PID tuning rules for minimum-time rest-to-rest transitions

Stefano Piccagli ∗ Antonio Visioli ∗

∗ Dipartimento di Ingegneria dell’ Informazione,
University of Brescia, Italy

Abstract: In this paper we propose new tuning rules for PID controllers so that the determined feedback controller can be employed with a properly designed feedforward command input in order to achieve a minimum-time transition from an equilibrium state to another (corresponding to a new desired process variable value) subject to constraints on the control and process variables. Simulation results demonstrate the effectiveness of the combined feedforward/feedback design methodology.

1. INTRODUCTION

Proportional–Integral–Derivative (PID) controllers are undoubtedly the most widely adopted controllers in industry owing to the advantageous cost/benefit ratio they are able to provide. In fact, they are capable to provide a satisfactory performance in many cases, despite their relative ease of use. Actually, many tuning rules have been proposed in the last seventy years in order to help the operator to select the controller gains to address given control specifications (O’Dwyer, 2006) and autotuning functionalities are almost always available in commercial products (Leva et al., 2001; Aström and Hägglund, 2006). However, it is also recognized that the performance of a PID control loop is determined also by the suitable implementation of those functionalities that have to (or can) be added to the basic PID control law in order to deal with practical issues (Visioli, 2006). In this context a particular attention has been paid by researchers to the synthesis of a suitable feedforward control action (Aström and Hägglund, 2006; Kuo, 1995) in order to improve the set-point following performance, which is of particular concern in many cases, especially in the control of batch processes when load disturbances are not critical.

Among the different approaches proposed in the literature (see, for example, (Araki, 1988; Hang and Cao, 1996; Visioli, 1999; Leva and Bascetta, 2007, 2008; Wallen, 2000; Wallen and Aström, 2002; Visioli, 2004; Piazzi and Visioli, 2006)), a methodology that takes into account explicitly the constraints on both the manipulated and the process variable has been proposed in (Piccagli and Visioli, 2009).

Therein, a feedback control system with an already tuned PID controller is considered. Then, a command input to be applied to the designed feedback control system, where the process is assumed to have a first-order-plus-dead-time (FOPDT) dynamics, is determined in order to provide a minimum-time rest-to-rest transition from an equilibrium state to another (corresponding to a process output transition from a set-point value to another) subject to minimum and maximum constraints for the manipulated variable as well as for the process output. A Chebyshev approach is employed for this purpose (Vlassenbroeck and Van Dooren, 1988; El-Gindy et al., 1995; Elnagar, 1997; Elnagar and Kazemi, 1998; Jaddu and Shimemura, 1999), namely, the state variables and the control variable are parametrized by Chebyshev series so that the system dynamics is transformed into a system of algebraic equations and therefore the minimum-time control problem is reduced into a constrained optimization problem. The same problem can be solved by exploiting the algorithm (based on discretization) proposed in (Consolini and Piazz, 2009), where the characterization of the open-loop control law as a generalized bang-bang (namely, either the input or the output saturates during the transient response) to achieve a minimum-time rest-to-rest transition for a single-input-single-output linear system has been presented. In fact, the method can be applied, with suitable simple modifications, in the context of a PID control loop, where, again, the PID parameters have been previously selected without taking into account the adoption of a minimum-time rest-to-rest feedforward strategy (Consolini et al., 2007).

In any case, it is evident that both in (Piccagli and Visioli, 2009) and (Consolini et al., 2007) a two stage design methodology is applied, because the feedback controller is designed first (by applying any tuning rules or a trial-and-error procedure) and the command input is determined subsequently. Since the state trajectories depend on the value of the PID parameters, the obtained rest-to-rest transition time actually depends on the PID gains themselves.

In this paper we propose to combine the design of the feedback controller with the design of the command input signal in order to achieve the minimum-time rest-to-rest transition. In particular, in addition to the command input function, the PID parameters are considered as argument of the optimization problem and tuning rules have been suitably devised for this purpose (note that this allows to preserve the ease of use of the overall method). In other words, we present tuning rules so that the feedback controller is designed in order to minimize the rest-to-rest transition time.

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The paper is organized as follows. In Section 2 the problem is formulated. Tuning rules for the PID controller are given in Section 3 and discussed in Section 4. Simulation results are given in Section 5 while conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

We consider the unity-feedback control loop shown in Figure 1, where the “true” process $P(s)$ to be controlled (assumed to be self-regulating) is modelled as a FOPDT transfer function:

$$ P(s) = \frac{K}{Ts + 1} e^{-Ls}, $$

where $K$ is the process gain, $T$ is the time constant and $L$ is the dead time. This is a typical choice in industrial practice, because this model can describe well the dynamics of many industrial processes and many techniques are available to approximate a high-order dynamics with a FOPDT model (Visioli, 2006). Further, it is assumed that the normalized dead time $L/T$ is less than or equal to one. Actually, it is well-known that if $L/T > 1$ a dead-time compensator should be employed to achieve a satisfactory performance (Åström and Hägglund, 2006).

The feedback controller is of (output-filtered) PID type, whose transfer function is:

$$ C(s) = K_p \left( 1 + \frac{1}{T_1 s + 1} \right) \frac{1}{T_2 s + 1} $$

where $K_p$ is the proportional gain, $T_1$ is the integral time constant, $T_2$ is the derivative time constant and $T_f$ is the time constant of a first-order filter that makes the transfer function proper. The value of $T_f$ can be selected easily, once the other parameters are determined, such that the filter dynamics does not influence the dynamics of the PID controller and the effects of the measurement noise are reduced as much as possible. For this reason, for the sake of simplicity, $T_f$ will not be included in the optimization problem posed hereafter and it will be selected a posteriori as $T_f = 0.1/\omega_n$ where $\omega_n$ is the natural frequency of the controller zeros. It is worth noting that, among the different PID configurations, the output-filtered one has been chosen because of its nice properties (Visioli, 2006), but the methodology proposed in this paper can be applied with any other PI(D) configuration.

The aim of the proposed design methodology is to determine the PID parameters and the signal $r(t)$ to be applied to the closed-loop system when a transition from an equilibrium point corresponding to a process output value $y_0$ to another equilibrium point corresponding to a process output value $y_f$ is required. In particular, a minimum-time rest-to-rest transition is required subject to given limits on the control variable and on the process variable. Hereafter, without loss of generality and for the sake of clarity, we will consider $y_0 = 0$ (null initial conditions), $K > 0$ and $y_f > 0$. We also denote as $t_f$ the transition time.

In order to pose the problem formally, it is convenient to approximate the process dead time by a first-order Padé approximation, namely,

$$ \hat{P}(s) = \frac{K}{Ts + 1 - \frac{T}{2} s + 1}, $$

and then to consider the closed-loop system

$$ F(s) := \frac{C(s) \hat{P}(s)}{1 + C(s) \hat{P}(s)} $$

and write a minimal state-space realization of it, that is:

$$ \frac{dx(t)}{dt} = Ax(t) + b r(t) $$

$$ y(t) = cx(t) $$

where

$$ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{T_f} & \frac{2K}{T_f} & K \\ 0 & 0 & 0 & 1 \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ \frac{K_p T_d}{T_f} \end{bmatrix} \quad c = \begin{bmatrix} 0 & 2K & 0 \end{bmatrix} $$

with this state space realization, the PID controller output is expressed as

$$ u(t) = \frac{K_p}{T_1 T_f} x_1(t) + \left( \frac{K_p}{T_f} - \frac{T_1 K_p}{T_f^2} \right) x_2(t) $$

$$ -\frac{2K K_p T_d}{T_1 T_f^2} x_3(t) + \frac{T_1 K_p K}{T_f^2} x_4(t) + \frac{K_p T_d}{T_f} r(t) $$

Then, denote as $x_0$ the equilibrium state corresponding to the process output $y_0 = 0$ and denote as $x_f$ the equilibrium state corresponding to the process output $y_f$.

The considered time-optimal feedforward constrained control problem can be therefore expressed as follows:

$$ \min_{K_p, T_1, T_2, r(t)} t_f $$

subject to:

$$ \frac{dx(t)}{dt} = Ax(t) + b r(t), \quad 0 \leq t \leq t_f $$

$$ x(0) = x_0, \quad x(t_f) = x_f $$

$$ u_{\min} \leq u(t) \leq u_{\max} \quad 0 \leq t \leq t_f $$

$$ y_{\min} \leq y(t) \leq y_{\max} \quad 0 \leq t \leq t_f $$

where $u_{\min}, u_{\max}$ and $y_{\min}, y_{\max}$ are evidently the constraints for the control variable and the process variable respectively.

It is worth noting that it has been demonstrated (see (Consolini and Piazz1, 2009)) that this time-optimal control problem has a solution if

$$ \left\{ 0, \frac{y_f}{K} \right\} \subset \left( y_{\min}, y_{\max} \right) and \left\{ 0, y_f \right\} \subset \left( y_{\min}, y_{\max} \right). \quad (14) $$

3. PID CONTROLLER TUNING RULES

The solution of the optimization problem (9)-(13) represents the feedforward/feedback control law that minimizes the rest-to-rest transition time of the process variable
subject to the given constraints. Indeed, the solution of this time-optimal control problem represents a generalized bang-bang, namely, the time-optimal control is characterized by the fact that either the control variable or the process variable saturates during the transient (Consolini and Piazzi, 2009).

Given $K_p$, $T_i$, and $T_d$, the command input function $r(t)$ that solves the optimization problem can be determined by employing a technique that exploits a discretization approach (Consolini et al., 2007) or a Chebyshev optimization (Piccagli and Visioli, 2009). However, the choice of the PID parameters affects the value of the optimal transition time $t_f$. For this reason, a tuning rule for the PID controller that determines the values of $K_p$, $T_i$, and $T_d$ that solve the optimization problem (9)-(13) has been determined by applying the following method.

First, in order to make the methodology easier to be used in practical cases, the following assumptions have been made:

- as there are time intervals in the transient response where the process output saturates at $y_{\text{min}}$ and time intervals where the process output saturates at $y_{\text{max}}$ (Consolini and Piazzi, 2009), the limits $y_{\text{min}}$ and $y_{\text{max}}$ of the process variable have been selected as $y_{\text{min}} = y_i - 0.01(y_f - y_i)$ and $y_{\text{max}} = y_f + 0.01(y_f - y_i)$ in order to obtain an almost monotonic response (actually, an undershoot and an overshoot of one percent is considered to be negligible) (Piccagli and Visioli, 2009);
- the value of $u_{\text{min}}$ has been selected as $-u_{\text{max}}$ in order to reduce the number of parameters involved in the optimization procedure and to make possible the determination of a tuning rule. In fact, it has to be taken into account that the lower limit of the actuator (recall that we have assumed $K > 0$ and $y_f > 0$) is attained for very small time intervals during the transient and therefore, from a practical point of view, having a different value for $u_{\text{min}}$ does not cause a significant decrement in the performance (see Section 5.1).

A tight grid has then been created to perform the optimization: each point of the grid corresponds to different values of the normalized dead time $L/T$ and to a different value of $u_{\text{max}}$ (and consequently $u_{\text{min}}$). In particular, the value of $u_{\text{max}}$ has been related to the steady-state value of the control signal at the end of the transient response, namely:

$$u_{\text{max}} = \frac{\bar{u} y_f}{K}, \quad \bar{u} > 1$$  \hspace{1cm} (15)$$

and different values of $\bar{u}$ have been considered.

For each pair $(\bar{u}, L/T)$, a genetic algorithm has been employed to select the optimal values of $K_p$, $T_i$, and $T_d$. For each trial required by the genetic algorithm, a bisection algorithm has been applied to select the optimal value of $t_f$. In this context, a linear programming procedure is employed to determine the feasibility of the considered set of parameters, namely, to return a boolean true or false value if constraints (10)-(13) can be satisfied or not, respectively. In the first case the command input function $r(t)$ has been also determined.

Eventually, the obtained optimal values of the PID parameters (namely, those for which the related command input function provides the minimum value of $t_f$) for the different considered processes and for the different values of $\bar{u}$ have been interpolated (Matlab, 2006) and the following tuning rules have been obtained:

$$K_p(K, T, L, \bar{u}) = \frac{(a_{11} \bar{u} + a_{12}) e^{(b_{11} \bar{u} + b_{12}) \frac{L}{T}}}{K}$$  \hspace{1cm} (16)$$

$$T_i(T, L, \bar{u}) = T (a_{21} \bar{u} + a_{22}) e^{(b_{21} \bar{u} + b_{22}) \frac{L}{T}}$$  \hspace{1cm} (17)$$

$$T_d(T, L, \bar{u}) = T (a_{31} \bar{u} + a_{32}) e^{(b_{31} \bar{u} + b_{32}) \frac{L}{T}}$$  \hspace{1cm} (18)$$

where 

$$\bar{u} := \min \{\bar{u}, 1.3\}$$  \hspace{1cm} (19)$$

and where the values of the parameters are reported in Table 1. It is worth stressing that, for the same process, the optimal values of the controller parameters are the same when the maximum limit of the control variable is greater of more than 30% than the steady-state value $y_f/K$ of the control variable at the end of the transient response. This is because of the strict limits $y_{\text{min}}$ and $y_{\text{max}}$ selected for the process variable.

The plots of the functions (16)-(18) are shown in Figures 2-4.

4. DISCUSSION

The provided tuning rules allow to simplify significantly the adoption of the optimal feedforward/feedback control

Table 1. Value of the parameters in the tuning rule (16)-(18)
law in practical cases. Indeed, once the PID controller parameters have been selected by means of the tuning rules (16)-(18), the command input can be determined by applying the discretization or the Chebyshev approach. In both cases, as already mentioned, a bisection algorithm can be employed to find the optimal value of the transition time. For each considered value \( t_f \), a simple linear programming procedure is applied to determine if there is a feasible solution (namely, if constraints (10)-(13) can be satisfied). Further, it is worth stressing that the devised tuning rules ensure a satisfactory level of robustness to the system, as it is demonstrated by the provided maximum sensitivity \( M_s \) (Åström and Hägglund, 1995) shown in Figure 5 for processes with different normalized dead time \( L/T \) and different saturation level \( \bar{u} \).

Finally, the command input determined by applying the Chebyshev optimization procedure can be applied to the closed-loop system as a polynomial function of the time. Alternatively, an approximated solution can be obtained as a step response of a linear filter, making the procedure more suitable to implement with standard control soft-

ware/hardware. Further details related to this technique can be found in (Gervasio et al., 2009).

5. SIMULATION RESULTS

In all the simulation examples that are considered hereafter, a process output transition from \( y_p = 0 \) to \( y_f = 1 \) is required. Further, the Chebyshev optimization procedure is employed to determine the command input signal \( r(t) \) to be applied to the closed-loop system (Piccagli and Visioli, 2009).

5.1 Example 1

As a first example, consider the following FOPDT system (Piccagli and Visioli, 2009):

\[
P(s) = \frac{1}{10s + 1} e^{-3s}
\]

The constraints for the control variable are selected as \( u_{\min} = -1.5 \) and \( u_{\max} = 1.5 \). By applying the tuning rules (16)-(18) we obtain \( K_p = 0.043, T_i = 0.50, T_d = 6.03 \), while the filter time constant has been selected as \( T_f = 0.19 \) (note that the corresponding maximum sensitivity is \( M_s = 1.6 \)). The Chebyshev optimization yields an optimal rest-to-rest transition time \( t_f' = 15.60 \) which is about half of the optimal transition time obtained with the PID parameters considered in (Piccagli and Visioli, 2009). Indeed, this confirms the usefulness of using a combined design for the feedback and the feedforward part. The command input signal \( r(t) \) that results from the Chebyshev optimization is plotted in Figure 6, while the corresponding process variable and control variable are shown in Figure 7 as a solid line. The case where the constraint on the control variable is actually \( u_{\min} = 0 \) (note that the same command signal as before has been employed) is plotted as a dashed line. This confirms that the presence of a limit \( u_{\min} \) on the control variable which is different from that considered in the tuning of the controller and in the optimization does not cause a significant decrement in the performance. The closed-loop system step response is also shown to give an idea of the employed PID controller tuning (the comparison of the
performance is obviously unfair). It is worth noting that, although the obtained response is not exactly a generalized bang-bang (because of the approximation introduced by the Padé approximation and by the finite Chebyshev series), the achieved performance is very satisfactory in any case.

If the constraints on the control variable are selected as \( u_{\min} = -1.1 \) and \( u_{\max} = 1.1 \), we have \( K_p = 0.22, T_i = 2.47, T_d = 5.60 (T_f = 0.37) \), the optimal transition time is \( t_f^* = 27.88 \), the control system maximum sensitivity is 1.45 and the command input signal is that plotted in Figure 8. The corresponding control system response is shown in Figure 9 (solid line) where again also the step response is shown (dotted line). In this latter case, the integrator windup phenomenon occurs and therefore the step response where the conditional integration anti-windup method (Åström and Hägglund, 2006; Visioli, 2006) is employed has also been determined (dashed line). Considerations similar to those done previously can be done also in this case, demonstrating that the technique is effective for all the possible given constraints.

5.2 Example 2

In order to verify the robustness of the proposed methodology, consider as a second example the process

\[
P(s) = \frac{1}{(s + 1)^4}
\]  

which has been approximated (by applying the method proposed in (Wang et al., 2000)) by a FOPDT transfer function with \( K = 1, T = 3.1 \) and \( L = 1.2 \). If the constraints on the control variable are selected as \( u_{\min} = -1.8 \) and \( u_{\max} = 1.8 \) the following PID parameters are obtained: \( K_p = 0.04, T_i = 0.18, T_d = 2.00 (T_f = 0.06) \) which yield a maximum sensitivity of 1.61. The optimal transition time resulting from the Chebyshev optimization is 4.98, and the corresponding command input is plotted in Figure 11. The obtained system response (together with the set-point step response) is shown in Figure 11. The process variable transition is also in this case very satisfactory despite the presence of an unmodelled dynamics.

6. CONCLUSIONS

In this paper we have proposed tuning rules for PID controllers to be employed together with a suitable command
input function in order to obtain a minimum-time rest-to-rest transition subject to constraints on the control and process variables. Simulation results have highlighted that, although the obtained response approximates the optimal one, the proposed technique is effective from a practical point of view. Further, it has been shown that the design of both the feedback and feedforward part plays a key role in achieving the required performance and therefore using a combined approach gives a significant advantage in this context. Future work will consist of the inclusion of slew-rate constraints in the control variable.

REFERENCES


