Unknown-input observation techniques in Open Channel Hydraulic Systems

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Abstract: This paper addresses a problem of state and disturbance estimation for an open channel hydraulic system. More precisely, a cascade of $n$ canal reaches, joined by gates, is considered, and, by using measurements of the water level in three points per reach, we design an observer capable of estimating both the infiltration and discharge in the middle point of each reach. To facilitate the observer design, the system dynamics is modeled by considering a linearized approximation of the underlying nonlinear dynamics around the subcritical uniform flow condition. The proposed solution is based on the unknown-input and proportional-integral observers theory. Simulation results are discussed to verify the effectiveness of the proposed schemes.

Keywords: Unknown-input observers, strong observability, open channel hydraulic system.

1. INTRODUCTION

Most open-channel hydraulic systems are currently manually operated by flow control gates. Medium-term research goals in this field are to operate those systems automatically in order to improve water distribution efficiency and safety. A key problem is to reconstruct all the information needed for control or monitoring purposes (water levels, flow rates, and infiltrations), some of which are intrinsically impossible or difficult to measure, by reducing the number of required sensors in the field. The aim of this paper is the development of new estimation algorithms for a cascade chain of open channel hydraulic systems subject to unmeasurable disturbances. Open channel hydraulic systems are described by two nonlinear coupled partial differential equations (Saint-Venant Equations). The two main methods used to solve Saint-Venant equations are basically the finite-difference method (Strelkoff et al. (1970)) and the finite-element method (Colley et al. (1976)). In (Besançon et al. (2001)) a three-point collocation-based nonlinear model of a single-reach irrigation canal was developed. In that model, the three points of interest where the system variables are evaluated are the canal start, middle and end points, respectively, and a constant uncertain infiltration is taken into account. An observer for the level variables and for the constant infiltration was designed by measuring the level in the middle of the reach and the upstream and downstream flows. Our developments take, as a starting point, the results presented in (Besançon et al. (2001)) for a single-reach canal. Here the main task is to consider a cascade of $n$ canal reaches instead of a single reach, and to relax some of the standing modeling assumptions made in Besançon et al. (2001).

More precisely we:

i.) dispense with the need of flow rate measurements by allowing only level measurements;

ii.) consider a time varying infiltration;

iii.) consider, in the case of absence of infiltration, a number of sensors less then those normally required.

Section 2 recalls the Saint-Venant equations that rigorously describe the dynamics of a canal reach. In Section 3 the three-point collocation based nonlinear model of a single reach presented in (Besançon et al. (2001)), is extended to a cascade of $n$ canal reaches. In Section 4, the two estimation problems addressed in the manuscript (called “Problem 1” and “Problem 2”) are stated. Problem 1 involves the simplifying assumption of absence of infiltration, and, as a counterpart, it considers certain level variables to be unavailable for measurements. The flow variables are wanted to be estimated along with the unmeasured level variables. Problem 2 deals with the more general non-zero infiltration case but requires the measurement of the level variables in all of the collocation points (three points per channel). The flow variables are wanted to be estimated again, along with the unknown infiltrations, after a finite-time estimation transient. Problem 1 and Problem 2 give rise to an observation problem for Linear Time-Invariant System with Unknown Inputs (LTISUI). In Section 5 a method for state estimation and unknown input reconstruction in LTISUI is recalled. The approach is based on the assumption of Strong Observability (Molinari et al. (1976); Hautus et al. (1983); Bejarano et al. (2007)), a structural geometric restriction involving the matrices of the LTISUI mathematical model. In the Section 6, a case study of a canal with rectangular section and three reaches in cascade is illustrated. In the successive Sections 7 and 8 the techniques described in the Section 5 are applied to solve the estimation Problem 1 and Problem 2, respectively. It is shown, in both cases, that the structural requirement of strong observability holds.
for the resulting models, and corresponding simulation results are illustrated and commented, which will confirm the expected performance of the suggested observers.

2. FORMULATION OF THE PROBLEM

Water flow dynamics in an open channel are governed by the Saint-Venant Equations

\[
\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/S)}{\partial x} + gS \left( \frac{\partial H}{\partial x} - I + J \right) = k_q(w) \frac{Q}{S} w
\]

(1)

\[
\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/S)}{\partial x} + gS \left( \frac{\partial H}{\partial x} - I \right) = \frac{Q}{S} - \frac{\partial Q}{\partial x}
\]

(2)

where \( x \in [0, L] \) is the spatial variable (\( L \) being the channel length), \( t \) being the time variable, and \( S(x, t), Q(x, t) \) and \( H(x, t) \) being the wet section, water flow rate and relative water level, respectively, and the term \( w = w(x, t) \) in the right-hand side of (1), (2) represents the infiltration. \( J \) represents the friction term, which has the expression

\[
J = \frac{Q^2}{2g}, \quad Di = kS \left( \frac{Q}{S} \right)^2, \quad \text{with} \quad k \quad \text{being Strickler friction coefficient}\]

and \( P(x, t) \) being the transversal wet length, and \( I \) is the canal slope. Finally the term \( k_q(w) \) is 0 if \( w \geq 0 \) and 1 if \( w < 0 \). In this paper we refer to the case of positive infiltration \((w \geq 0)\) and to canals with rectangular section, so if \( B \) is the constant canal width one has that \( S = BH \) and \( P = 2BH \).

Thus, model (1)-(2) can be rewritten in terms of \( Q \) and \( H \) variables only, and, in particular, eq. (1) modifies as

\[
\frac{\partial H}{\partial t} = \frac{1}{B} \frac{\partial Q}{\partial x} + \frac{1}{B} w
\]

(3)

If the slope of the canal is low, as it is the case, e.g., in irrigation channels, it can be assumed subcritical flow condition which makes it reasonable to complement (1) with the Dirichlet boundary conditions (BCs)

\[
Q(0, t) = Q_U(t), \quad Q(L, t) = Q_D(t).
\]

(4)

\[
H(0, t) = H_0, \quad H(L, t) = H_L
\]

(5)

Initial conditions are given by \( H(x, 0) = H^0(x) \) and \( Q(x, 0) = Q^0(x) \), which fulfill the considered BCs (4)-(5).

3. COLLOCATION-BASED FINITE-DIMENSIONAL MODEL

In (Dulhoste et al. (2001)) it was shown that the Saint-Venant equations can be approximated by ordinary differential equations of finite dimension using a collocation point Galerkin method (Fletcher et al. (1984)). It has been also shown that three collocation points located at the canal upstream, middle, and downstream points (say points \( A, M, \) and \( B \), respectively) lead to a sufficiently accurate representation for observation and control purposes. Consider a cascade of \( n \) canal reaches connecting the two upstream and downstream reservoirs, separated by \( n + 1 \) adjustable gates, and subject to infiltration losses, as represented in the Figure 1. By choosing three collocation points for each channel, and using the same notation as before to denote the resulting points \( A_i, M_i, B_i \) (\( i = 1, 2, \ldots, n \)) the equation (3) can be discretized as follows (Besancon et al. (2001)):

\[
\dot{H}_{A_i} = \frac{1}{B_i L_i} \left[ -4Q_{M_i} + 3Q_{A_i} + Q_{B_i} \right] + \frac{w_i}{B_i}
\]

(6)

\[
\dot{H}_{M_i} = \frac{1}{B_i L_i} \left[ Q_{A_i} - Q_{B_i} \right] + \frac{w_i}{B_i}
\]

\[
\dot{H}_{B_i} = \frac{1}{B_i L_i} \left[ 4Q_{M_i} - Q_{A_i} - 3Q_{B_i} \right] + \frac{w_i}{B_i}
\]

(7)

where \( H_{A_i}, H_{M_i}, \) and \( H_{B_i} \) (\( i = 1, 2, \ldots, n \)) are the state variables, \( Q_{A_i}, Q_{M_i}, \) and \( Q_{B_i} \) (\( i = 1, 2, \ldots, n \)) denote the flow at the collocation points, \( w_i \) (\( i = 1, 2, \ldots, n \)) is the infiltration in the \( i \)-th reach, \( \eta_i \) and \( \Sigma_i \) (\( i = 1, 2, \ldots, n + 1 \)) are the discharge coefficients and the opening sections of the \( i \)-th gate, and \( Q_{C_i} \) is the withdrawal request form the users. \( H_{B0} \) and \( H_{A_{n+1}} \) represent the constant levels in the upstream and downstream reservoirs. Considering (7) and (8) into (6) one obtains a more compact expression for the system’s nonlinear dynamics:

\[
\dot{H}_{A_i} = \frac{1}{B_i L_i} \left[ f_A(H_{B_{i-1}}, H_{B_i}, H_{A_i}, H_{A_{i+1}}, \Sigma_i, \Sigma_{i+1}) - 4Q_{M_i} + Q_{C_i} \right] + \frac{w_i}{B_i}
\]

(9)

\[
\dot{H}_{M_i} = \frac{1}{B_i L_i} \left[ f_M(H_{B_{i-1}}, H_{B_i}, H_{A_i}, H_{A_{i+1}}, \Sigma_i, \Sigma_{i+1}) - Q_{C_i} \right] + \frac{w_i}{B_i}
\]

\[
\dot{H}_{B_i} = \frac{1}{B_i L_i} \left[ f_B(H_{B_{i-1}}, H_{B_i}, H_{A_i}, H_{A_{i+1}}, \Sigma_i, \Sigma_{i+1}) + 4Q_{M_i} - 3Q_{C_i} \right] + \frac{w_i}{B_i}
\]

with implicit definition of the nonlinear functions \( f_A(\cdot), f_M(\cdot) \) and \( f_B(\cdot) \). The dynamic relationship between the middle point flow variable \( Q_{M_i} \) and the remaining system variables can be derived by generalizing the “single-reach canal” relationship (Dulhoste et al. (2001)) as follows:

\[
Q_{M_i} = \psi_i Q_{A_i}, Q_{B_i}, Q_{M_{i-1}}, H_{A_i}, H_{M_i}, H_{B_i}, w_i
\]

(10)

\[
= g_{i+1} H_{M_i} \left( 1 + \frac{H_{A_i} - H_{B_i}}{L_i} \right) + \frac{\left( 2Q_{M_i} - Q_{B_i} \right)}{B_i L_i} \frac{Q_{M_i}}{H_{M_i}} + \frac{\left( H_{B_i} - H_{A_i} \right)}{B_i L_i} \frac{g}{K^2 B_i H_{M_i} \left( \frac{B_i H_{M_i}}{B_i H_{M_i} + 2B_i H_{M_i}} \right)} Q_{M_i}^2
\]

3.1 Linearized Model

The nonlinear model (9) can be linearized in a vicinity of the uniform flow condition (Corriga et al. (1983)). Let \( \overline{Q}_i \) (\( i = 1, 2, \ldots, n \)) denote the flow value in the \( i \)-th channel in the uniform flow condition. Let also \( \overline{\Sigma}_i \) (\( i = 1, 2, \ldots, n \)) be the corresponding water levels, and \( \overline{H}_i \) (\( i = 1, 2, \ldots, n + 1 \)) be the corresponding values for the gates opening sections. Define the corresponding variables \( h_{A_i}, h_{M_i}, h_{B_i}, q_{M_i}, \) and \( \sigma_i \) like the deviation of \( H_{A_i}, H_{M_i}, H_{B_i}, Q_{M_i}, \) and \( \Sigma_i \) from
the uniform condition, then relation (7) can be linearized as follows in a vicinity of the uniform flow condition (Corriga et al. (1983))

\[ Q_{Ai} = Q_i + a_i \sigma_i(t) + b_i (h_{Bi-1}(t) - h_{Ai}(t)) \]  

(11)

with the coefficients \(a_i\) and \(b_i\) as follows

\[ a_i = \sqrt{2g/(H_{i-1} - H_i)}; \quad b_i = \frac{\eta_i \sqrt{2g}}{\sqrt{2(H_{i-1} - H_i)}} \]  

(12)

The corresponding linearized form for (8) can be derived from the next continuity equation

\[ Q_{Bi} = Q_{Ai+1} + Q_{Ci}, \quad i = 1, \ldots, n \]  

(13)

which leads to

\[ Q_{Bi} = Q_{Ci} + \frac{\sigma_i}{\sigma_{i+1}} + a_{i+1} \sigma_{i+1} + b_{i+1} (h_{Bi} - h_{Bi+1}) \]  

(14)

with \(i = 1, 2, \ldots, n\) and the deviation variables \(h_{An+1}\) and \(h_{B0}\) customarily set both to zero as a consequence of the fact that the water level in the upstream and downstream reservoirs is supposed to keep constant

\[ h_{An+1} = 0; \quad h_{B0} = 0 \]  

(15)

Substituting (11)-(14) into (6)-(8), considering the continuity condition and the definitions

\[ Q_i = Q_{i+1} + Q_{Ci}; \quad i = 1, \ldots, n \]  

(16)

\[ h = [h_{A1} h_{M1} h_{B1} \ldots h_{An} h_{Mn} h_{Bn}]^T, h \in \mathbb{R}^{2n} \]  

(17)

\[ \sigma = [\sigma_1 \sigma_2 \ldots \sigma_{n+1}]^T, \quad \sigma \in \mathbb{R}^{n+1} \]  

(18)

\[ q_M = [q_{M1} q_{M2} \ldots q_{Mn}]^T, \quad q \in \mathbb{R}^{n} \]  

\[ w = [w_1 w_2 \ldots w_n]^T, \quad w \in \mathbb{R}^{n} \]  

(19)

one can rewrite the system (9) in the compact state-space form

\[ \dot{h} = Ah + M_\sigma \sigma + M_q q_M + M_w w \]  

(20)

with implicitly defined constant matrices \(A, M_\sigma, M_q\) and \(M_w\) of appropriate dimension. Vector \(q_M\) is obtained solving the nonlinear differential equation

\[ \dot{q}_M = \psi(h, q_M, \sigma, w) = (\psi_{q_1}(\cdot), \psi_{q_2}(\cdot), \ldots, \psi_{q_n}(\cdot))^T \]  

(21)

with the functions \(\psi_q(\cdot)\) \((i = 1, 2, \ldots, n)\) are defined in (10). Note that the nonlinear dynamics (22) needs not to be linearized since vector \(q_M\) is going to be treated as an unknown input of the systems, rather than as a part of the system state. It is also possible to consider a reduced-order version of system (21) where the state vector \(h\) is replaced by the reduced-order version

\[ \tilde{h} = [h_{A1} h_{B1} h_{A2} \ldots h_{An} h_{Bn}]^T, \quad \tilde{h} \in \mathbb{R}^{2n} \]  

(23)

The corresponding reduced-order state space model is given by

\[ \dot{\tilde{h}} = \tilde{A} \tilde{h} + \tilde{M}_\sigma \sigma + \tilde{M}_q q_M + \tilde{M}_w w, \]  

(24)

whose matrices \(\tilde{A}, \tilde{M}_\sigma, \tilde{M}_q, \tilde{M}_w\) can be trivially derived by removing selected rows and columns from the matrices \(A, M_\sigma, M_q, M_w\) of the full-order model (21).

4. FLOW AND INFILTRATION ESTIMATION

PROBLEM STATEMENT

In this paper we make reference to the linearized dynamics (21) and we address two distinct state and disturbance estimation problems under the common constraint that only level measurements are allowed. Vector \(\sigma\) is supposed to be known, while vectors \(w\) and \(q_M\) are both unmeasurable. We cast the following problems:

**Problem 1.** By measuring only a portion of the reduced-order state vector \(\tilde{h}\), and assuming no infiltrations \((w = 0)\), estimate the flow vector \(q_M\) and reconstruct the unmeasured part of vector \(\tilde{h}\).

**Problem 2.** By measuring the full vector \(h\), reconstruct the infiltration vector \(w\) and the flow vector \(q_M\) in finite time.

Both Problems 1 and 2 will be solved by making use of unknown-input observers (UIO) under the requirement of “strong observability” (Molinari et al. (1976); Hautus et al. (1983)) for certain subsystems that shall be specified later on.

5. STRONG OBSERVABILITY AND UIO DESIGN FOR LINEAR SYSTEMS WITH UNKNOWN INPUTS

Consider the linear time invariant dynamics

\[ \begin{align*}
\dot{x} &= Ax + Gu + F\xi \\
y &= Cx
\end{align*} \]  

(25)
where \(x \in \mathbb{R}^n\) and \(y \in \mathbb{R}^p\) are the state and output variables, \(u(t) \in \mathbb{R}^h\) is a known input to the system, \(\xi(t) \in \mathbb{R}^m\) is an unknown input term, and \(A, G, F, C\) are known constant matrices of appropriate dimension. Let us make the following assumptions:

**A1.** The matrix triplet \((A, F, C)\) is strongly observable.

**A2.** \(\text{rank}(CF) = \text{rank} F = m\).

If conditions A1 and A2 are satisfied then it can be systematically found a state coordinates transformation together with an output coordinates change which decouple the unknown input \(\xi\) from a certain subsystem in the new coordinates. Such a transformation is outlined below. For the generic matrix \(J \in \mathbb{R}^{n_r \times n_e}\) with \(\text{rank} J = r\), we define \(J^+ \in \mathbb{R}^{n_r \times r}\) as a matrix such that \(J^+J = 0\) and \(\text{rank} J^+ = n_r - r\). Matrix \(J^+\) always exists and, furthermore, it is not unique \(^1\). Let \(\Gamma^+ = [\Gamma^T \Gamma]^{-1}\Gamma^T\) denote the left pseudo-inverse of \(\Gamma\) and it is such that \(\Gamma^+ \Gamma = I_n\), with \(I_n\) being the identity matrix of order \(n_e\). Consider the following transformation matrices

\[
T = \begin{bmatrix} \frac{F^+}{(CF)^+} \\ \frac{C}{(CF)^+} \end{bmatrix}, \quad U = \begin{bmatrix} (CF)^+ \\ (CF)^+ \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}
\]

and the transformed state and output vectors

\[
\hat{x} = Tx, \quad \hat{y} = Uy
\]

Consider the following partitions of vectors \(\hat{x}\) and \(\hat{y}\)

\[
\hat{x} = \begin{bmatrix} T_1x \\ T_2x \end{bmatrix} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}, \quad \hat{x}_1 \in \mathbb{R}^{n-m} \quad \hat{x}_2 \in \mathbb{R}^m
\]

\[
\hat{y} = \begin{bmatrix} U_1y \\ U_2y \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix}, \quad \hat{y}_1 \in \mathbb{R}^{n-m} \quad \hat{y}_2 \in \mathbb{R}^m
\]

After simple algebraic manipulations the transformed system in the new coordinates take the form:

\[
\begin{align*}
\dot{\hat{x}}_1 &= \hat{A}_{11}\hat{x}_1 + \hat{A}_{12}\hat{x}_2 + F^+Gu \\
\dot{\hat{x}}_2 &= \hat{A}_{21}\hat{x}_1 + \hat{A}_{22}\hat{x}_2 + (CF)^+CGu + \xi \\
\dot{\hat{y}}_1 &= \hat{C}_1\hat{x}_1 \\
\dot{\hat{y}}_2 &= \hat{x}_2
\end{align*}
\]

with the matrices \(\hat{A}_{11}, ..., \hat{A}_{22}\) and \(\hat{C}_1\) such that

\[
\begin{bmatrix} \hat{A}_{11} \\ \hat{A}_{21} \end{bmatrix} = TAT^{-1}, \quad \hat{C}_1 = (CF)^+CT_1
\]

It turns out that the triple \((A, C, F)\) is strongly observable if, and only if, the pair \((\hat{A}_{11}, \hat{C}_1)\) is observable (Molinarì et al. (1976); Hautus et al. (1983)). In light of the Assumption A1, this property, that can be also understood in terms of a simplified algebraic test to check the strong detectability of a matrix triple, opens the way to design stable observers for the state of the transformed dynamics (31). The peculiarity of the transformed system (31) is that \(\hat{x}_2\) constitutes a part of the transformed output vector \(\hat{y}\). Hence, the observation of the state of systems (31) can be accomplished by estimating \(\hat{x}_1\) only, whose dynamics is not affected by the unknown input vector. The observability of the \((\hat{A}_{11}, \hat{C}_1)\) permits the implementation of the following Luenberger observer for the \(\hat{x}_1\) subsystem of (31):

\[
\dot{\hat{x}}_1 = \hat{A}_{11}\hat{x}_1 + \hat{A}_{12}\hat{y}_2 + F^+Gu + L(\hat{y}_1 - \hat{C}_1\hat{x}_1)
\]

which gives rise to the error dynamics

\[
\dot{e}_1 = (\hat{A}_{11} - L\hat{C}_1)e_1, \quad e_1 = \hat{x}_1 - \hat{x}_1
\]

whose eigenvalues can be arbitrarily located by a proper selection of the matrix \(L\). Therefore, with properly chosen constant matrix \(L\) we have that \(\hat{x}_1 \to \hat{x}_1\) as \(t \to \infty\), which implies that the overall system state can be reconstructed by the following relationships

\[
\hat{x} = T^{-1}[\hat{x}_1 \; \hat{y}_2]^T
\]

Note that the convergence of \(\hat{x}_1\) to \(\hat{x}_1\) is exponential and can be made as fast as desired.

### 5.1 Reconstruction of the unknown inputs

An estimator can be designed which gives an exponentially converging estimate of the unknown input vector \(\xi\). Consider the following estimator dynamics

\[
\dot{\check{x}}_2 = \hat{A}_{21}\check{x}_1 + \hat{A}_{22}\check{y}_2 + (CF)^+CGu + v(t)
\]

with the estimator injection input \(v(t)\) yet to be specified. Let it can be found a constant \(\Xi_d\) such that

\[
|\hat{\xi}(t)| \leq \Xi_d
\]

Define the estimator sliding variable as

\[
s = \hat{x}_2 - \hat{y}_2 = \check{x}_2 - \check{x}_2
\]

By (36) and (31), the dynamics of the sliding variable \(s\) takes the form

\[
\dot{s} = f(t) - v(t), \quad f(t) = \hat{A}_{21}\check{e}_1(t) + \xi(t)
\]

Considering (34), the time derivative of the uncertain term \(f(t)\) can be evaluated as

\[
\dot{f}(t) = \hat{A}_{21}(\hat{A}_{11} - L\hat{C}_1)e_1(t) + \hat{\xi}(t)
\]

where \(e_1(t)\) is exponentially vanishing. Then, considering (37), by taking any \(\Xi > \Xi_d\), the next condition

\[
|\dot{f}(t)| \leq \Xi, \quad t > T^*, \quad T^* < \infty
\]

will be established starting from a finite time instant \(t = T^*\) on. As shown in (Levant et al. (1993)), if the injection input \(v(t)\) is designed according to

\[
v(t) = \lambda |s|^{1/2} \text{signs} + v_1; \quad \dot{v}_1(t) = \text{asign} s, \quad \alpha > \Xi, \quad \lambda > \frac{1 - \theta}{1 + \theta} \frac{\|\hat{\xi}\|}{\|\hat{\xi}\| + \alpha}, \quad \theta \in (0, 1)
\]

then the finite-time convergence to zero of, both, the sliding variable \(s\) and its time derivative \(\dot{s}\) is ensured. Therefore, condition

\[
v(t) = \xi(t) + \hat{A}_{21}(\hat{A}_{11} - L\hat{C}_1)e_1(t)
\]

holds after a finite transient time. It readily follows from the contraction property of \(e_1(t)\) that the second term

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\(^1\) A Matlab instruction for computing \(M_h = M^+\) for a generic matrix \(M\) is \(\text{null}(M)^T\).
in the right-hand side of (44) is exponentially vanishing, which implies that $|v(t) - \xi(t)| \to 0$ as $t \to \infty$ and, furthermore, that such convergence takes place exponentially. Therefore, under the condition (37), the estimator (36), (38), (42)-(43) allows one to reconstruct the unknown input vector $\xi$ acting on the original system (25).

6. CASE STUDY

We shall consider a test canal with rectangular section and three reaches in cascade having constant width of 2 m and length, respectively, of 4 km, 5 km and 2 km. The discharge coefficient is $\eta = 0.6$, the roughness is $K_s = 50m^2/m^2$, and the constant slope is $I = 0.001$. The water level in the upstream reservoir is $H_{B0} = 3m$, and in the downstream reservoir it is $H_A = 1m$. The withdrawals are $Q_{C1} = 2m^3$, $Q_{C2} = 3m^3$, $Q_{C3} = 1m^3$. The opening section of the 4-th gate is kept constant to the value $\Sigma_4 = 0.538m^2$. The uniform flow condition is characterized by the next operating points for the flow rates, level and opening section variables, respectively: $Q_1 = 6.01m^3$, $Q_2 = 4.00m^3$, $Q_3 = 1.96m^3$, $\Pi_1 = 2.40m$, $\Pi_2 = 1.72m$, $\Pi_3 = 0.99m$, $\Sigma_1 = 2.92m^2$, $\Sigma_2 = 1.85m^2$, $\Sigma_3 = 0.86m^2$. The opening flow converges to the actual one. The estimation performance for the flow variables $q_{M1}$ and $q_{M2}$ is pretty equivalent and it is not shown for brevity. The reconstruction of the unmeasured level variable $h_{A2}$ is shown in the Figure 3. The left and right plot show the transient and long term behaviour of the actual and estimated variables, respectively.

7. FLOW ESTIMATION WITH PARTIAL LEVEL MEASUREMENTS AND NO INfiltrATION (PROBLEM 1)

Consider the reduced-order linearized dynamics (24) by assuming no infiltration (i.e., $w = 0$) according to the statement of Problem 1:

$$\dot{\hat{h}} = \tilde{A}\hat{h} + \tilde{M}_\sigma \sigma + \tilde{M}_q q_M$$

Only five elements of vector $\hat{h}$ are supposed to be measured, according to the next output equation

$$\tilde{y} = \tilde{C}\hat{h}; \quad \tilde{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It is worth noting that system (46)-(47) is a special case of the general dynamics (25) with the modified notation $\hat{h} = x$, $\sigma = u$, $q_M = \xi$. It is easy to check that the matrix triplet $(\tilde{A}, \tilde{M}_q, \tilde{C})$ is strongly observable, hence the design method previously described can be applied to reconstruct, both, the unknown vector $q_M$ and the unmeasured level variable $h_{A2}$. By computing the matrices of the transformed system dynamics, it can be readily verified that $(\tilde{A}_1, \tilde{C}_1)$ is an observable pair. Therefore, it can be implemented the suggested scheme (33),(36), (42)-(43), with the observer gain matrix

$$\tilde{L} = 10^{-3} \begin{bmatrix} -1.7 & 2.1 \\ -0.6 & -3.0 \\ -3.3 & -0.9 \end{bmatrix}$$

that has been computed in order to assign the observation error matrix $\tilde{A}_1 - \tilde{L}\tilde{C}_1$ the desired spectrum of eigenvalues $[-0.05, -0.05, -0.005]$. The gain parameters $\alpha$ and $\lambda$ of the unknown input reconstruction algorithm are set as $\alpha = 1.5\sqrt{3}, \lambda = 5$. The performance of the observer is tested by means of simulations made in the Matlab-Simulink environment. The system and the observers are integrated by fixed step Runge-Kutta method, with the integration step $T_s = 10^{-4}s$. The system’s initial conditions are: $\tilde{h}(0) = [0, 0, 0, 0, 0]$. All the observer’s initial conditions are set to zero. For simulation purposes the actual $Q_M$ profiles are generated by solving the corresponding system of nonlinear differential equations (22), with the initial conditions $Q_M(0) = [6.01, 4.00, 1.96]$. The next Figures 2 show the actual and estimated profiles of the unknown flow variable $q_{M1}$ during the TEST 1, of duration 500 seconds. The left and right plot show the transient and long term behaviour. After a transient of about twenty seconds, the estimated flow converges towards the actual one. The estimation performance for the flow variables $q_{M2}$ and $q_{M3}$ is pretty equivalent and it is not shown for brevity. The reconstruction of the unmeasured level variable $h_{A2}$ in the TEST 1

8. FLOW AND INFILTRATION ESTIMATION WITH FULL LEVEL MEASUREMENTS (PROBLEM 2)

We consider here the full-order model (21), and we assume the availability for measurement of the entire state vector $h \in \mathbb{R}^{3n} \equiv \mathbb{R}^9$, i.e. the considered output is $y = h$. 

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Fig. 2. Actual and estimated flow variable $q_{M1}$ in the TEST 1

Fig. 3. Actual and estimated level variable $h_{A2}$ in the TEST 1
The problem here is to reconstruct after a finite-time observation transient the unknown input terms, namely the discharge $q_M$ in the middle of the channels and the infiltration vector $w$. System (21) along with the considered output equation $y = h$ can be rewritten as

$$\dot{h} = Ah + M_\sigma \sigma + [M_q \ M_w] \cdot [q_M \ w]^T$$

$$y = Ch$$

(49)

which belongs to the class of dynamics (25) with the modified notation $h = x$, $\sigma = u$, $[q_M^T \ w]^T = \xi$, $M_\sigma = G$ and $[M_q \ M_w] = F$, and with the output transformation matrix $C = I$ ($I$ being the identity matrix of 9 - th order). It is easy to check that the matrix triplet $(A, [M_q \ M_w], I)$ is strongly observable. Since the state vector is supposed to be fully available, a simplified version of the design methodology previously described can be applied to reconstruct the unknown vectors $q_M$ and $w$. Since the state vector is already available, only the observer (38)-(43) for reconstructing the unknown input is necessary. The gain parameters $\alpha$ and $\lambda$ of the observation algorithm are set as $\alpha = 1.5\sqrt{5}, \lambda = 5$. The performance of the observer are tested by means of simulations. For simulation purposes the actual $Q_M$ profiles are generated by solving the corresponding system of nonlinear differential equations (22), with the initial conditions $Q_M(0) = [6.01, 4.00, 1.96]$ The next Figures 4 show the actual and estimated profiles of the unknown flow variable $q_{M2}$ during the TEST 2, of duration 100 seconds. The left and right plot show the transient and long term behaviour. After a transient of about half a second, the estimated flow converges towards the actual one. The estimation performance for the flow variables $q_{M1}$ and $q_{M3}$ is pretty equivalent and it is not shown for brevity. The reconstruction of of the unknown infiltration variable $w_3$ is shown in the Figure 5. The left and right plot show the transient and long term behaviour, which show a satisfactory behaviour according to the presented analysis results.

**Fig. 4.** Actual and estimated flow variable $q_{M2}$ in the TEST 2

**Fig. 5.** Actual and estimated infiltration variable $w_3$ in the TEST 2

9. CONCLUSIONS

A linear UIO and a nonlinear sliding mode disturbance observer (DO) have been combined to reconstruct water level, discharge and infiltration variables in open channel irrigation canals connected in cascade and subject to unknown time varying infiltrations. The underlying, collocation based, nonlinear dynamics are linearized around the subcritical uniform flow condition. The UIO and DO design procedures are constructively illustrated along the paper. Simulation results using realistic data are discussed to verify the effectiveness of the proposed schemes. The decentralizedization of the scheme (e.g. by consensus-based methodologies), and/or its use to address observer-based controller design problems, could be interesting tasks for future research efforts.

REFERENCES


