Iterative design using IMC for input time-delay systems with disturbance observer

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Abstract: This paper discusses the iterative design method of closed-loop identification and internal model control (IMC) for input time-delay systems with a disturbance observer. The IMC is known as an effective control method for time-delay systems; however, a precisely identified model is required for achieving effective control performance. High closed-loop performance is achieved by the iteration of the IMC and the closed-loop identification. Moreover, the disturbance observer requires the same precise model as the IMC. In this paper, an iterative design method for the IMC combined with a disturbance observer for the time-delay system is proposed and confirmed by experimental results.

Keywords: Time delay, Iterative methods, Internal model control, Closed-loop identification, Disturbance observer

1. INTRODUCTION

The internal model control (IMC) is an effective design method for the process control system [Morari (1989)]. The IMC has a few free design parameters, and they are directly related to the performance, for example, to the response and the robustness of the closed-loop. However, the precise model is required to directly use the plant model for the controller design.

For the iterative design method of IMC for lumped parameter systems, the windsurfer approach is used [Lee (1995)]. The closed-loop identification and the IMC are alternately executed in the technique. The windsurfer approach uses the fractional representation approach for the closed-loop identification [Hansen (1980)]. When the IMC structure with the fractional representation approach is used for an input time-delay system, the re-identified model does not belong to the same class of input time-delay systems, and thus the model cannot be used to directly design the IMC controller. However, this problem has been solved using the ARX model with input time delay and the prediction error method [Abe (1999)]. The time delay was decided by searching for the value such that the sum of square of the prediction error was minimized in a specified time delay range. The effect was confirmed by temperature control experimentation [Abe (2003)].

The disturbance observer has been applied to several mechanical systems such as the servo system and the robot manipulator control [Ohnishi (1987)]. The main aim of the disturbance observer is to decrease external unknown or uncertain disturbances without the use of an additional sensor by using the inverse model. It was extended to the time-delay system on the basis of the Smith predictor and was applied to the pneumatic servo system [Noritsugu (1994)].

The precise model is important to both the IMC and the disturbance observer in order to directly use the model. Despite both methods treating the model directly, there are few studies that collectively treated the IMC and the disturbance observer. Gorez (1991) considered the filter of the disturbance observer and the IMC. Hashimoto (2007) used the IMC with the disturbance observer for the high-precision stage.

This paper proposes that an iterative design method using the IMC with the disturbance observer for the input time-delay system improves the closed-loop property and the disturbance suppression.

The model, which is provided by the conventional iterative design method, is adopted as the design of the disturbance observer. In other words, after the iterative design was completed, the obtained model implemented the disturbance observer. The purpose of the iterative design method is to obtain a better model to improve the closed-loop performance; it does not always identify the exact model. Therefore, the provided model using the conventional iterative design method will not be suitable for the disturbance observer.

In this paper, we propose an iterative design method, in which the plant combined with the disturbance observer is regarded as a new closed-loop identification object. The closed-loop performance is improved by repeating the closed-loop identification and the IMC for the system that included the disturbance observer. We apply the proposed method to a temperature control system with time delay and confirm the effect by experimentation.

The summaries of the IMC and the disturbance observer for time-delay systems are presented in sections 2 and 3, respectively. We propose the iterative design method combined with the disturbance observer in section 4.
Experimental results and discussion are shown in section 5.

2. INTERNAL MODEL CONTROL

One feature of the IMC is that the plant model is included in the feedback control loop. If the plant and the design model are completely equal, the output of the plant and the model are canceled and it becomes an open-loop control. Fig. 1 shows the block diagram of the IMC.

Fig. 1. Structure of IMC system

The closed-loop property is given by

\[ y = \frac{PCz^{-D/T}}{1 + C(Pz^{-D/T} - P_mz^{-D_m/T})} r \]  

(1)

where the plant \( P \), the model \( P_m \), and controller \( C \) are described in the discrete system. \( D, D_m \) and \( T \) denote time delay of the plant, time delay of the model, and the sampling time of the control, respectively. The block \( D \) and \( D_m \) in Fig. 1 express \( z^{-D/T} \) and \( z^{-D_m/T} \). Suppose that the plant \( P \) and model \( P_m \) here are the minimum phase systems.

The IMC controller \( C \) for the step reference input \( r \) is provided in (2), where, \( F \) is called the IMC filter. The parameter \( \lambda \) in the IMC filter is the only design parameter, and when \( \lambda \) is set smaller, then the bandwidth becomes larger.

\[ C = FP_m^{-1} \]  

(2)

\[ F = 1 - \alpha \]  

(3)

If the plant \( P = P_m \) and time delay \( D = D_m \), then the \( H_2 \) norm cost function (4) is minimized [Morari (1989)]. Here, \( K \) denotes a compensator expressed in the IMC structure as the unity feedback system.

\[ J_{imc}(Pz^{-D/T}, F) = \left\| \frac{Pz^{-D/T}K}{1 + Pz^{-D/T}K} - F \right\|_2 \]  

(4)

\[ K = \frac{C}{1 - Pz^{-D/T}C} \]  

(5)

3. DISTURBANCE OBSERVER

When the response of IMC is improved, the influence of the disturbance in the response increases. Therefore, it is necessary to consider the disturbance suppression. The disturbance observer is a technique that uses a model same as the IMC. When disturbance is added to a system, the difference between the system and model outputs is fed back to the input signal to compensate for the disturbance. The disturbance observer adds input to offset the output, which fluctuates due to disturbance.

Fig. 2 shows the structure of the disturbance observer for the time-delay system [Noritsugu (1994)], which was proposed based on the Smith compensator.

![Disturbance Observer](image)

For later analysis, Fig. 2 is transformed to the equivalent feedback structure form shown in Fig. 3. The low-pass filter in Fig. 2 is the same as the IMC filter \( F \) (3).

![Equivalent Disturbance Observer](image)

Since the disturbance is estimated after the time delay, it cannot be completely removed; however, the disturbance will be effective in the step disturbance for the small time delay. If time delay becomes long, the reverse response, which arises from the windup phenomenon of the time delay, increases after the disturbance is suppressed.

We consider an augmented plant with the disturbance observer as the loop transfer function and assume it to be equivalent to the plant \( P_{eq} \).

\[ P_{eq} = \frac{(1 - D_mF)^{-1}Pz^{-D/T}}{1 + Pz^{-D/T}(1 - D_mF)^{-1}FP_m^{-1}} \]  

(6)

Note that \( P_{eq} \) has a time-delay element in the numerator and denominator; however, the \( P_{eq} \) is equal to the real plant \( Pz^{-D/T} \) when \( P = P_m \) and \( D = D_m \).

The total system structure combined with the disturbance observer in the IMC is shown in Fig. 4.

4. ITERATIVE DESIGN METHOD

4.1 Cost function

In the iterative design method, the model, which is used in the IMC and the disturbance observer, is updated by
We consider \( P_{eq} \) the controlled plant for the IMC, the cost function of the identification \( J_{id} \) is evaluated to be the same as the IMC-only case [Abe (1999)].

\[
J_{id}(P_{eq}, K) := \left\| \frac{P_{eq}K}{1 + P_{eq}K} - \frac{P_m z^{-D_m/T}K}{1 + P_m z^{-D_m/T}K} \right\|_2 \quad (7)
\]

We consider a variation of the norm of the system with the disturbance observer as an additive perturbation \( \Delta \).

\[
\Delta := \left\| \frac{P_{eq}K}{1 + P_{eq}K} - \frac{P_{eq}K}{1 + P_{eq}K} - \frac{P_m z^{-D_m/T}K}{1 + P_m z^{-D_m/T}K} \right\|_2 \quad (8)
\]

Then (7) and \( J_{imc} \) (4) are rewritten using the following, respectively:

\[
J_{id}(P_{eq}, K) = \left\| \frac{P_m z^{-D_m/T}K}{1 + P_m z^{-D_m/T}K} - F + \Delta \right\|_2 \quad (9)
\]

\[
J_{imc}(P_{eq}, F) = \left\| \frac{P_{eq}K}{1 + P_{eq}K} - F \right\|_2 \quad (10)
\]

\[
J_{imc}(P_{eq}, F) \leq J_{id}(P_{eq}, K) + \Delta \quad (10)
\]

\[
J_{imc}(P_{eq}, F) \leq J_{imc}(P_{eq}, F) + \Delta \quad (11)
\]

From (10), (11) and (12),

\[
J_{imc}(P, F) + \Delta \leq J_{id}(P, K) + J_{imc}(P_m, F) + \Delta
\]

\[
J_{imc}(P, F) \leq J_{id}(P, K) + J_{imc}(P_m, F) \quad (13)
\]

The first term on the left side of (13) is minimized by the closed-loop identification, and the second term becomes small by the IMC controller. It is expected to make \( J_{imc} \) smaller indirectly by iterating control and identification.

4.2 Closed-loop identification

We consider the equivalent plant \( P_{eq} \) as the following ARX model with time delay:

\[
y_m = \frac{B(z)}{A(z)} z^{-D_m/T} u + H(z) w \quad (14)
\]

where \( A(z) \) and \( B(z) \) are polynomial. \( H := \frac{1}{\tilde{A}} \) and \( w \) denote noise model and white noise, respectively. We define the one step prediction error \( \varepsilon \) and prefilter \( L' \):

\[
\varepsilon := y - y_m \quad (15)
\]

\[
L' = \frac{H_m}{1 + \frac{B}{\tilde{A}} z^{-D_m/T} K} \quad (16)
\]

Then, the square of the error that is multiplied by the pre-filter is described as

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \{L'(\varepsilon(k))\}^2 = J_{id}^2(P_{eq}, K). \quad (17)
\]

Therefore, it is easy to apply the basic prediction error method by using the pre-filtered input-output data of the closed-loop [Abe (1999)], if the time delay \( D_m \) is known.

4.3 Time-delay estimation

It is necessary that the identified model be chosen as the input time-delay system to minimize the cost function (7). When the time delay is unknown, one of the simplest methods for deciding the time delay is a coefficient comparison method for the high-order identification model [Ljung (1987)]. However, it is not suitable to minimize the prediction error.

First, we fix the order of the model \( P_m = B/A \), and set a range for the expected time delay in which the cost function is minimized. There are a number of candidates for the model time delay in the range depending on the identification sampling time. We then estimate all the
models of each time delay in the range and derive all loss function, which denote a sum of the square of the prediction error with the cross validation. Finally, we select the model time delay $D_m$, the model $P_m$, and the noise model $H$ by the smallest value of the loss function.

Fig. 6 shows the loss function of the third iteration, which is combined with the disturbance observer as an example. The time delay that minimizes the loss function is 70 [s], and thus we estimate the time delay at 70 [s] in this example. Note that the decided time delay is not the exact value but the loss function is minimized at the time delay.

Note that the convergence of the iterative design method has not been proved.

5. EXPERIMENTAL RESULTS

The experiment equipment is a cooling device with a frigistor module in the thermostatic chamber. The control objective is to cool an aluminum piece to the target temperature. The target temperature is -5°C from the initial chamber temperature, which is described at 0 °C in graphs. However, there is a small time delay, and thus we insert an artificial time delay of 30 [s] using software in order to make the effectiveness clear. The inserted time delay is not so longer than the system time constant.

We show a step response result in Fig. 7.

Based on the above discussion, the procedure of the iterative design method is as follows.

**Step 1 (open-loop identification):** Estimate the model using open-loop identification.

**Step 2 (IMC compensator design and control):** Set the time constant $\lambda$ of the IMC filter (3) and derive the IMC compensator (2). If the closed loop of the IMC with the disturbance observer achieves desirable performance, then the iterative step ends.

**Step 3 (closed-loop identification):** Identification signal (Pseudo Random Binary Sequence: PRBS) is input from $r$. Input $u$ and output $y$ of the closed loop (Fig. 5) is filtered by the pre-filter $L'$ (16). The new model $P_{eq}$, which includes $P_m$ and the model time delay $D_m$, is identified by the prediction error method and the time-delay estimation method. Then return to Step 2.

The first-order continuous model is obtained from the step response. The discrete model is derived by the bilinear transformation from the continuous model with a sampling time of control $T = 0.05$[s].

$$G_m(s) = \frac{-27.96}{275.75s + 1} e^{-30s}$$

(18)

$$G_m(z) = -\frac{0.002535z - 0.002535}{z - 0.9998}z^{-30/T}$$

(19)

We chose the model as a proper second-order discrete system with the identification sampling time of 1 [s]. We were able to choose a high-order model, but a provided model may have unstable zeros. When unstable zero exists, an all-pass term is added, however, the response property deteriorates generally. Thus, the high-order model is not suitable for the design of the IMC.

5.1 Iterative experimentation

The design parameter $\lambda$ of the IMC filter was set smaller with 250, 210, 170, and 130 for each iteration. It was decided that control performance was provided adequately in $\lambda = 130$ [s], which is about half of the time constant. It was also the control objective that the closed-loop had a suppressive performance against the step disturbance, increasing 3°C in the thermostatic chamber. The step disturbance was added from 2,000 [s], at which the output reached the steady state.

We show the experimental results of the IMC-only case in Figs. 8-11, and the results with the disturbance observer in Figs. 12-15, respectively.
Fig. 8. Output with pre-identified model

Fig. 9. Output with iteration i = 1

Fig. 10. Output with iteration i = 2

Fig. 11. Output with iteration i = 3

Fig. 12. Output with pre-identified model

Fig. 13. Output with iteration i = 1

Fig. 14. Output with iteration i = 2

Fig. 15. Output with iteration i = 3
In these graphs, the simulation lines are not based on a theoretical model; rather they are based on a simulation of the identified model. We show the identified time delay for each iteration and the value of the design parameter \( \lambda \) in Table 1.

**Table 1. Identified time delay and \( \lambda \)**

<table>
<thead>
<tr>
<th>( D_m [s] )</th>
<th>( D_m [s] )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only IMC</td>
<td>with DO</td>
<td></td>
</tr>
<tr>
<td>( i=0 )</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>( i=1 )</td>
<td>42</td>
<td>35</td>
</tr>
<tr>
<td>( i=2 )</td>
<td>56</td>
<td>61</td>
</tr>
<tr>
<td>( i=3 )</td>
<td>53</td>
<td>70</td>
</tr>
</tbody>
</table>

In the IMC-only case and the IMC with the disturbance observer case, the response performance improved. Since we decreased the parameter \( \lambda \) for each iteration, the convergence of the disturbance occurred slightly early. This shows that the proposed iterative design method for the system with the disturbance observer was applicable.

Comparing the disturbance suppressive performance, the case combined with the disturbance observer is considerably suppressed. When there was no disturbance observer, temperature increased approximately 1°C, but using the disturbance observer, it was able to suppress until 0.4°C.

The convergence is good; however, there is some overshoot that is not observed in the IMC-only case. The overshoot was reduced even when we applied the large low-pass filter of the disturbance observer. This is a windup phenomenon of the time delay. The identification of the time delay obtained was longer than the true value at each iteration (Table 1). It was caused by the fact that we identified the system with the disturbance observer; however, also in the IMC-only case, the time delay was longer.

In the experimentation of the mechanical system, a similar result was found for the identification of the time delay using the predictive error method [Iwasaki (2009)]. One reason for this may be that the noise model in the ARX model does not have adequate order. Therefore, the error influenced the time-delay value. In the proposed method, the disturbance observer is included in the identified model, and thus more errors occurred, and time delay was longer than that in the IMC-only case. A future study will examine this in detail.

Even in the case of longer time delay, this method is applicable; however, the time-delay windup increases.

When we used the model for the disturbance observer, which was derived by the iterative design method in the IMC-only case, a large overshoot occurred in response (Fig.16). This result shows that it is necessary to consider the disturbance observer in the closed-loop identification procedure when the disturbance observer is used in the iterative design method.

### 6. CONCLUSION

In this study, we examined the iterative design method by using the IMC with a disturbance observer for a time-delay system. The disturbance was suppressed and it maintained good performance of the IMC in the experimental results.

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**REFERENCES**