Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Differential Game Missile Guidance with Impact Angle and Time Constraints ⋆

Seonhyeok Kang* H. Jin Kim**

* *School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea, (e-mail: hkkang@snu.ac.kr)*
** *School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea, (e-mail: hjinkim@snu.ac.kr)*

Abstract: Modern anti-ship missiles are required for enhancement of the survivability against anti-air defense systems of warships. In this paper, the linear quadratic differential game missile guidance law to control impact angle and time is proposed, which enables to take advantages of vulnerability of warships. The closed-form solution based on linear engagement and quadratic cost has the form of combination of the optimal guidance to control impact angle and an additional command to control impact time. Nonlinear simulation demonstrates the satisfactory homing performance in terms of impact angle and time error as well as feasibility of the proposed guidance law for a salvo attack.

Keywords: Anti-ship missile, survivability, linear quadratic differential game, impact angle and time control

1. INTRODUCTION

Recent development of anti-air defense systems of warships, such as close-in weapon systems (CIWS) which can detect and destroy incoming missiles at short range, pose a great challenge in enhancing the survivability of anti-ship missiles. The weaving or barrel-roll maneuvers that can reduce the possibility of attackers being hit by the projectiles of CIWS guns have been applied to real missile-target engagement. Obviously such maneuvers are significantly limited especially in sea-skimming missiles. Given the particular threat of anti-air missiles, a salvo attack which takes advantages of vulnerability of CIWS would be a promising candidate for enhancing the survivability of anti-ship missiles. A salvo attack means that multiple missiles are required to attack the same target simultaneously, which can introduce many-to-one engagements for missile defense systems. Typical warships have two or four CIWS systems to cover certain defense zone limited in range and azimuth. Thus, to maximize the effectiveness of a salvo attack, all missiles need to home to the target at the specified impact time as well as with the required impact angle.

A number of solutions of the missile homing problems with terminal impact angle and time constraints have been mainly obtained by using optimal control guidance and modified proportional navigation guidance (PNG). A simple rendezvous of an interceptor and a stationary target is solved, where quadratic penalties on terminal miss and velocity components perpendicular to the specified rendezvous course are introduced (Bryson and Ho (1975)). It can be used to impose terminal angle constraints by choosing the rendezvous course as the predetermined collision course. An optimal control law for the minimization of intercept angle error and miss distance is proposed for the vertical plane for reentry vehicle (Kim and Grider (1973)). The generalized energy optimal guidance law is also proposed for constant speed missiles with an arbitrary system order while achieving the desired impact angle as well as zero miss distance (Ryoo et al. (2005)). It is extended by introducing the time-to-go weighted energy cost function and gain sets of higher values which can improve the robustness to external disturbances or uncertainties are also obtained (Ryoo et al. (2006)). A biased PN law using time-varying component for an impact with the specified missile attitude angle against a moving target is proposed (Kim et al. (1998)). As an alternative way, the backstepping control method is used to design a integrated guidance and control algorithm with the terminal angle constraint. (Shin et al. (2008))

Although a number of techniques considering the impact angle constraint have been proposed, most of the impact time controller proposed in the literature thus far have been focused on finding specific feedback command to reduce impact time error, rather than being developed from control frameworks. This is because it is quite hard to mathematically represent the time-to-go in some sort of performance index. The suboptimal missile guidance law that can be represented by the combination of a feedback loop of the impact time error with the traditional optimal guidance is proposed to control impact time (Jeon et al. (2006)). The guidance law to control both impact time and impact angle is also proposed in a similar way, which comprises optimal guidance law to meet the impact angle requirement with zero effort miss and an additional command to control impact time (Lee et al. (2007)).

Differential game theory can be well suited to the engagement between a missile and a target as a conflict of two

⋆ The authors are grateful to Dr. Jin-Ik Lee for his comments and suggestions that helped to improve the manuscript.

objects. The guidance law to find the optimal strategies of the dual controls for the end-game phase of the interception is derived using a differential game (Shima and Golan (2007)). Linear quadratic differential game guidance is presented to control the predetermined intercept angle (Shaferman and Shima (2008)). Theoretic conditions for the existence of a saddle-point solution are also derived, which show that imposing the terminal angle constraints requires a higher maneuverability advantage from the missile. The cooperative linear quadratic differential game guidance law is also developed, in which three agents are involved in a two team problem between an evading aircraft protected by a defending missile and an attacking missile trying to intercept the aircraft and to avoid the defending missile (Perelman et al. (2010)).

In this paper, the differential game missile guidance law with terminal angle and time constraints is proposed by using the linearized missile-target engagement and the quadratic cost function. Instead of finding the solutions for both constraints in one optimization problem, the guidance law for impact angle control is derived first in the optimal strategy and extended for both impact angle and time control by employing an time-to-go estimation. The rest of this paper is organized as follows: Section 2 describes the formulation of the homing problem. The optimal solutions to control impact angle is discussed in section 3. Section 4 describes the way to determine the additional command to control desired impact time. The results of the numerical simulations are given in section 5. The conclusions and future research directions are presented thereafter.

2. PROBLEM FORMULATION

The relative dynamics between the missile and target is shown in Fig. 1 where X-Y is a Cartesian inertial reference frame. The flight path angles and positions of both missile and target are denoted as γ and $(X,\ Y)$. And A is acceleration magnitude perpendicular to the velocity vector V. The subscript M and T denote the missile and target, respectively.

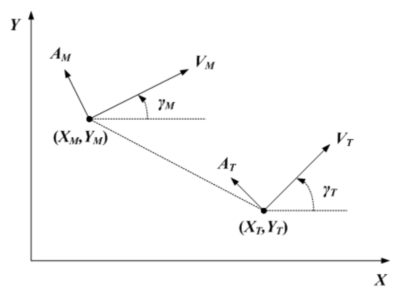


Fig. 1. Two-dimensional missile-target engagement geometry

The equations of motion can be written as

$$dX_i/dt = V_i \cos\gamma_i \quad (1a)$$
$$dY_i/dt = V_i \sin\gamma_i \quad (1b)$$
$$d\gamma_i/dt = A_i/V_i \quad (1c)$$

where t denotes time and $i = M,\ T$. The initial conditions are given by

$$X_i(t_0) = X_{i_0},\ Y_i(t_0) = Y_{i_0}$$
$$\gamma_i(t_0) = \gamma_{i_0},\ A_i(t_0) = A_{i_0}$$

For convenience, all the state variables are nondimensionalized as

$$x_i = X_i/L_i$$
$$y_i = Y_i/L_i$$
$$a_i = A_i/(V_i/t_f)$$
$$\tau = t/t_f$$

where t_f is the final time and $L_i = V_i t_f$ is the length of flight. The initial conditions are expressed as

$$y_i(x_0) = y_{i_0},\ \gamma_i(x_0) = \gamma_{i_0},\ a_i(x_0) = a_{i_0}$$

In order to control the final time, the rate of acceleration that provides the additional degree of freedom can be introduced as

$$dA_M(t)/dt = J(t) \quad (4)$$

where $J(t) = j(t) + j_0$. While $j(t)$ is to control terminal angle constraints, which is a function of time, j_0 is an arbitrary constant to be determined to meet the impact time constraints. The acceleration rate commands are also nondimensionalized as

$$\eta = j/(V_M/t_f^2)$$
$$\eta_0 = j_0/(V_M/t_f^2)$$

Under the assumption that γ_i are small and V_i are constant, the corresponding equations of motion can be written as

$$\xi' = \mathrm{E}\xi + \mathrm{F}\Xi + \mathrm{G}a_T \quad (6)$$

where the prime $'$ denotes the derivative with respect to x, $\Xi = \eta + \eta_0$ and $\xi = [\ z\ \gamma_M\ \gamma_T\ a_M\ ]^T$. z is the relative distance between the missile and target in Y-direction and E, F, and G are given by

$$\mathrm{E} = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \mathrm{F} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},\ \mathrm{G} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

3. IMPACT ANGLE CONTROL GUIDANCE

As mentioned in section 2, the control scheme of this study is to find the optimal solution η for the terminal angle constraints and then specific additional command η_0 is added to control the impact time. In this section, the optimal strategy of both missile and target to satisfy the terminal angle constraints for the missile is derived.

The quadratic cost function for the finite horizon game can be expressed as

$$J = \frac{1}{2b} z_f^2 + \frac{1}{2c}(\gamma_{M_f} - \gamma_{M_c})^2 + \int_{x_0}^{x_f} \left(\eta^2(s) - \mu a_T^2(s)\right) ds \quad (7)$$

where the subscript f denotes the state variable at homing and γ_{M_c} is the desired terminal angle of the missile. The weights b and c are nonnegative weighting on the terminal miss and angle, respectively, and μ represents the design weighting on the maneuverability of the target relative to that of the missile. If the agile target is considered, smaller values of μ should be chosen, which allows to obtain larger gains, and vice versa.

In order to solve this optimization, the Hamiltonian can be written as

$$H = \frac{1}{2}\eta^2 - \frac{\mu}{2}a_T^2 + \lambda_z(\gamma_T - \gamma_M) + \lambda_{\gamma_M} a_M + \lambda_{\gamma_T} a_T + \lambda_{a_M}(\eta + \eta_0) \tag{8}$$

The adjoint equations are given by

$$\lambda_z' = 0,\ \lambda_{\gamma_M}' = \lambda_z,\ \lambda_{\gamma_T}' = -\lambda_z,\ \lambda_{a_M}' = -\lambda_{\gamma_M} \tag{9}$$

where

$$\lambda_{z_f} = \frac{1}{b}z_f,\ \lambda_{\gamma_{M_f}} = \frac{1}{c}(\gamma_{M_f} - \gamma_{M_c}),\ \lambda_{\gamma_{T_f}} = 0,\ \lambda_{a_{M_f}} = 0 \tag{10}$$

Integrating (9) yields the solution

$$\lambda_z = \frac{1}{b}z_f \tag{11a}$$

$$\lambda_{\gamma_M} = -\frac{1}{b}z_f x_{\rm go} + \frac{1}{c}(\gamma_{M_f} - \gamma_{M_c}) \tag{11b}$$

$$\lambda_{\gamma_T} = \frac{1}{b}z_f x_{\rm go} \tag{11c}$$

$$\lambda_{a_M} = -\frac{1}{2b}z_f x_{\rm go}^2 + \frac{1}{c}(\gamma_{M_f} - \gamma_{M_c})x_{\rm go} \tag{11d}$$

where $x_{\rm go} = x_f - x$.

From the first-order necessary condition, the optimal solution of the missile and the target are given by

$$\eta = \frac{1}{2b}z_f x_{\rm go}^2 - \frac{1}{c}(\gamma_{M_f} - \gamma_{M_c})x_{\rm go} \tag{12a}$$

$$a_T = \frac{1}{\mu b}z_f x_{\rm go} \tag{12b}$$

In order to implement (12), the optimal command needs to be expressed in terms of the current state variables instead of the final state variables. Substituting (12) with η_0 into (7) and integrating yield the explicit expressions as

$$\begin{aligned} a_M(x) &= a_{M_f} + \int_{x_f}^{x} (\eta(s) + \eta_0)\, ds \\ &= -\frac{1}{6b}z_f x_{\rm go}^3 + \frac{1}{2c}(\gamma_{M_f} - \gamma_{M_c})x_{\rm go}^2 + a_{M_f} - \eta_0 x_{\rm go} \end{aligned} \tag{13a}$$

$$\begin{aligned} \gamma_T(x) &= \gamma_{T_f} + \int_{x_f}^{x} a_T(s) ds \\ &= -\frac{1}{2\mu b}z_f x_{\rm go}^2 + \gamma_{T_f} \end{aligned} \tag{13b}$$

$$\begin{aligned} \gamma_M(x) &= \gamma_{M_f} + \int_{x_f}^{x} a_M(s) ds \\ &= \frac{1}{24b}z_f x_{\rm go}^4 - \frac{1}{6c}(\gamma_{M_f} - \gamma_{M_c})x_{\rm go}^3 - a_{M_f}x_{\rm go} \\ &\quad + \gamma_{M_f} + \frac{1}{2}\eta_0 x_{\rm go}^2 \end{aligned} \tag{13c}$$

$$\begin{aligned} z(x) &= z_f + \int_{x_f}^{x} (\gamma_M(s) - \gamma_T(s))\, ds \\ &= \frac{1}{120b}z_f x_{\rm go}^5 - \frac{1}{24c}(\gamma_{M_f} - \gamma_{M_c})x_{\rm go}^4 + \frac{1}{6\mu b}z_f x_{\rm go}^3 \\ &\quad - \frac{1}{2}a_{M_f}x_{\rm go}^2 + \gamma_{M_f}x_{\rm go} - \gamma_{T_f}x_{\rm go} + z_f - \frac{1}{6}\eta_0 x_{\rm go}^3 \end{aligned} \tag{13d}$$

Solving the above four algebraic equations for the unknown terminal values and substituting the solution into (12), the optimal control with the terminal angle constraint is obtained as

$$\eta = K^T\zeta + C\eta_0 \tag{14}$$

where

$$K = k^{-1}\begin{bmatrix} 120\mu x_{\rm go}^5 + 1440\mu c x_{\rm go}^2 \\ -36\mu x_{\rm go}^6 - 960x_{\rm go}^4 + 2880\mu b \\ -24\mu x_{\rm go}^7 + 960x_{\rm go}^5 - 720\mu c x_{\rm go}^4 - 2880\mu b x_{\rm go}^2 \end{bmatrix} \tag{15a}$$

$$C = k^{-1}\left[-2\mu x_{\rm go}^8 + 480x_{\rm go}^6 - 240\mu c x_{\rm go}^5 - 1440\mu b x_{\rm go}^3 \right] \tag{15b}$$

$$\begin{aligned} k &= 3\mu x_{\rm go}^8 - 320x_{\rm go}^6 + 144\mu c x_{\rm go}^5 + 960\mu b x_{\rm go}^3 \\ &\quad - 960c x_{\rm go}^3 + 2880\mu bc \end{aligned} \tag{15c}$$

and $\zeta = [\ (z - \gamma_M x_{\rm go} + \gamma_T x_{\rm go})\ \gamma_{\rm go}\ a_M\ ]^T$ and $\gamma_{\rm go} = \gamma_{M_c} - \gamma_M$.

If the terminal angle constraint is not imposed, that is $c \to \infty$, the gains of the optimal controller are degenerated to

$$K(c \to \infty) = \frac{1}{k(c \to \infty)}\begin{bmatrix} 30\mu x_{\rm go}^2 \\ 0 \\ -15\mu x_{\rm go}^4 \end{bmatrix} \tag{16a}$$

$$C(c \to \infty) = \frac{1}{k(c \to \infty)}\left[-5\mu x_{\rm go}^5 \right] \tag{16b}$$

$$k(c \to \infty) = 3\mu x_{\rm go}^5 - 20x_{\rm go}^3 + 60\mu b \tag{16c}$$

It should be noted that the gain for $\gamma_{\rm go}$ is zero, since the constraint of a terminal angle is not considered.

For zero effort miss and angle, it is required that $b, c \to 0$, yielding the following controller gains:

$$K(b,\ c \to 0) = \frac{1}{k(b,\ c \to 0)}\begin{bmatrix} 120\mu x_{\rm go}^5 \\ -36\mu x_{\rm go}^6 - 960x_{\rm go}^4 \\ -24\mu x_{\rm go}^7 + 960x_{\rm go}^5 \end{bmatrix} \tag{17a}$$

$$C(b,\ c \to 0) = \frac{1}{k(b,\ c \to 0)}\left[-2\mu x_{\rm go}^8 + 480x_{\rm go}^6 \right] \tag{17b}$$

$$k(b,\ c \to 0) = 3\mu x_{\rm go}^8 - 320x_{\rm go}^6 \tag{17c}$$

By assuming a non-maneuvering target with $\gamma_T = 0$, the game solution degenerates to the one-sided optimal control solution (Lee et al. (2007))

$$\eta = \frac{40u_{\rm PN}}{3x_{\rm go}} - \frac{12\gamma_{\rm go}}{x_{\rm go}^2} - \frac{8a_M}{x_{\rm go}} - \frac{2}{3}\eta_0 \tag{18}$$

where $u_{\rm PN} = 3(z - \gamma_M x_{\rm go})/x_{\rm go}^2$ which is an approximation of the proportional navigation command with navigation constant of 3 for $|x_{\rm go}| \gg |z|$.

4. IMPACT TIME CONTROL GUIDANCE

This section presents a way to determine the additional command η_0 to control impact time. Since the solutions will be trivial when the interception between the missile and target is not obtained, the perfect intercept and perfect intercept angle are assumed, that is $b = c = 0$. If the target is stationary with $\gamma_T = 0$, the impact time can be expressed as a path constraint in terms of the flight path angles of the missile, which is estimated by the closed-form solution (13c). Thus, the estimation of time-to-go both with and without η_0 allows to derive the equation of impact time error.

After Taylor series expansion without the higher order terms, the estimation of the time-to-go can be written as (Jeon et al. (2006))

$$\begin{aligned}\tau_{\text{go}} &= \int_x^{x_f} \sqrt{1+\gamma_M(s,\eta_0=0)}ds \\ &\approx \int_x^{x_f} \left(1+\frac{1}{2}\gamma_M^2\right) ds \\ &= x_{\text{go}} + \frac{1}{10368b^2} z_f^2 x_{\text{go}}^9 + \frac{1}{504c^2}\left(\gamma_{M_f}-\gamma_{M_c}\right)^2 x_{\text{go}}^7 \\ &+\frac{1}{6}a_{M_f}^2 x_{\text{go}}^3 + \frac{1}{2}\gamma_{M_f}^2 x_{\text{go}} + \frac{1}{1152} z_f \gamma_{M_f} x_{\text{go}}^8 \\ &-\frac{1}{144} z_f a_{M_f} x_{\text{go}}^6 + \frac{1}{120} z_f \gamma_{M_f} x_{\text{go}}^5 \\ &+\frac{1}{30c}\left(\gamma_{M_f}-\gamma_{M_c}\right) a_{M_f} x_{\text{go}}^5 \\ &-\frac{1}{24c}\left(\gamma_{M_f}-\gamma_{M_c}\right)\gamma_{M_f} x_{\text{go}}^4 - \frac{1}{2} a_{M_f}\gamma_{M_f} x_{\text{go}}^2 \end{aligned} \tag{19}$$

The unknown terminal values in (19) can also be determined by solving four algebraic equations (13).

However, the time-to-go with the additional command η_0, which is the second-order function with respect to that, can be given by

$$\begin{aligned}\bar{\tau}_{\text{go}} &= \int_x^{x_f} \sqrt{1+\gamma_M(s,\eta_0)}ds \\ &= (\alpha/\delta)\eta_0^2 + (\beta/\delta)\eta_0 + \tau_{\text{go}} \end{aligned} \tag{20}$$

where

$$\begin{aligned}\alpha &= \mu^2 x_{\text{go}}^9 - 180\mu x_{\text{go}}^7 + 345600 x_{\text{go}}^5 \\ \beta &= \left(15a_M x_{\text{go}}^8 - 117 x_{\text{go}}^7\gamma_{\text{go}} + 15 x_{\text{go}}^6(z-\gamma_M x_{\text{go}})\right)\mu^2 \\ &\quad - \left(7200 a_M x_{\text{go}}^6 + 77760\gamma_M x_{\text{go}}^5 - 2160\gamma_{M_c} x_{\text{go}}^5 \right. \\ &\quad \left. -302400 z x_{\text{go}}^4 - 2160\right)\mu \\ &\quad - \left(3456000 a_M x_{\text{go}}^4 + 13132800\gamma_M x_{\text{go}}^3 \right. \\ &\quad \left. +11059200\gamma_{M_c} x_{\text{go}}^3\right) \\ \delta &= 11340(3\mu x_{\text{go}}^2 - 320)^2 \end{aligned}$$

The solutions of (20) are given by

$$\eta_0 = -\frac{\beta}{2\alpha} \pm \sqrt{\left(\frac{\beta}{2\alpha}\right)^2 + \frac{\delta}{\alpha}\varepsilon_\tau} \tag{21}$$

where $\varepsilon_\tau = \bar{\tau}_{\text{go}} - \tau_{\text{go}}$ is an impact time error in terms of nondimensionalized value. Since $\eta_0 = 0$ when $\varepsilon_\tau = 0$, the additional command can be rewritten as

$$\eta_0 = -\frac{1}{2}\eta_L\left(1-\sqrt{1+\eta_E/\eta_L^2}\right) \tag{22}$$

where $\eta_L = \beta/\alpha$ and $\eta_E = 4(\delta/\alpha)\varepsilon_\tau$. If a nonmaneuvering target is considered, that is $\mu \to \infty$, the gains are also rewritten as (Lee et al. (2007))

$$\eta_L = \frac{5}{x_{\text{go}}} u_{\text{PN}} - \frac{117}{x_{\text{go}}^2}\gamma_{\text{go}} + \frac{15}{x_{\text{go}}} a_M \tag{23}$$

$$\eta_E = \frac{408240}{x_{\text{go}}^5}\varepsilon_\tau \tag{24}$$

It is noted that the existence of a positive impact time error implies that the additional command η_0 should always be added to the solution for the terminal angle control. Negative impact time error should not be allowed because the desired impact time is chosen to be larger than the expected time-to-go in most cases. It means that the impact time error is set to be zero if it is smaller than a certain small value.

5. SIMULATION RESULTS

The proposed guidance law is applied to a two-dimensional nonlinear engagement scenario described in section 2. The constant velocity of the missile is set to be 200 m/s starting from the origin with the initial flight path angle of 30 deg. The initial acceleration command $A(0)$ is zero and the location of the stationary target is 5 km apart from the missile along the X-axis. The desired impact angle is 0 deg.

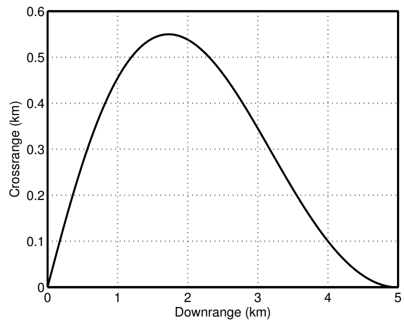
Figure 2 shows the time histories for the impact angle control guidance. As mentioned before, the weights b, c, and μ should be carefully chosen to reflect a tradeoff between miss distance, impact angle error and allowable maneuverbility. The values of $b = c = 10^{-3}$ and $\mu = 10^5$ are used for this simulation. In figure 2(a), satisfactory homing performance is shown. Also, as shown in 2(b), the flight path angle of the missile is approached to the desired terminal angle zero. The acceleration command which is obtained from the integration of the jerk command is shown in figure 2(c). In this case, it is observed that the impact time is about 25.84 sec. The error of impact time estimation according to (19) is about 0.006 sec at homing, which is appropriate to implement this estimation.

Figure 3 shows the simulation results with both terminal angle and time constraints for various desired impact time. The scenario is the same as the impact angle control guidance while the values of weights are set to be $b = c = 0$ and $\mu = 10^5$. It is obvious that the missile generates the longer trajectories if the larger impact time is imposed as shown in figure 3(a). As can be seen in figures 3(b) and 3(d), respectively, both terminal angle and time constraints are satisfied. The errors of impact time are within 0.05 sec for all cases. The acceleration commands are also shown in figure 3(c) where the abrupt change about 10 sec is caused by the change of sign of the impact time error.

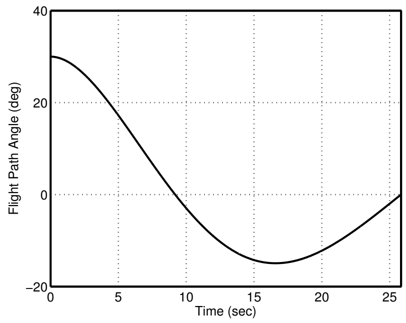
The proposed guidance law is applied to a salvo attack scenario under the assumption that three missiles attack a single stationary target with built-in CIWS. Each missile has the same velocity of 200 m/s and the desired impact time is set to be 30 sec. The scenario for a salvo attack scenario is summarized in table 1. Figure 4 illustrates the salvo attack trajectories of three missiles. Each missile that uses only impact angle control guidance reaches the target at 26.36, 26.59 and 25.80 sec, respectively, whose trajectories are represented by dash-dot line. On the other hand, as shown by solid line, the final angle and time constraints of three missiles are all achieved by using impact angle and time control guidance. The final impact time errors are all within 0.03 sec.

Table 1. Scenario for a salvo attack

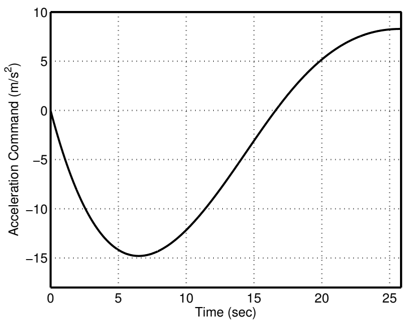
	$(X_0,\ Y_0)$(km)	$(\gamma_{M_0},\ \gamma_{M_c})$(deg)
Target	$(0,0)$	–
Missile1	$(-5,0.5)$	$(30,0)$
Missile2	$(-4,-2.5)$	$(90,-30)$
Missile3	$(-2,-4.5)$	$(100,-60)$



(a) Trajectory



(b) Flight path angle

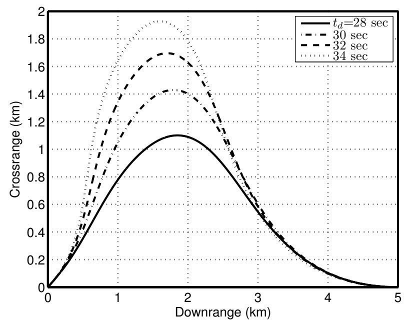


(c) Acceleration command

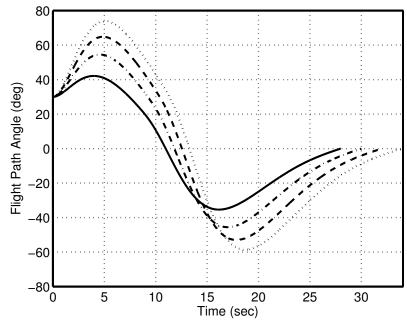
Fig. 2. Time histories by impact angle control guidance with the values of $b = c = 10^{-3}$ and $\mu = 10^5$.

6. CONCLUSION

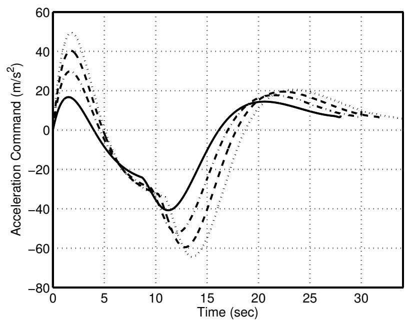
A useful suboptimal missile guidance law which incorporates impact angle and time constraints is presented. A proposed guidance law has the form of combination of the impact angle control guidance loop and the feedback loop of impact time error. The guidance law based on optimal control techniques is evaluated in nonlinear simulations of various engagements. The simulation results show that the proposed law performs well in terms of accurate achievement of impact angle and time requirements. The capability of this guidance law demonstrated by the salvo attack could greatly improve the survivability and the warhead lethality of anti-ship missiles. The analytic conditions for the existence of a saddle point solution in the differential game are issues that require further study to understand well characteristics of the proposed guidance law.



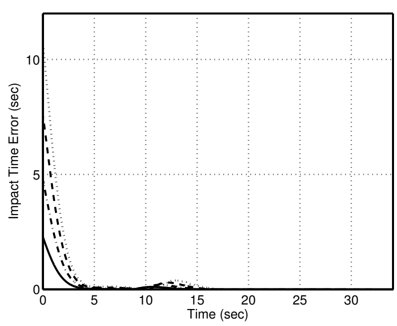
(a) Trajectory



(b) Flight path angle



(c) Acceleration command



(d) Impact time error

Fig. 3. Time histories by impact time and angle control guidance with the values of $b = c = 0$ and $\mu = 10^5$ in which the desired impact time t_d are set to be 28, 30, 32, and 34 sec, respectively.

REFERENCES

Bryson, E.A. and Ho, C.Y. (1975). *Applied Optimal Control*. Hemisphere, New York.

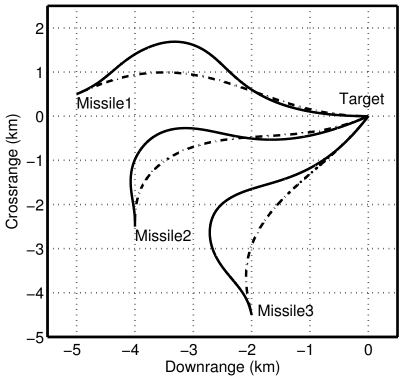


Fig. 4. Salvo attack: impact angle control guidance (dash-dot), impact angle and time control guidance (solid)

Jeon, I.S., Lee, J.I., and Tahk, M.J. (2006). Impact-time-control guidance law for anti-ship missiles. *IEEE Transactions on Control Systems Technology*, 14(2), 260–266.

Kim, B.S., Lee, J.G., and Han, H.S. (1998). Biased png law for impact with angular constraint. *IEEE Transactions on Aerospace and Electronic Systems*, 34(1), 277–288.

Kim, M. and Grider, K.V. (1973). Terminal guidance for impact attitude angle constrained flight trajectories. *IEEE Transactions on Aerospace and Electronic Systems*, AES-9(6), 852–859.

Lee, J.I., Jeon, I.S., and Tahk, M.J. (2007). Guidance law to control impact time and angle. *IEEE Transactions on Aerospace and Electronic Systems*, 43(1), 301–309.

Perelman, A., Shima, T., and Rusnak, I. (2010). Cooperative differential games strategies for active aircraft protection from a homing missile. *AIAA Guidance, Navigation, and Control Conference.*

Ryoo, C.K., Cho, H.J., and Tahk, M.J. (2005). Optimal guidance laws with terminal impact angle constraint. *Journal of Guidance, Control, and Dynamics*, 28(4), 724–732.

Ryoo, C.K., Cho, H.J., and Tahk, M.J. (2006). Time-to-go weighted optimal guidance with impact angle constraints. *IEEE Transactions on Control Systems Technology*, 14(3), 483–492.

Shaferman, V. and Shima, T. (2008). Linear quadratic guidance laws for imposing a terminal intercept angle. *Journal of Guidance, Control, and Dynamics*, 31(5), 1400–1412.

Shima, T. and Golan, O.M. (2007). Linear quadratic differential games guidance law for dual controlled missiles. *IEEE Transactions on Aerospace and Electronic Systems*, 43(3), 834–842.

Shin, H., Hwang, T., Tsourdos, A., Whit, B., and Tahk, M. (2008). Integrated intercept missile guidance and control with terminal angle constraint. *International Congress of The Aeronautical Sciences.*