Gain-Scheduled Yaw Control for Aircraft Ground Taxiing

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Abstract: A gain scheduled yaw controller is proposed to control low speed rolling and manoeuvring of an aircraft on ground. This method is based on local linearisation of a non-linear model and the synthesis of a family of local controllers. A parallel feedforward action is added to achieve the setpoint tracking and let the feedback controller perform the regulation task. Simulation results show attractive and robust performances regard to the adopted control method.

Keywords: Aircraft control ; Gain-Scheduling ; Feedforward ; Model reference control

1. INTRODUCTION

Currently, the on-ground motion of the aircraft is manually driven by coordinating actions on each individual system. The pilot has six controls: the thrust of the left and right engines, the rudder deflection, the left and right brake pressure, and the nose wheel steering angle. This piloting task is quite demanding since those systems have heterogeneous and highly non-linear dynamics. Moreover, the aircraft behaviour dramatically varies with external conditions (runway contamination, wind, etc.). Those inconveniences divert the pilot from its prime task: guiding and maintaining the aircraft in a safe situation with regard to infrastructure and neighbour vehicles (Fig. 1). Then, a need arises for a piloting aid in order to let the human pilot is unable, in poor visibility for instance. It would also improve the ground traffic capacity of airports by communicating with a traffic manager which would optimize the vehicle flow on the airport area (Villaume, 2002).

In addition, air transportation research has focused on overall reduction of pollutants and especially on minimisation of ground noise and exhaust gas emissions. One topic considers the benefits of driving the aircraft by means of electromechanical actuators located in the main landing gears (MLG) instead of main engines (Teo et al., 2008). This new propulsion system has the advantage of being silent and offers a much better efficiency than traditional jet engines. Work is ongoing to install such systems on each wheel of the main landing gears. Thereby, the rolling dynamics is greatly affected since the new driving system exhibits lower torque capacity but faster dynamics than the older. Differential driving and braking can now support the nose landing gear (NLG) steering system to control the lateral motion of the vehicle. They have complementary properties, the steering system provides a large yaw moment but is fairly slow, whereas differential driving has a limited moment capacity but exhibits fast dynamics (Fig. 2). This paper presents a method for controlling the yaw dynamics of an aircraft. First the model used for the synthesis is introduced in section 2. Section 3 exhibits the control design and its robustness is analysed in section 4. Finally, some simulation results, performed on a reference model, are shown in section 5 and section 6 gives a short conclusion.

LIST OF SYMBOLS

α_{N(M)} Slip angle of NLG (MLG)
C_{N(M)} Cornering stiffness of NLG (MLG)
P_{yN(M)} Cornering force at NLG (MLG) contact point
I_z Yaw moment of inertia of the vehicle

Fig. 1. Aircraft at taxiing

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Fig. 2. Envelope of both static and dynamic actuator capacity at providing yaw moment 

\[ Lx_{N(M)} \] Longitudinal distance between CG and NLG (MLG) 
\[ m \] Mass of the vehicle 
\[ Mz \] Global yaw moment: \[ Mz = Mz_N + Mz_{Md} \] 
\[ Mz_N \] Yaw moment provided by NLG 
\[ Mz_{Md} \] Yaw moment provided by MLG differential driving 
\[ r \] Yaw rate of the vehicle (state variable) 
\[ \theta_N \] Steering angle of NLG 
\[ V_{N(M)} \] Velocity vector at NLG (MLG) contact point 
\[ V_{CG} \] Velocity vector at CG 
\[ V_x \] Longitudinal velocity at CG 
\[ V_y \] Lateral velocity at CG (state variable) 

2. SYSTEM MODELLING

A grounded short haul aircraft is a tricycle vehicle with symmetric MLG and a unique steerable NLG (Fig. 3). MLG wheels are both driven and braked and NLG ones are free rolling. The taxiing phase is performed at fairly low speed therefore roll dynamics appears negligible. A simplified control-oriented model will be developed in this section. This simplified model only considers the lateral dynamics of the vehicle.

Fig. 3. Schematic short haul aircraft viewed from top

2.1 Lateral vehicle model

The rolling vehicle can be described by the well known planar bicycle model (Fig. 4). All wheels located on a same axis are considered as a unique wheel lying on the median plane of the vehicle. The rolling system is assumed to be parallel to the ground and neither roll, nor pitch, nor vertical dynamics are considered. Aerodynamic effects are assumed negligible since low speed rolling is considered here, thus only ground contact forces are modelled. The friction contact force acting at each point is denoted \( F_{YN} \) and \( F_{YM} \) for the NLG (resp. MLG). \( V_N, V_M \) and \( V_{CG} \) denote the velocity vector at the nose and main contact points and at the center of gravity of the vehicle (CG). The longitudinal distance between the CG and the contact points is denoted by \( Lx_N \) and \( Lx_M \). \( \theta_N \) represents the nose wheel steering angle. The lateral velocity \( V_y \) and the yaw rate \( r \) are chosen as state variables expressed at the CG. The longitudinal velocity \( V_x \) is considered as an input exogenous parameter known by measurement or estimation. The equations of lateral dynamics are given here below:

\[ I_z \cdot \dot{r} = L_{xN} \cdot F_{YN} \cdot \cos(\theta_N) - L_{xN} \cdot F_{YM} + M_{zMd} \]
\[ m \cdot (\dot{V_y} + V_x \cdot r) = F_{YN} \cdot \cos(\theta_N) + F_{YM} \]  

(1)

\( I_z \) is the yaw moment of inertia, \( m \) is the mass of the vehicle and \( M_{zMd} \) stands for the yaw moment produced by differential driving of the MLG.

2.2 Contact force model

In equation 1, the cornering forces \( (F_{YN}, F_{YM}) \) are assumed to be linear with the slip angle \((\alpha_N, \alpha_M)\) and with the equivalent cornering stiffness \((C_N, C_M)\). This is true while the slip angles remain in a bounded region around zero.

\[ F_{YN} = -C_N \cdot \alpha_N, \quad C_N > 0 \]  
\[ F_{YM} = -C_M \cdot \alpha_M, \quad C_M > 0 \]  

(2)

(3)

Given :

\[ \tan(\alpha_N + \theta_N) = \frac{V_y + L_{xN} \cdot r}{V_x} \]  
\[ \tan(\alpha_M) = \frac{V_y - L_{xM} \cdot r}{V_x} \]  

(4)

(5)

The vehicle lateral dynamics is highly non-linear because of its longitudinal velocity and slip angles dependency. Note that, even at low speed \( (V_x \in [0; 10] \text{ m/s}) \), the non-linearity with regard to velocity is important (Fig. 5). A widespread method is to linearise the slip angle expressions (4) and (5) for small slip quantities and small steering angle (Zheng and Anwar, 2009). The latter assumption cannot be made during low speed taxiing and manoeuvring where the steering angle reaches 75 deg. To cope with the dynamics dispersion, a linearisation was made over a discrete domain of both the longitudinal velocity \( V_x \) and the lateral equilibrium, instead of a single point close to zero.
2.3 Model linearisation

Noting $\Phi = (V_y e; r; \theta_N e)^T$ an equilibrium point of the dynamical system (1). A change of control variable has been performed to use the NLG yaw moment $M_{2N}$ as command instead of the steering angle. The first order linearisation, at this point $\Phi$, gives the local linear state space form:

$$
\begin{bmatrix}
\delta r \\
\delta V_y
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\delta r \\
\delta V_y
\end{bmatrix}
+ 
\begin{bmatrix}
1 \\
B_2
\end{bmatrix}
\begin{bmatrix}
\delta M_{2N} \\
M_{2Md}
\end{bmatrix}
$$

with $\delta r$ and $\delta V_y$ the new state variables representing the variation of the local system in the vicinity of the equilibrium point. $\delta M_{2N}$ is the additional yaw moment produced by the NLG. The state matrix terms express as follow:

$$
A_{11} = -\frac{1}{Iz \cdot Vx}
\left( \frac{C_M \cdot Lx \cdot M^2}{1 + \Delta_M^2} + \frac{C_N \cdot Lx \cdot N^2 \cdot \cos(\theta_N e)}{1 + \Delta_N^2} \right)
$$

$$
A_{12} = \frac{1}{Iz \cdot Vx}
\left( \frac{C_M \cdot Lx \cdot M}{1 + \Delta_M^2} - \frac{C_N \cdot Lx \cdot N \cdot \cos(\theta_N e)}{1 + \Delta_N^2} \right)
$$

$$
A_{21} = \frac{1}{m \cdot Vx}
\left( \frac{C_M \cdot Lx \cdot M}{1 + \Delta_M^2} - \frac{C_N \cdot Lx \cdot N \cdot \cos(\theta_N e)}{1 + \Delta_N^2} \right) - Vx
$$

$$
A_{22} = \frac{1}{m \cdot Vx}
\left( \frac{C_M}{1 + \Delta_M^2} - \frac{C_N \cdot \cos(\theta_N e)}{1 + \Delta_N^2} \right)
$$

$$
B_2 = \frac{Iz}{m \cdot Lx
$$

The terms $\Delta_N$ and $\Delta_M$ reflect the state matrix dependency on the operating point $(\Phi, Vx)^T$:

$$
\Delta_N(\Phi, Vx) = \frac{V_y e + Lx \cdot N \cdot r}{Vx}
$$

$$
\Delta_M(\Phi, Vx) = \frac{V_y e - Lx \cdot M \cdot r}{Vx}
$$

These terms tend to zero as the cornering slip of the wheels vanishes. Then the state matrix corresponds to the one obtained assuming small slip angle (Zheng and Anwar, 2009).

**Remark 1.** The linearised system (6) features a second order dynamics which turns out to be quite close to a first order system, because of one pole-zero compensation. Moreover, if the second input $M_{2Md}$ may improve the system performance in a MIMO control loop, it is powered by the differential driving. The system is already employed by the longitudinal task and has a very limited action on the yaw moment (fig. 2). That is why it not considered in the rest of the article and will be studied in a future work. Thereby, the SISO first order model will be used for synthesis in the following section.

Figure 5 depicts both the static gain $G_{sd}$ and time constant $\tau_{sd}$ of the equivalent first order model. The time constant range is fairly large (1:10 ratio) and highly depends on the vehicle velocity. This characteristic will have a great impact on the controller design.

2.4 Model validation

This synthesis model (remark 1) is compared to a complete 3-dimensions model which counts 16 degrees of freedom (Slama, 2009) and more accurate tyre models (Albero, 2009). Three simulations were performed at distinctive setpoints. The figure 6 shows a good adequateness between each response. At low speed ($Vx = 1 \text{ m/s}$) and high steering angle ($\theta_N e = 50 \text{ deg}$), the linearised model depicts a higher static gain than the reference one. Nevertheless, the dynamics seem to be fairly close to each other. This is the most important characteristic to match since any steady state error could be cancelled by an integral term in the controller. The model is aimed at synthesising a controller, not at performing feedforward control. At higher velocity and smaller steering angle, the synthesis model tightly fits the reference one. The dynamics difference in transient response comes from dynamics and slew rate limitation of the steering system which are not taken into account in the linearised model.

3. CONTROL STRUCTURE DESIGN

3.1 Control specifications

Though flight control has been studied for decades, aircraft rolling dynamics is a relatively new issue of interest.
Ground interactions are highly non-linear and, above all, quite uncertain. (Duprez et al., 2004) proposed a yaw rate control by feedback linearisation with a particular logic to extend the method to vanishing velocity. (Roos and Biannic, 2006) adopted an LFT representation to design an adaptive anti-windup suitable to handle saturations of systems and tire adhesion. A prolific literature exists in the road vehicles domain where studies have widely focused on lateral control at relatively high speed (Zheng and Anwar, 2009). The robotic domain also counts tremendous papers dealing with mobile robot control (Mauder, 2008). However those works usually consider kinematic vehicle model or even ideal dynamic models with nonholonomic constrains (Pourboghrat and Karlsson, 2002). This hypothesis cannot be made for heavy vehicles such as planes rolling on various and potentially slippery surfaces. The yaw control has to fit in a general control architecture suitable for interfacing with both a pilot and a future automatic guiding unit and with existing on-board systems (Jacquet, 2008). Then it must satisfy the requirements for automatic path tracking while being compatible with pilots feeling. The latter are familiar with a common vehicle handling and the assistance control has not to bother them. Hence the control method must ensure no steady state error and a settling time comparable to the natural response of the vehicle with no overshoot over the whole operating span. For testing purpose, the pilot shall still be able to manually steer the NLG while the aircraft is stationary. Moreover, to improve the aircraft manoeuvrability and to support the standard steering system in case of failure or lack of efficiency, differential driving is foreseen by means of the wheel actuators and brakes. The controller should cope with this additional yaw actuator in parallel to the NLG steering system.

All these considerations lead to implement a control method combining a feedforward action in parallel to an additive feedback non-linear controller. The feedforward term offers a similar control to the current open loop method and brings a naturally stable contribution. The feedback controller adds a local regulation action to ensure the cancellation of the steady state error.

3.2 Feedforward Command

A natural lateral vehicle handling is to control the direction of travel, i.e. the angle $\beta_N$ between the longitudinal axis of the vehicle and the direction the pilot is pointing toward. On the other hand, a suitable reference variable for automatic control is the yaw rate of the vehicle. The two variables can easily be linked by an ideal kinematic model (Fig. 7) assuming no side slip of the wheels (7). This approximation remains reasonable while the aircraft operates in safe conditions. The kinematic model enables to command an open loop term $\theta_{ff}$ from the equality $\theta_{ff} = \beta_N$. Thereby, in manual control, the pilot commands the sight angle $\beta_N$ whereas, in automatic piloting, the guiding unit sets the yaw rate demand $r_c$. The latter is converted into a feedforward command $\theta_{ff}$ by solving equation (7) for $\beta_{Nc}$ and substituting $\theta_{ff}$ for $\beta_{Nc}$. Then a proper processing is made on the measured longitudinal velocity $V_{x_m}$ to ensure no division by zero. This treatment must be robust to both measurement bias and noise amplitude. In fact, the limitation on $V_{x_m}$ is part of the non-holonomic constrains of the rolling vehicle since it cannot rotate at a standstill. Therefore, the yaw task is itself not defined at this singular point.

$$ r_c = \frac{V_{x_m}}{L_N X_N + L_M X_M} \tan(\beta_{Nc}) $$

This open loop command $\theta_{ff}$ acts as a tracking command and brings the vehicle yaw rate in the neighbourhood of the demand. This action has the benefit of being intrinsically stable. In parallel to the feedforward action, the pilot seat angle demand $\beta_N$ is translated into a yaw rate remand $r_c$ with the equation (7). Then, the controller can operate at the setpoint as a disturbance compensator to keep the measured yaw rate $r_m$ equal to the demand $r_c$. Assuming the feedforward action steers the system close to the yaw rate demand, the equilibrium point can be worked out from the demand. This allows for adapting the controller to the current operating point.

3.3 Local controller design

The system (1) is assumed to be in the vicinity of an equilibrium point $\Phi$. In order to ensure the steady state error cancellation, a PI-controller is proposed. The global yaw moment $M_z = \delta M_{ZN} + M_{ZM}$ is chosen as the controller output. This variable is independent from any
The linear PI-controller yields the required yaw moment \( \delta M_{SN} \) to cancel the yaw rate error \( \tau_c - \tau_m \). The corrective NLG steering angle \( \theta_{fb} \) is then worked out using the non-linear model of section 2.1:

\[
\theta_{fb} = \frac{I_z}{C_N \cdot L_{xN} \cdot \cos(\theta_N)} \cdot M_{SN} \tag{9}
\]

Either the measured steering angle or the speed vector orientation angle can be used for \( \theta_{fb} \) in equation (9). This term carries the non-linearity. It explicitly depends on the total steering angle, therefore it is valid over the whole operating range.

### 3.4 Gain Scheduling

The aforementioned local controller is to be extend to the whole operating range of the vehicle. The gain scheduling method may be one of the most popular approach of non-linear control. It has successfully been applied on a broad range of applications and a variety of methods have been proposed. One can refer to (Leithhead, 1999) for an extended review of the literature. The classic approach is adopted here, i.e. a family of local controllers is synthesized at each equilibrium points and the non-linear controller is realized by linear interpolation of the gains (Rugh and Shamna, 2000). The span of the open loop dynamics (Fig. 5) is too large to achieve a unique close loop time response for all operating points. Indeed, it could be annoying for the pilot to lose the span of the lateral dynamics, especially the velocity dependence. That is why the close loop time response \( \tau_{cl} \) is also scheduled with regard to the operating point in order to halve the open loop free settling time. Referring to figure 5, the non-linearity variation appears to be sharper at low speed, thus the controller grid is chosen finer below 5 m/s than above. The lateral operating range exhibits a fairly regular shape, thus the grid is evenly distributed along this axis. Thus, the non-linear controller relies on twenty local models parametrized by two scheduling variables, the measured longitudinal velocity \( Vx \) and the yaw rate demand \( r_c \). The figure 8 exhibits the resulting non-linear gains \( Kp(\tau_c, Vx) \) and \( Ki(\tau_c, Vx) \).

Finally, figure 9 outlines the global control structure for the automatic piloting configuration.

An anti windup action is applied to the controller integral term in order to cope with the angular rate saturation of the steering system. This term freezes the integral state when the maximum current is delivered to the steering servovalve.

### 4. ROBUSTNESS ASSESSMENT

The local stability of the controlled system has been studied by varying the uncertain parameters. The table 1 depicts the variation range of these parameters. The controller turns out to be very robust to parameters uncertainty. As figure 10 shows, the controller dynamics does not vary in a large scale and the smallest phase margin is over 60 deg. The open loop phase does not cross the - 180 deg axis, it reveals that the system remains conditionally stable and has infinite gain margin. This local stability margin is widely sufficient to cope with the potential...
vehicle dynamics variations. The system appears to be fairly under stressed, this is voluntary to cope with
the saturation aspects that are not taken into account in the synthesis model. The global stability has been assessed
from simulation results.

![Open-loop gain (dB)](chart1.png)

**Fig. 10.** Black chart of the open-loop controlled system

5. SIMULATION RESULTS

The control law has been validated on the complete 3-dimensions model. The figure 11 exhibits a two steps
response of 4 deg/s at a longitudinal speed of 2 m/s with the heaviest aircraft on a dry surface. This simulation
makes the controller operate at an interpolated point in between four models (Fig. 8). The propulsion is achieved by
the wheel actuators but no differential driving is applied. The yaw rate response $r_m$ reaches the reference $r_c$ in an
acceptable period of time with no steady state error and only a slight overshoot for the second step. The settling
time is mostly driven by the steering rate saturation as the second subplot shows. Between 1 and 3.5 sec, and latter between 6 and 8.5 sec, the slew rate saturation of the steering system limits the response. The commanded
angle $\theta_c$ reaches high magnitudes because of the combina-
tion of the feedforward term $\theta_ff$ and the controller action. Nevertheless, the latter additional command does not influence the system since the steering is already in saturation. Simulations performed at different speeds and several yaw rate demands give similar results.

6. CONCLUSION

The proposed yaw controller, based on local model pursuit and gain scheduling, achieves satisfactory performances on the entire operational range. The combination of feed-forward tracking and local regulation gives comfortable robustness to parameter uncertainty. The reference adaptation makes the solution suitable for both manual driving and automatic piloting. The results, partially validated on a complete 16-D.O.F. model, must be evaluated by real ground tests. Additional MLG differential driving will be investigated to support the steering action and improve the overall yaw control performances and robustness.

![Response to a step demand of 4 deg/s at a speed of 2 m/s](chart2.png)

**Fig. 11.** Response to a step demand of 4 deg/s at a speed of 2 m/s

REFERENCES


Duprez, J., Mora-Camino, F., and Villaume, F. (2004). Aircraft-on-ground lateral control for low speed maneuver-
ers. In 16th IFAC Symposium on Automatic Control in Aerospace. St. Petersburg, Russia.


Mauder, M. (2008). Robust tracking control of non-holonomic dynamic systems with application to the bistee-


10.1016/S0005-1098(00)00058-3.


Teo, A., Rajashekara, K., Hill, J., and Simmers, B. (2008). Examination of aircraft electric wheel drive taxiing
