On the Uniform Positive Definiteness of the Estimated Inertia for Robot Manipulators

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Abstract: This paper is dedicated to seeking a projection algorithm that can guarantee the uniform positive definiteness of the manipulator estimated inertia during parameter adaptation. To achieve this objective, we first investigate the convexity property of the parameter space which corresponds to the inertia parameters of a single rigid body, extending the existing result to a more general case. Then we propose a new projection algorithm to avoid the escape of estimated parameters from the pre-specified parameter region, while this cannot be ensured by the conventional projection approach. The second part of this paper is on how to guarantee the uniform positive definiteness of the estimated manipulator inertia matrix. We show that if we can guarantee the uniform positive definiteness of each link’s estimated inertia, the manipulator estimated inertia can be considered to be the inertia of a physically existing robot manipulator, and hence must be uniformly positive definite. Based on this idea and the proposed new projection algorithm, we can obtain a uniformly positive definite estimated robot inertia. Under the frame work of the recursive passivity-based controller, we illustrate the performance of the proposed approach using a six-DOF manipulator.

Keywords: Parameter projection; uniform positive definiteness; estimated manipulator inertia.

1. INTRODUCTION

Adaptive control of robot manipulators has been extensively studied in the past over two decades and numerous adaptive control strategies have been proposed, see Berghuis and Nijmeijer (1993); Brogliato, Landau, and Lozano (1991); Craig, Hsu, and Sastry (1987); Dawson and Lewis (1991); Peng and Palaniwami (1993); Lozano-Leal and Canudas (1990); Middleton and Goodwin (1988); Slotine and Li (1987, 1988, 1989); Spong and Ortega (1990), and the survey (Ortega and Spong, 1989). These approaches can be generally classified in two categories: passivity based control (Slotine and Li, 1987, 1988, 1989; Brogliato, Landau, and Lozano, 1991; Lozano-Leal and Canudas, 1990) and inverse dynamics control (Craig, Hsu, and Sastry, 1987; Dawson and Lewis, 1991; Peng and Palaniwami, 1993; Middleton and Goodwin, 1988; Spong and Ortega, 1990). One general assumption that inverse dynamics adaptive control approach requires is the uniform invertibility of estimated inertia. While it is shown that passivity based control has some attractive advantages over inverse dynamics approach, e.g., without requiring the existence of inversion of estimated inertia, the assumption that the inversion of estimated inertia should exist is a prerequisite in passivity based adaptive observer design (Berghuis and Nijmeijer, 1993; Srouspour and Salcudean, 2001; Liu, Cheah, and Slotine, 2006). More recently, it is found that applying passivity based adaptive control to free-floating space manipulators also needs the existence of inversion of the estimated spacecraft inertia (Wang and Xie, 2009).

During the early research of adaptive inverse dynamics control, some researchers attempted to avoid using inversion of the estimated inertia (Dawson and Lewis, 1991; Spong and Ortega, 1990), but the final results given in Dawson and Lewis (1991) demonstrate that this kind of modification technique to the original adaptive inverse dynamics control has to pay a relatively large price. Another idea to tackle this problem is proposed in Li and Slotine (1989). Their basic idea is that if we can guarantee that the estimated object grasped by the manipulator be a physical object (e.g., both of its mass and moment of inertia about its center of mass are greater than or equal to zero), and further based on an assumption that the inertia of the manipulator itself is accurately known and thus must be positive definite, then we will obtain a positive definite estimated inertia. It is also shown that the parameter region which guarantees positive semi-definiteness of the mass and moment of inertia about the center of mass is convex (Li and Slotine, 1989), which allows the application of the classical projection algorithms (Ioannou and Sun, 1997).

Physical features of the manipulator inertia matrix such as its uniform positive definiteness, its lower or upper bound and its relation with the inertias of the manipulator links have been extensively studied in Slotine and Li (1991); Ghorbel, Srinivasan, and Spong (1993, 1998); Featherstone (2004); Gunawardana and Ghorbel (1999); Featherstone (2008). However, none of these has answered the question...
that what kind of properties may hold for the estimated manipulator inertia matrix in adaptive case.

Inspired by the work of Li and Slotine (1989), this paper investigates the more general case that all robot inertia parameters are unknown. In this case, we should seek an estimation algorithm to guarantee the positive definiteness (rather than just positive semi-definiteness) of the estimated inertia. Furthermore, we need to investigate an approach that is able to ensure the positive definiteness of the whole robot estimated inertia (different from the case of estimating the object only (Li and Slotine, 1989)). Another problem that needs to be resolved is how to avoid the escape of the estimated parameters from the pre-specified domain. In conventional projection algorithms, as presented in Ioannou and Sun (1997), non-escape of the estimated parameters is ensured by the approximation techniques, but however they may incur instability of the closed-loop system. Here, we propose a new approach that can both guarantee the non-escape of the estimated parameters and retain stability of the closed-loop system. As an illustrative example, we show the effectiveness of the proposed approach under the framework of recursive passivity based adaptive controller (Wang, 2010; Huo, Gao, and Cheng, 1994; Niemann and Slotine, 1991).

2. CONVEXITY OF THE PARAMETER SPACE

A prerequisite for parameter projection algorithm is the convexity of the parameter space (Ioannou and Sun, 1997). In particular, for the purpose of guaranteeing the positive definiteness of the estimated inertia, we should first show that the parameter region which corresponds to a series of positive definite inertias is convex. And of course, these inertias should contain the actual inertia of the manipulator.

We start by showing that for a single rigid body, the inertia parameter space that forms positive definite inertias is convex, the main idea is similar as Li and Slotine (1989). A rigid body’s inertia property can be described by ten inertia elements, which include six inertia elements, three elements related to the location of the center of mass (\(mc_x, mc_y, mc_z\)), and the mass \(m\). Given a body-fixed frame, denote the inertia of the rigid body about this frame by \(H\), and then the inertia of the rigid body about its center of mass can be expressed as (following the standard formulation in Theoretical Mechanics),

\[
H_C = H - m \left( e^T c I - c e^T \right)
\]

where \(e = \begin{bmatrix} c_x & c_y & c_z \end{bmatrix}^T\) is the position vector from the origin of the body-fixed frame to the center of mass.

Now consider the following matrix,

\[
\begin{align*}
\bar{H} &= mH_C = mH - (p^T p I - pp^T) \\
\end{align*}
\]

(2)

It can be clearly seen that if we can guarantee the positive definiteness of the matrix \(\bar{H}\) and the mass \(m\), then the inertia \(H_C\) must be positive definite. Next, we define a region \(\Omega\) satisfying,

\[
\begin{align*}
 & \{ mH - [p^T p I - pp^T] \geq mD \} \\
 & m \geq \beta
\end{align*}
\]

where \(p = mc\), \(D = D^T > 0\) is a positive definite matrix specifying the lower bound of the inertia \(H_C\), and \(\beta > 0\) specifies the lower bound of the mass, respectively.

Now we present the following theorem, which can be thought to be an extension of the work of Li and Slotine (1989).

**Theorem 1.** The parameter space \(\Omega\) is convex with respect to the inertia parameter vector \(a\) defined by

\[
 a = \{ H_{xx}, H_{xy}, H_{xz}, H_{yy}, H_{yz}, H_{zz}, mc_x, mc_y, mc_z, m \}
\]

**Proof.** First let us define a generalized point \(\theta\) in the parameter space,

\[
\theta' \sim \left( \frac{m_1 + m_2}{2}, \frac{p_1 + p_2}{2}, \frac{H_1 + H_2}{2} \right)
\]

Since \(\theta_i (i = 1, 2)\) belongs to the space \(\Omega\), we have,

\[
m_i \geq \beta
\]

(4)

\[
m_i H_i - (p_i^T p_i I - p_i p_i^T) \geq m_i D
\]

(5)

Equation (4) quickly leads to the following result,

\[
\frac{m_1 + m_2}{2} \geq \beta
\]

(6)

Now consider the following matrix \(L\),

\[
L = \frac{m_1 + m_2}{2} \left( \begin{array}{c} H_1 + H_2 \\
\end{array} \right)
\]

\[
- \frac{1}{4} \left[ (p_1 + p_2)^T (p_1 + p_2) I - (p_1 + p_2) (p_1 + p_2)^T \right]
\]

(7)

The matrix \(L\) can be further expressed as,

\[
L = \frac{1}{4} (L_1 + L_2)
\]

(8)

where \(L_1\) and \(L_2\) are defined as,

\[
L_1 = \left[ m_1 H_1 - (p_1^T p_1 I - p_1 p_1^T) \right] + \left[ m_2 H_2 - (p_2^T p_2 I - p_2 p_2^T) \right]
\]

(9)

\[
L_2 = m_1 H_2 + m_2 H_1 - (2p_1^T p_2 I - p_1 p_2^T - p_2 p_1^T)
\]

(10)

From equation (5), we obtain,

\[
L_1 \geq (m_1 + m_2) D
\]

(11)

Using \(m_1 m_2\) to multiply both sides of equation (10) yields,

\[
m_1 m_2 L_2 = m_1^2 m_2 H_2 + m_2^2 m_1 H_1 - 2(m_2 p_2)^T (m_1 p_1) I + (m_2 p_1) (m_2 p_1)^T + (m_1 p_2) (m_1 p_2)^T
\]

(12)

Substituting (5) into the above equation gives,

\[
m_1 m_2 L_2 \geq (m_1^2 m_2 + m_2^2 m_1) D + (m_1 p_2)^T (m_1 p_2) I - (m_1 p_2) (m_2 p_2)^T + (m_2 p_1) (m_2 p_1)^T - (m_2 p_1) (m_2 p_1)^T + (m_1 p_2) (m_1 p_2)^T
\]

(13)
Further formulation of (13) results in the following,
\[ m_1 m_2 L^2 \geq \left( m_2^1 m_2 + m_2^2 m_1 \right) D + (m_1 p_2 - m_2 p_1)^T (m_1 p_2 - m_2 p_1) T \]
\[ \geq \left( m_2^2 m_2 + m_2^1 m_1 \right) D \]
\[ = m_1 m_2 (m_1 + m_2) D \]  
(14)
This implies that \( L^2 \geq (m_1 + m_2) D \). Therefore, the matrix \( L \) satisfies,
\[ L \geq \frac{m_1 + m_2}{2} D \]  
(15)

Equations (6) and (15) lead to the conclusion that the parameter space \( \Omega \) is convex with respect to the generalized point \( \theta \), and of course also convex with respect to the parameter vector \( a \).

Remark 1. Compared with the results of Li and Slotine (1989), the proposed theorem seems more general, which is reflected in two aspects. First, the parameter region \( \Omega \) is more general, e.g., the lower bound of \( H_C \) is a positive definite matrix rather than a zero matrix as presented in Li and Slotine (1989). In addition, from a physical viewpoint, the proposed lower bound of \( H_C \) is much more physically motivated, which can guarantee the positive definiteness of \( H_C \). Also, designating a lower bound in matrix form allows much flexibility in dealing with different rigid bodies in different dimensions and mass properties. For example, the inertia of a very long but thin rigid body about its center of mass should be very unbalanced, e.g., the inertia elements in different directions differ greatly.

3. ESTIMATING THE INERTIA PARAMETERS USING A NEW PROJECTION ALGORITHM

3.1 The Standard Projection Algorithm
Let \( N \) be the parameter updating direction of the estimated parameter \( \hat{a} \) in the case that no projection is applied, i.e.,
\[ \dot{\hat{a}} = N \]  
(16)
Our objective is to restrict the estimated parameter \( \hat{a}(t) \) always belonging to the convex space \( \Omega \), which is defined as \( \Omega = \{ a | f(\hat{a}) \leq 0 \} \). In standard projection techniques (Ioannou and Sun, 1997), if the current estimated parameter \( \hat{a}_k \) (at the time instant \( t = t_k \)) is on the boundary of the region \( \Omega \) and if the angle between the direction of \( N \) and that of \( \nabla h (\nabla h^T \nabla h)^{-1} \nabla h^T N \) is less than or equal to \( \pi / 2 \), the parameter updating direction \( N \) will be projected to the surface which is normal to \( \nabla h \), e.g.,
\[ \dot{\hat{a}} (t_k) = N - \nabla h (\nabla h^T \nabla h)^{-1} \nabla h^T N \]  
(17)
Implementation of the above projection parameter estimation algorithm using a digital computer is of the following form (with Euler integration),
\[ \hat{a}_{k+1} = \hat{a}_k + \dot{\hat{a}} (t_k) \Delta t \]  
(18)
where \( \Delta t \) is the sampling period. In most cases, it is expected that \( \hat{a}_{k+1} \) will escape from the region \( \Omega \) as shown in Fig. 1, which is very undesirable, and in addition this

![Fig. 1. Projection of the estimated parameter](image)

problem is inherent and thus it is impossible to solve this issue by simply resorting to more complex and accurate numerical integration algorithms.

3.2 A New Projection Algorithm
To overcome the deficiencies of the conventional projection algorithm, we will attempt to present a new projection approach.

It is well-known that an effective projection algorithm should satisfy two requirements. First, the algorithm should not destroy the stability of estimation algorithm. Second, it should keep the estimated parameter belonging to the desired parameter region as best as it can.

The projection approach we propose is of the following form,
\[ \dot{\hat{a}} (t_k) = prj (N) - \lambda^* \nabla h \]  
(19)
or its equivalent discrete-time form,
\[ \hat{a}_{k+1} = \hat{a}_k + (prj (N) - \lambda^* \nabla h) \Delta t \]  
(20)
where \( prj (N) = N - \nabla h (\nabla h^T \nabla h)^{-1} \nabla h^T N \) is the projection of \( N \), the positive number \( \lambda^* \) is the minimum solution of the following equation with respect to the to-be-determined variable \( \lambda \),
\[ f (\hat{a}_k + prj (N) \Delta t - \lambda \nabla h \Delta t) = 0 \]  
(21)
With (19) updating the estimated parameter, \( \hat{a}_{k+1} \) will be ensured to be in the region \( \Omega \), as illustrated in Fig. 1. In addition, the stability of the estimation algorithm can be guaranteed. In fact,
\[ \Delta a^T \dot{\hat{a}} = \Delta a^T N - \Delta a^T \nabla h (\nabla h^T \nabla h)^{-1} \nabla h^T N - \lambda^* \Delta a^T \nabla h \]  
(22)
Since \( \Delta a^T \nabla h \geq 0 \), then the additional term \(- \lambda^* \Delta a^T \nabla h \) simply makes \( \Delta a^T \dot{\hat{a}} \) more negative and thus maintains the estimator’s stability.

Remark 2. One projection scheme to guarantee the estimated parameters in the region \( \Omega \) is using the approximation technique (Ioannou and Sun, 1997), as shown in Fig. 1. The estimated parameter is finally determined as the intersection point between the line from \( \hat{a}_{k+1} \) to \( a_0 \) and the boundary of \( \Omega \), where \( a_0 \) is some known point in the region \( \Omega \). However, this will not ensure the estimator’s stability and may cause the estimator unstable in some cases.
Fig. 2. A case that cannot be tackled

Remark 3. A case that cannot be dealt with by the proposed approach happens when the parameter updating is too fast. In that case, for all λ, \( f(\dot{\hat{a}}_k + prj(\hat{N}) \Delta t - \lambda \nabla h_\lambda \Delta t) > 0 \), and therefore it is impossible to project the estimated parameter into the region \( \Omega \). This situation is shown in Fig. 2. A sub-optimal choice is to project the estimated parameter to a point on the line directed from \( \hat{a}_{k+1} \) along \(-\nabla h_\lambda \) which is nearest to the region \( \Omega \). An alternative approach to avoid this problem is to slow down the parameter estimation.

4. GUARANTEEING THE UNIFORM POSITIVE DEFINITENESS OF THE ESTIMATED INERTIA

4.1 The basic idea

The basic idea is that if we can guarantee the uniform positive definiteness of each link’s estimated inertia by the previously presented parameter projection techniques, we will obtain a uniformly positive definite estimated inertia for the multi-link robot manipulator.

The above fact can be further interpreted as follows: Since the estimated inertia (including the estimated mass) of each link is ensured to be uniformly positive definite, we can find a physical object in the real world that has exactly the same inertia properties as that estimated inertia. And hence, we are able to find \( n \) physical objects, each of which corresponds to one of the manipulator links’ estimated inertias. Now, we naturally arrive at the conclusion that the estimated manipulator inertia matrix is exactly identical to the inertia matrix of a manipulator which is composed of those \( n \) physical objects. This implies that the estimated inertia must be uniformly positive definite since it is also the inertia matrix of a physically-existing manipulator.

4.2 Implementation Scheme

From the above discussion, we know that what we need to do is to guarantee the uniform positive definiteness of the estimated inertia of each manipulator’s link. This goal can be achieved by restricting the estimated inertia parameter always in the region \( \Omega \) defined by equation (3).

In this case specific involving estimated parameters, based on equation (3), we can define an estimated parameter region \( \Omega \) satisfying the following,

\[
\left\{ \begin{array}{l}
\det_i \left( \hat{m} \hat{H} - \hat{m} \hat{D} - \hat{p}^T \hat{p} I + \hat{p} \hat{p}^T \right) \geq 0, i = 1, 2, 3 \\
\hat{m} \geq \beta 
\end{array} \right.
\]  

(23)

where \( \det_i (\cdot) \) is the determinant of the \( i \)-th order primary sub-matrix of \( \cdot \). Equation (23) can be rewritten in a more compact form,

\[
f_i \leq 0, i = 1, 2, 3, 4
\]

(24)

where \( f_i = -\det_i \left( \hat{m} \hat{H} - \hat{m} \hat{D} - \hat{p}^T \hat{p} I + \hat{p} \hat{p}^T \right), i = 1, 2, 3, f_4 = -\hat{m} + \beta. \]

Now the gradient vector at the parameter \( \dot{a} \) which is on the boundary defined by \( f_i = 0 \) is written as,

\[
\nabla h_i = \frac{\partial f_i}{\partial \hat{a}}, i = 1, 2, 3, 4
\]

(25)

Then, the projection estimation algorithm is as follows,

\[
\dot{\hat{a}}(t_k) = \begin{cases} 
prj_i (\hat{N}) - \lambda_i^* \nabla h_i & \text{if } \hat{a}_k \in S \left( \hat{\Omega} \right), f_i = 0 \\
\text{and } N^T \nabla h_i & \text{otherwise}
\end{cases}
\]

(26)

where \( prj_i (\hat{N}) = N - \nabla h_i (\nabla h_i^T \nabla h_i)^{-1} \nabla h_i^T N, S \left( \hat{\Omega} \right) \) is the boundary of \( \hat{\Omega} \), and \( \lambda_i^* \) is the minimal solution of the following equation with respect to \( \lambda_i \),

\[
f(\dot{a}_k + prj_i (\hat{N}) \Delta t - \lambda_i \nabla h_i \Delta t) = 0
\]

(27)

Scaled version of the projection estimation algorithm (26) can be written as,

\[
\dot{\hat{a}}(t_k) = \begin{cases} 
prj_i (\Gamma N) - \lambda_i^* \nabla h_i & \text{if } \hat{a}_k \in S \left( \hat{\Omega} \right), f_i = 0 \\
\text{and } N^T \nabla h_i & \text{otherwise}
\end{cases}
\]

(28)

where \( prj_i (\Gamma N) = \Gamma N - \nabla h_i (\nabla h_i^T \nabla h_i)^{-1} \nabla h_i^T \Gamma N, \Gamma \) is a positive definite estimation gain matrix, and in this case, \( \lambda_i^* \) should be the minimal solution of the equation with respect to \( \lambda_i \),

\[
f(\dot{a}_k + prj_i (\Gamma N) \Delta t - \lambda_i \nabla h_i \Delta t) = 0
\]

(29)

5. A CASE STUDY

In this section, we examine the performance of the proposed projection algorithm via the well-known passivity based adaptive control algorithm. A six-DOF manipulator (Wang, 2010) is employed as a simulation example, as shown in Fig. 3, where the link frames has been illustrated. The gravitational acceleration is assumed to be \( g_{acc} = 9.8 \cdot m \cdot s^{-2} \). The physical parameters of the six-DOF manipulator are shown in Table 1.

For a six-DOF complex manipulator, spatial-vector based recursive adaptive algorithm (Wang, 2010; Huo, Gao, and Cheng, 1994; Niemeyer and Slotine, 1991) is much more favorable, please refer Wang (2010) for detail. With the proposed projection approach, adaptation law of \( i \)-th manipulator’s link inertia parameters should be as follows,

\[
\dot{\hat{a}}_i(t_k) = \begin{cases} 
prj_i (-\gamma_i \hat{e}_i^T \hat{e}_i) - \gamma_i \lambda_i^* \nabla h_i & \text{if } f_i (\dot{X}_i(t_k)) = 0 \\
(\gamma_i \hat{e}_i^T \hat{e}_i)^T \nabla h_i & \text{otherwise}
\end{cases}
\]

(26)

\[
l = 1, 2, 3, 4
\]

The desired joint trajectory of the manipulator is chosen as \( q_{di} = 30^\circ (1 - \cos \pi t) \). The manipulator is initially at
Table 1. Physical parameters of the six-DOF manipulator

<table>
<thead>
<tr>
<th>i-th body</th>
<th>$m_i$ (kg)</th>
<th>$I_{c,i}$ (kg · m²)</th>
<th>$r_{i,c,i}/c_i$ (m)</th>
<th>$r_{i,i+1}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10</td>
<td>0.5f</td>
<td>0.5, 0, 0.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.4f</td>
<td>0.5, 0, 0.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.2f</td>
<td>0, −0.2, 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.2f</td>
<td>0, −0.2, 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.2f</td>
<td>0, 0, 2</td>
<td></td>
</tr>
<tr>
<td>6 &amp; tool</td>
<td>diag(0.7, 0.8, 0.9)</td>
<td>0.5, 0.5, 0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the framework of recursive passivity based adaptation, a six-DOF manipulator was employed as a simulation example.

The lower bounds for the estimated mass and moment of inertia with respect to the center of mass are all determined to be $\beta = 0.5$, $D = 0.5I$.

In standard projection case, simulation results of the adaptive controller are shown in Fig. 4 and Fig. 5. The positive definiteness of the estimated inertia cannot be guaranteed (See Fig. 5), which is mainly caused by the escape of the estimated parameters from the pre-specified parameter space $\Omega$.

The performance of the proposed new projection algorithm is shown in Fig. 6 and Fig. 7, where the solution of the equation (31), e.g. $\lambda^*_i$, is obtained via Newton’s tangent approach, which is a recursive numerical method,

$$\lambda_{n+1} = \lambda_n - \frac{f_1(\lambda_n)}{f_1'(\lambda_n)}, i = 1, 2, 3, 4$$

where $\lambda_0 = 0, f'_1(\lambda) = \frac{df_1(\lambda)}{d\lambda}$. We use $\lambda_1$ as the final $\lambda^*_i$ in simulation. From Fig. 7, we see that the new projection algorithm achieve the goal of guaranteeing the positive definiteness of the estimated inertia. Furthermore, the proposed new projection gives much smoother time history of the minimum eigenvalue of the estimated manipulator inertia.

In this paper, we have examined the problem of ensuring the uniform positive definiteness of the estimated manipulator inertia during parameter adaptation. We first studied the convex property of the parameter space that corresponds to a single rigid body’s positive definite inertia, where we extend the existing result to the more general case. Then, a new projection algorithm has been proposed to restrict the estimated parameters into the pre-designated region such that the estimated rigid body’s inertia is uniformly positive definite. The proposed projection approach avoids the escape of the estimated parameters from pre-designated parameter space and meanwhile maintains the estimator’s stability. The second part of this paper is concerned about how to ensure the uniform positive definiteness of the estimated robot inertia matrix. We show that if each link’s estimated inertia is guaranteed to be uniformly positive definite, then the manipulator’s estimated inertia must be positive definite. The idea behind is that if each link’s estimated inertia is positive definite, the manipulator estimated inertia can be considered to be the inertia matrix of a physically existing robot manipulator, and hence must be uniformly positive definite. Under the framework of recursive passivity based adaptation, a six-DOF manipulator was employed as a simulation example.

Fig. 3. A six-DOF manipulator with unknown load

Fig. 4. Joint position tracking errors (Standard projection)

Fig. 5. Minimum eigenvalue of the estimated inertia (Standard projection)

6. CONCLUSION
to show the performance of the proposed projection approach. It is shown that the proposed projection algorithm can give a uniformly positive definite estimated inertia.

REFERENCES


