A Two-Dimensional Spatial-based Wiener Printer Model Embedded Into Model-based Digital Halftoning

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Abstract: This paper presents a two-dimensional Wiener model in spatial domain for a class of laser electrophotographic printers, and embeds it into a framework of model-based digital halftoning. The Wiener model comprises an FIR filter followed by a nonlinear static mapping. The nonlinear static mapping is synthesized using a fuzzy system to maximize its capability to approximate nonlinearity of high degree. A set of systematic steps for parametric optimization and evaluation of the Wiener printer model is proposed. Due to the parameters being determined off-line, the Wiener model demands much less computation and memory when executing on-line. Another advantage is that the structure complexity of the Wiener model is adjustable via altering the order of the FIR filter and the number of membership functions in the fuzzy system. Experimental results as well as a comparative study show that the proposed model-based halftoning approach is superior (in terms of image quality) to conventional ones and comparable to the one with a comprehensive printer model.

Keywords: Electrophotography; Fuzzy system; Halftoning; Neural network; Printer; Two-dimensional system; Wiener model.

1. INTRODUCTION

Digital halftoning refers to the process of transforming a continuous-tone image into one with a finite number of intensities. Using a pattern of pixels with a limited number of intensities as the building block, the transformed image (so-called halftone image) is perceived as a continuous-tone image when seen by the human eye from a distance. The success of halftoning is a result of the eye being a spatial low-pass filter that blurs the rendered pixel pattern. Digital halftoning is often used to manifest continuous-tone images in media where direct rendition of the tones is impossible. The most common example of such media is ink or toner on paper, and the most common rendering devices for such media are printers.

Based on the type of computation involved, halftoning algorithms can be classified into three categories (Allebach, 1999): point algorithms (e.g., screening or dithering), neighborhood algorithms (e.g., error diffusion), and iterative algorithms (e.g., least squares and direct binary search). Point algorithms conduct a pixel-by-pixel comparison with a spatially varying threshold to obtain the binary value of the halftone image. In neighborhood algorithms, in addition to pixel-by-pixel comparison with a threshold to determine the binary value of each pixel in the halftone image, an error is also calculated and diffused ahead to a set of pixels in the continuous-tone image. Iterative algorithms determine the binary value of each pixel by processing the image in multiple passes and searching for the best halftone, which minimize a metric of perceived error. Although iterative algorithms yield images with superior quality to those generated by neighborhood and point processes, the quality is usually achieved at the sacrifice of algorithmic complexity.

For digital printing, traditional halftoning algorithms assume that printers print square dots and there is no interaction between neighboring dots. Therefore, the absorbance (the ratio of the total absorbed light to the incident light) of a printed image should be proportional to the number of dots printed in the image. However, real printers do not obey such assumption and induce phenomena well known as dot overlap and dot gain. A recent approach to address such issues is to embed the models of printers and human visual system (HVS) in existing halftoning algorithms. Halftoning techniques incorporating the models of printers and HVS are often categorized as model-based halftoning algorithms (Pappas and Neufiff, 1995, 1999; Nishida, 1999; Kim and Allebach, 2002; Kacker et al., 2002; Baqai and Allebach, 2003). The most widely used printer model is the circular dot-overlap model (Roetling and Holladay, 1979). The model is insensitive to small variation in dot gain so that it is applicable to a wide range of printers. Several researchers have integrated such model into their works (Pappas and Neuhoff, 1995, 1999; Neuhoff, 1997; Mehta and Allebach, 2003). Another class of printer models which is based on direct (macroscopic or microscopic) measurement of the absorbance of specifically designed test patterns and is invariant to printer technologies and resolution is the tabular model (Baqai and Allebach, 2003; Pappas et al., 1993). A variation of the tabular model which may further reduce the
number of parameters is the offset-centered model (Wang et al., 1994). More analytic and comprehensive printer models (specifically for laser electrophotographic printers), most of which were developed independent of halftoning algorithms, have been proposed by many researchers, e.g., (Chen et al., 2003, 2008; Chen and Weng, 2009) and the references therein. As demonstrated by Kacker and Allebach in their work, combination of a detailed electrophotographic process model with the halftoning algorithm of direct binary search produces halftone and printed images of very high quality. However, those models are so complex in general that they demand very intensive on-line computation and memory.

In this paper, we present a new class of printer models, i.e., two-dimensional Wiener models in the spatial domain (Haber and Unbehauen, 1990), which originate from system identification of nonlinear dynamic systems in the time domain. A methodology for carrying out model identification and evaluation is also proposed. Since a Wiener model is architecturally simpler than a comprehensive printer model and the parameters of which can be determined via an off-line system identification or training process, it demands much less computation and memory when incorporated into conventional halftoning algorithms (running online). The proposed Wiener model comprises an FIR (finite impulse response) filter followed by a nonlinear static mapping. The nonlinear static mapping is constructed using a fuzzy system with Gaussian membership functions, which may be further represented as a feedforward neural network. A set of specifically designed test images serve as input. Those images were 'printed' using a comprehensive model of monochrome laser electrophotographic printer (Chen et al., 2003, 2008; Chen and Weng, 2009), which generate corresponding reference output images. Gradient or back propagation algorithm along with those input and desired output images is then implemented to optimize the parameters of the Wiener model. Finally, the optimized model along with an HVS model is embedded into a typical digital halftoning algorithm, i.e., error diffusion. Experimental results including a comparative study show that, in terms of image quality, the proposed model-based halftoning approach is superior to conventional ones and comparable to the one utilizing a comprehensive printer model.

2. HVS MODEL

Many sophisticated models of human visual system have been suggested, and some have been successfully incorporated into halftoning algorithms such as Bayer’s screen (Bayer, 1973), Floyd-Steinberg error diffusion (Floyd and Steinberg, 1976), and Ulichney’s void and cluster algorithm (Ulichney, 1987). A specific HVS model will be used in this work, which can be expressed in the frequency domain as

$$H_s(u,v) = a L^2 \exp\left(-\frac{1}{s(\phi) c \log(L)+d}\right)$$

where $u$ and $v$ takes the units of cycles/degree subtended at the retina, $L$ is the average luminance in candela/m², $\phi$ is the angular frequency defined as $\phi = a \tan(u/v)$, and $s(\phi)$ is a designable function. Denote $h_s(x,y)$ as the inverse Fourier transform of $H_s(u,v)$, $h_s(T_{x},T_{y})$ as the discretized version of $h_s(x,y)$, and $H(o_k,o_l)$ as the discrete Fourier transform of $h_s(T_{x},T_{y})$. Then, we have

$$H(o_k,o_l) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} H_s(o_k-2\pi k, o_l-2\pi l) \frac{2\pi T}{2\pi T}$$

(2)

For this study, $a=131.6$, $b=0.3188$, $c=0.525$, $d=3.91$, $s(\phi) \approx 1$, and $L=10$ cd/m². Suppose that $R$ is the print resolution (in dot per inch or dpi) and $D$ is the viewing distance (in inches). Since a length of $x$ inches, when viewed at a distance of $D$, subtends an angle of $\theta = \tan^{-1}(x/D) \approx x/D$ radians for $x << D$, the spacing of the dots or the sampling period $T$ will be

$$T = \frac{180^\circ}{\pi \frac{D}{RD}}$$

(3)

A value of $T = 0.0165$ will be adopted along with a print resolution of 300 dpi and a viewing distance of approximately 11.5 inches. The frequency unit can be further converted using the following relationship

$$(u,v) = \left(\frac{\pi D}{180^\circ \bar{n}}, \frac{\pi D}{180^\circ \bar{v}}\right)$$

(4)

where $\bar{n}$ and $\bar{v}$ takes the units of cycles/inch with respect to a viewing distance of $D$.

3. COMPREHENSIVE PRINTER MODEL

A comprehensive printer model, which integrates the models of motor/organic photoconductor subsystem, charging, exposure, and development process, will be employed to ‘print’ the test samples and generate reference output images. This model is given by

$$M(x,y) = \frac{Z_v(V_s(x,y) - V_c)}{Q/M} \rho_\rho 8\varepsilon_0$$

(5)

where $M(x,y)/A$ is the toner mass per unit image area at position $(x,y)$, $V_s(x,y)$ is the OPC surface potential after exposure, $Z_v$ is the toner concentration, $V_c$ is the developing roller voltage, $\nu$ is the speed ratio between the developing roller and the OPC, $\rho_o$ is the ratio of the single projected area to the total projected area of the carrier on the OPC, $\rho_v$ and $\rho_c$ are the toner particle density and the carrier particle density, respectively, and $R_s$ is the toner particle radius, $Q/M$ is the average value of the charge-to-mass ratio, and $\varepsilon_0$ is the permittivity of free space. Readers interested in details on derivation and verification of this model may refer to (Chen and Weng, 2009) and the references therein.

4. TEST IMAGE DESIGN

The test images were 'printed' using a comprehensive model of monochrome laser electrophotographic printer (Chen et al., 2003, 2008; Chen and Weng, 2009), which generate corresponding reference output images. A set of specifically designed test images serve as input. Those images were 'printed' using a comprehensive model of monochrome laser electrophotographic printer (Chen et al., 2003, 2008; Chen and Weng, 2009), which generate corresponding reference output images. Gradient or back propagation algorithm along with those input and desired output images is then implemented to optimize the parameters of the Wiener model. Finally, the optimized model along with an HVS model is embedded into a typical digital halftoning algorithm, i.e., error diffusion. Experimental results including a comparative study show that, in terms of image quality, the proposed model-based halftoning approach is superior to conventional ones and comparable to the one utilizing a comprehensive printer model.
Adopting the concept of conducting system identification using sinusoidal signals, two-dimensional spatial-based test images \( \text{Img}(x, y) \) of \( 100 \times 100 \) pixels are designed based on the following formula:

\[
\text{Img}(x, y) = B + A \cos \left( 2\pi x \frac{f_x}{\text{DPI}} + 2\pi y \frac{f_y}{\text{DPI}} \right)
\]

(6)

where \( B \) is the absorptance of background, \( A \) is the amplitude of the sinusoid, \( f_x \) is the frequency of the sinusoid in \( x \) direction, \( f_y \) is the frequency of the sinusoid in \( y \) direction, and \( \text{DPI} \) is the print resolution (set to 600 for this study). Note that the frequency unit is cycles/inch.

Table 1 lists the set of test images with various combinations (around two hundred) of background absorptance, sinusoid amplitude, and frequencies in \( x \) and \( y \) directions. Fig. 1 depicts some of the test images, which have been enlarged for better visibility.

**Fig. 1.** Test images with different combinations of background absorptance, sinusoid amplitude and frequencies

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG75</td>
<td>VM15</td>
</tr>
<tr>
<td>BG100</td>
<td>VM15, VM35</td>
</tr>
<tr>
<td>BG125</td>
<td>VM25, VM45, VM55, VM65</td>
</tr>
<tr>
<td>BG150</td>
<td>VM25, VM45, VM55, VM95</td>
</tr>
<tr>
<td>BG175</td>
<td>VM15, VM35, VM55, VM75</td>
</tr>
<tr>
<td>BG200</td>
<td>VM25, VM45, VM55</td>
</tr>
<tr>
<td>BG225</td>
<td>VM15, VM25</td>
</tr>
</tbody>
</table>

**Table 1. Parameters of the test images**

5. IDENTIFICATION/OPTIMIZATION OF THE WIENER PRINTER MODEL

The main idea for conducting parametric identification or optimization of the proposed Wiener model is illustrated in Fig. 2. The Wiener model consists of an FIR filter and a nonlinear static mapping. Although conventional approaches of system identification are available for determining the parameters of the nonlinear static mapping of a Wiener model, this paper will exploit the feasibility and effectiveness of utilizing a fuzzy system as a universal approximator. It is expected that the fuzzy system should be able to approximate any nonlinear static mapping with high degree of nonlinearity.

In Fig. 2, \( x(m,n) = u(m,n) \) represents a test image. \( \hat{y}(m,n) \) and \( y(m,n) \) are the outputs of the comprehensive printer model and the Wiener model, respectively. Therefore, an error function or objective function \( E \) can be specified as

\[
E = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{1}{2} (y(m,n) - \hat{y}(m,n))^2
\]

(7)

where \( M \) and \( N \) are the vertical and horizontal resolutions of the image, respectively. An optimization search algorithm or update law (to be described later) will adjust the parameters of the Wiener model iteratively such that \( E \) is minimized. The output \( w(m,n) \) of the FIR filter can be expressed by

\[
w(m,n) = \sum_{m=-k}^{k} \sum_{n=-t}^{t} h(s,t)u(m+s, n+t)
\]

(8)

**Layer Four**

**Layer Three**

**Layer Two**

**Layer One**

**Fig. 3.** Fuzzy neural network for nonlinear static mapping

where \( h(s,t) \) is a unit impulse response matrix \([h_{st}]\) of \( n \times n \). For example, \([h_{st}] = 3 \times 3\) implies that

\[
h(s,t) = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}
\]

(9)

The nonlinear static mapping is synthesized by a fuzzy neural network of four layers with Gaussian membership functions as activation functions (see Fig. 3). In the following, we briefly summarize the operation for each layer of the network. Note that most of the variables correspond to two-dimensional spatial-based signals.

**Fig. 4.** Gaussian membership function (a type of radial basis functions)

Let \( q_i^{(1)} \) represent the output of the \( i \)th layer. The output of the first layer is
\[ q_{2}^{(i)} = w(m,n) \]

The output of the second layer is

\[ q_{2}^{(i)} = \exp \left\{ -\left( \frac{q_{2}^{(i)} - p_{j}}{\delta_{j}} \right)^{2} \right\} \]

(11)

where \( p_{j} \) and \( \delta_{j} \) are, respectively, the adjustable center and width of the radial basis function (see Fig. 4). The output of the third layer is

\[ q_{3}^{(i)} = \frac{\sum_{j=1}^{q} q_{2}^{(i)} r_{j}}{\sum_{j=1}^{q} q_{2}^{(i)}} \]

(13)

where \( \beta \) is the number of membership functions, and \( r_{j} \) determines the weight of a link. For 8-bit digital image, its intensity level can vary only between 0 and 255. Hence, further constraints need to be imposed, i.e.

\[ 0 \leq p_{j} \leq 255, \quad \delta_{j} \neq 0, \quad 0 \leq r_{j} \leq 255 \]

(14)

\[ 0 \leq w(m,n) \leq 255 \]

(15)

\[ 0 \leq \exp \left\{ -\left( \frac{w(m,n) - p_{j}}{\delta_{j}} \right)^{2} \right\} \leq 255 \]

(16)

To minimize the error function defined previously, the gradient or back-propagation algorithm is implemented to iteratively adjust the parameters of the FIR filter and the Wiener model (known as the training process) based on the update laws given below:

\[ p_{j}(t+1) = p_{j}(t) - \eta \frac{\partial E(t)}{\partial p_{j}} \]

(17)

\[ \delta_{j}(t+1) = \delta_{j}(t) - \eta \frac{\partial E(t)}{\partial \delta_{j}} \]

(18)

\[ r_{j}(t+1) = r_{j}(t) - \eta \frac{\partial E(t)}{\partial r_{j}} \]

(19)

\[ h_{\text{MN}}(t+1) = h_{\text{MN}}(t) - \eta \frac{\partial E(t)}{\partial h_{\text{MN}}} \]

(20)

where \( t \) corresponds to a certain iteration. Detailed derivations of the partial derivatives are omitted due to page limit.

For this study, the size of the FIR filter is set to 13×13 and the number of membership functions is set to 30. The training of the Wiener model is conducted on an epoch-by-epoch basis. Each epoch consists of presentation of all test images and corresponding reference images one by one to the Wiener model in a randomized order. Eleven epochs are performed for this study, which ends up with a total of 2200 test images and approximately 9500 iterations. The results are shown in Fig. 5. As expected, the error fluctuates up and down in response to altering of the test image during each epoch. From epoch to epoch, however, it can be seen that the error follows a macroscopic trend of decreasing. Fig. 6 and Fig. 7 depict the frequency response of the FIR filter and the nonlinear static mapping, respectively, after the training is completed.

To evaluate the effectiveness of the identified Wiener model in approximating the comprehensive printer model, a comparative study is performed (see Fig 8). As first step, a set of 15 continuous-tone image samples are transformed into halftone images using a typical error diffusion algorithm. Next, the halftone images are 'printed' using both the comprehensive and the Wiener printer models. Third, the two groups of 'printed' images and the original grayscale images are filtered by the HVS model to generate 'perceived' images. Finally, the RMS (root mean square) errors described by

\[ \text{RMS Error} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \left( \text{img}_{\text{op}}(m,n) - \text{img}_{\text{p}}(m,n) \right)^{2} \]

(21)

are evaluated and summarized in Table 2. The closeness of the RMS errors indicates that the identified Wiener model is a good approximation of the comprehensive printer model. This can be further justified by looking at the image results. Fig. 9 demonstrates the results corresponding to two of the image samples.
6. MODEL-BASED ERROR DIFFUSION
INTEGRATING THE WIENER PRINTER MODEL

The identified Wiener model is integrated with an existing and widely used halftoning algorithm, i.e., error diffusion, to form a new class of model-based halftoning techniques. This is illustrated in Fig. 10. Also shown is how to evaluate the performance of a specific type of model-based halftoning algorithms. The proposed model-based halftoning approach is compared with three other approaches, i.e., traditional error diffusion (assuming all-pass filter printer model), error diffusion with dot overlap printer model, and error diffusion with comprehensive printer model. A set of 43 image samples are used for this study. The results are summarized in Table 3. It is seen that, in all cases excepting samples 11, 18, and 29, error diffusion based on either the comprehensive or the Wiener model performs better than the other two conventional approaches. Another observation is that, in one quarter of the cases, error diffusion based on the Wiener model even outperforms one based on the comprehensive model. Fig. 11 and Fig. 12 demonstrate the halftone images and printed images corresponding to three of the image samples.

Table 2. RMS errors for comprehensive (CP) and Wiener (WP) printer models

<table>
<thead>
<tr>
<th>Printer Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>CP</td>
<td>92.1</td>
<td>98.9</td>
<td>101.4</td>
<td>113.7</td>
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<td>WP</td>
<td>90.5</td>
<td>95.3</td>
<td>96.6</td>
<td>105.2</td>
<td>97.9</td>
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</table>

<table>
<thead>
<tr>
<th>Printer Model</th>
<th>CP</th>
<th>WP</th>
<th>CP</th>
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<td>106.9</td>
<td>87.9</td>
<td>129.1</td>
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</tr>
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</table>

Table 3. Compare the RMSE among different model-based halftoning algorithms ((ED: traditional error diffusion, Md_dot: dot overlap, Md_Wm: Wiener model, Md_Vr: comprehensive printer model)

<table>
<thead>
<tr>
<th>Printer Model</th>
<th>1</th>
<th>2</th>
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<td>97.9</td>
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Fig. 10. Configuration of model-based digital halftoning

7. CONCLUSIONS

A new class of model-based digital halftoning algorithms is introduced in this paper. We study the feasibility and effectiveness of embedded a two-dimensional spatial domain Wiener printer model into a conventional halftoning algorithm.
technique. We propose systematic procedures to perform parametric identification/optimization and evaluation of the Wiener printer model. Since the parameters of the Wiener model are determined off-line, it demands much less computation and memory when running on-line. Another advantage is that the architecture complexity of the Wiener model can be adjusted by altering the order of the FIR filter and the number of membership functions in the nonlinear static mapping. A comparative study shows that the proposed model-based halftoning approach is superior (in terms of image quality) to conventional ones and comparable to the one based on a comprehensive printer model.

Fig. 11. Halftone images corresponding to three of the image samples (1, 6, 11)

Fig. 12. Printed images corresponding to three of the image samples (1, 6, 11)

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