}
\]
This task is a nonlinear optimization problem because of the $Q$ prediction weights. With fixed $Q$ prediction weight (11) becomes a quadratic optimization problem. Its solution is in Coleman and Li (1996); Gill et al. (1981).

- **Optimization 2: The difference between momentary reference velocity and modified reference velocity must be minimized, i.e., $|v_{\text{ref}},0 - \bar{\xi}_0| \rightarrow \text{Min!}$**
  The optimal solution can be achieved in a relatively easy way since the vehicle tracks the predefined velocity if the predicted road conditions are not considered. Consequently the prediction weights are $\bar{Q} = 1 - W$ and $\bar{\gamma}_i = 0, i \in [1,n]$.

The two optimization criteria lead to different optimal solutions. In the first criterion the predicted road inclinations and speed limits are taken into consideration by using appropriately chosen weights $\bar{Q}, \bar{\gamma}_i$. At the same time the second criterion is optimal if the predicted information is neglected. In the latter case the prediction weights are noted by $\bar{Q}, \bar{\gamma}_i$.

Finally, a balance between the two performances must be achieved, which is based on a tuning of the designed prediction weights. Several methods can be applied in this task. In the proposed method two further performance weights, $R_1$ and $R_2$, are introduced. Performance weight $R_1$ ($0 \leq R_1 \leq 1$) is related to the importance of the minimization of the longitudinal control force $F_{l1}$ (Optimization 1) while performance weight $R_2$ ($0 \leq R_2 \leq 1$) is related to the minimization of $|v_{\text{ref}},0 - \bar{\xi}_0|$ (Optimization 2). There is a constraint according to the performance weights $R_1 + R_2 = 1$. Thus the performance weights, which guarantee a balance between the optimizations tasks, are calculated in the following expressions:

\[
Q = R_1 \bar{Q} + R_2 \bar{Q} = R_1 \bar{Q} + R_2
\]

\[
\gamma_1 = R_1 \bar{\gamma}_1 + R_2 \bar{\gamma}_1 = R_1 \bar{\gamma}_1
\]

\[
\vdots
\]

\[
\gamma_n = R_1 \bar{\gamma}_n + R_2 \bar{\gamma}_n = R_1 \bar{\gamma}_n
\]

Based on the calculated performance weights the modified reference velocity can be determined by using (6).

The tracking of the leader vehicle is necessary to avoid a collision, therefore $W$ is not reduced only to save energy. If the leader accelerates, the following vehicle must accelerate as well. As a velocity increases so thus the braking distance, therefore the following vehicle strictly tracks the velocity of the leader. On the other hand it is necessary to prevent that the velocity of the vehicle from increasing above the official speed limit. Therefore the tracked velocity of leader vehicle is limited by the maximum velocity regulation. If the leader vehicle accelerates and exceeds the speed limit the following vehicle may fall behind.

### 4. LPV-BASED CONTROL DESIGN METHOD

Velocity tracking requires a controller, which generates the longitudinal force. Both the driveline and braking systems have delays in their operations. The delay is caused by different reasons such as the inertia of the driveline, the burning processes and injection, the turbo lag at driving, and the inertia of the hydraulic (or pneumatic) system in the braking system. The actuator dynamics is approximated by a first-order system: $\bar{F}_{l1} = (\bar{F}_{l1} - \bar{F}_{l1})/\tau$ where $\bar{F}_{l1}$ is the realized force, $\bar{F}_{l1}$ the desired force of the vehicle, and $\tau$ is the delay of the system. Moreover, the delay parameter differs at driving ($\tau_d$) and at braking ($\tau_b$).

The equations of the longitudinal dynamics and actuator dynamics are transformed into the following state-space representation form:

\[
\dot{x} = \begin{bmatrix} 0 & 1/m \\ 0 & -1/\tau \end{bmatrix} x + \begin{bmatrix} -1/m \\ 0 \end{bmatrix} F_{d1} + \begin{bmatrix} 0 \\ 1/\tau \end{bmatrix} \bar{F}_{l1}
\]

with $x = [\bar{x}_0, F_{l1}]^T$. In this description the operations of the driving and the braking are handled simultaneously. Although the delay parameters are different (in practice $\tau_d > \tau_b$) the model is able to separate the driving and braking cases depending on $\tau$. Selecting the scheduling variable $\tau$ the model can be transformed into an LPV model:

\[
\dot{x} = A(\rho)x + B_1 F_{d1} + B_2(\rho) \bar{F}_{l1}
\]

where $\rho = \tau$ is the scheduling variable: $\rho = \tau_d$ in case of driving or $\rho = \tau_b$ in case of braking. Based on this model the LPV control design is applied. The advantage of LPV methods is that the controller meets robust stability and nominal performance demands in the entire operational interval, since the controller is able to adapt to the current operational conditions.

Equation (8) leads to a tracking problem. The aim of tracking is to ensure that the system output follows a reference value with an acceptable small error, which is the performance of the system. The mathematical description of the optimization problem is as follows:

\[
(\bar{\xi}_0 - \lambda) \rightarrow \text{Min!}
\]

where the parameter $\lambda$ is the reference value. In the velocity tracking problem, $z = \bar{x}_0 - \lambda$ is the performance output.

The closed-loop interconnection structure, which includes the feedback structure of the model $P$ and controller $K$, is shown in Figure 4. The control design is based on a weight-
Adaptive Cruise Control (ACC) system is compared to the introduced method using two different performance weighting. Different performance weights for the ratio $R_1 : R_2$ are analyzed, i.e., 1 : 1 (Controller 1) and 10 : 1 (Controller 2). Figure 6 (c) shows the velocity of the vehicle along the trip. It can be established that in conventional ACC case the vehicle tracks the constant 70 km/h reference velocity accurately, while the velocity of vehicle changes according to road conditions. In the proposed method less longitudinal control force is enough compared to the conventional ACC system, see Figure 4 (d). Figure 6 (b) shows that the vehicle which considers the predicted road conditions also saves energy compared to the conventional ACC system. In order to qualify the controllers, the necessary absolute values of actuated control energies are added up in time ($\sum |F_l|$). The ACC controller which does not take into consideration the road conditions require 12% more control energy than Controller 1 and 23% more than Controller 2. The lower energy requirement of Controller 2 shows that the strategy seems to be more economical than in Controller 1 but saving energy causes lower velocity on the downhill slope, which is unfavourable in terms of cruising. For this reason a balance between saving energy and a decrease in velocity must be obtained since more braking force is used.

![Fig. 6. ACC systems on hilly road](image)

![Fig. 7. ACC systems with a compulsory speed limit](image)
velocity must be reduced to $60 \text{ km/h}$. Figure 7 shows the simulation results. If the controller is designed by using the road slope the reference velocity changes continuously as Figure 7(a) shows. The tendency of change depends on the proportion of performance weights, at $R_1 : R_2 = 10 : 1$ the change of velocity increases. At the same time, the velocity of vehicle in which the controller uses constant reference velocity (ACC) decreases rapidly. It can be stated that the knowledge of predicted road conditions control energy can be saved compared to ACC controller (see Figure 7 (b)). When $R_1 : R_2 = 1 : 1$ the saved energy is 19.7%, when $R_1 : R_2 = 10 : 1$ this value is 40%.

The third example analyses the incidence when another vehicle overtakes the vehicle with an adaptive cruise control proposed in the paper. In order to avoid a collision, the role of weight $W$ is important since the control must focus on the velocity instead of energy saving. In the first part of the simulation the leader vehicle is slower, however, in the second part its velocity is higher than that of the follower vehicle. Furthermore, in the example the leader also exceeds the official speed limit ($110 \text{ km/h}$). Figure 8(a) and 8(b) show that in the first part of the simulation the follower vehicle approaches the leader taking the braking distance into consideration, while in the second part the follower vehicle avoids exceeding the speed limit and it falls behind. This velocity control is achieved by using the value of $W$ as it is shown in Figure 8(d). In the first part of the simulation the weight is increased by the risk incident while in the second part it is reduced by the increasing distance. The solution requires radar information which is available in a conventional adaptive cruise control vehicle. This simulation example shows that the designed control system is able adapt to external circumstances.

Fig. 8. ACC systems with a leader vehicle

6. CONCLUSION

The paper has proposed the design of an adaptive cruise control system which is able to consider the inclinations of the road, keep compulsory speed limits, and it is able to adapt to the traffic environment. The controller is able to perform velocity tracking, tolerates sensor noise disturbances, minimizes the actuator forces, thus reduces fuel consumption. The control design is based on the robust LPV method, in which both performance specifications and model uncertainties are taken into consideration. Thanks to the integration of vehicle actuators and road conditions the simulation results show that the designed control reduces the energy required by the actuators.

REFERENCES

Bokor, J. and Balas, G. (2005). Linear parameter varying systems: A geometric theory and applications. 16th IFAC World Congress, Prague.


