Online Parameter Estimation and Compensation of Preisach Hysteresis by SVD updating

L. Lei * K. K. Tan ** S. Huang *** T. H. Lee ****

* NUS Graduate School for Integrative Sciences and Engineering, National University of Singapore, 28 Medical Drive, 117456, Singapore (e-mail: leiliu@nus.edu.sg).
** Corresponding author, Department of Electrical and Computer Engineering, National University of Singapore, 117576, Singapore, (e-mail: kktan@nus.edu.sg)
*** Department of Electrical and Computer Engineering, National University of Singapore, 117576, Singapore, (e-mail: elehsn@nus.edu.sg)
**** Department of Electrical and Computer Engineering, National University of Singapore, 117576, Singapore, (e-mail: elelethe@nus.edu.sg)

Abstract: Preisach model can be used to describe the hysteresis in smart actuators. In this paper, online parameter identification by singular value decomposition (SVD) updating is presented for Preisach hysteresis. Firstly, SVD in least squares sense is used to obtain the pseudo inverse. However, it is time-consuming to compute singular values for large matrix, and PE condition is not always satisfied. Secondly, the SVD of large matrix is transformed to low order thin matrix, then, based on the initial result of offline SVD, online SVD updating is given by rotating basis which needs to be revised when new data is large. Finally, inverse Preisach feedforward is used to compensate the hysteresis.

Keywords: Preisach hysteresis, least squares, singular value decomposition, online updating, inverse Preisach.

1. INTRODUCTION

Nanometer (nm) scale is required in some precision engineering. The position errors and drift need to be controlled less than 0.1 nm for sensitive instruments. Smart materials such as piezoceramic materials, SMA materials, magnetostrictive materials can be used. However, hysteresis is one of the main nonlinearities which affects control accuracy. In the open loop, the maximum error from hysteresis might be 10-15% of the total displacement of smart actuators (Ge (1996)). Hysteresis Models can be classified into physical models and mathematical models (Mayergozy (2003); Davino (2004); Visintin (1994); Janaiden (2008)). The physical model is built using physical laws applied to the phenomenon of hysteresis and thus this model is intuitive to understanding, but it is typically in a complex form which is difficult to be identified and used for control purposes. Conversely, the mathematical model represent an input-output relationship of the actual system and it is usually more amenable to practical use for identification and control. In the current literature, Preisach mathematical model is frequently used to describe hysteresis in smart materials, see Mayergozy (2003).

Approximate Preisach density functions can be identified through a discretized Preisach plane by transforming the double integral of density functions to a numerical summation. Song (2005) and Mayergozy (2003) identified Preisach density functions by differentiating the measurements. However, the identified functions may be sensitive to measurement noise. Hu (2003) and Ge (1996) studied the tracking control of PZT actuators using an offline computation of the difference of measured data. Iyer (2005) and Tan (2005) developed recursive schemes for parameter identification and designed a closet match algorithm for the compensation of the Preisach hysteresis. Henze (2002) presented some approaches for identification of the Preisach function based on different distribution characteristics, but these approaches rely on assumptions of the form of the density functions and they are typically not amenable to be used for the purpose of producing a model for hysteresis compensation and control.

In order to get an accurate and smooth approximation of the continuous density functions from a discretized plane, a large number of the split lattices will be needed, then a large number of model parameters is to be determined. This needs sufficiently exciting data to satisfy a persistent excitation (PE) condition for parameter identification, see Tan (2005) and Astrom (1994). With limited data, this issue is equivalent to solving an ill-conditioned inverse problem of an large singular matrix (Aster (2005); Lebedev (2002)), as shown in equation (1)

\[ Ax = y \]

where \( x \) and \( y \) belongs to normed linear spaces, and \( A \) is a deficient matrix mapping \( x \) to \( y \). There will be infinite solutions in the least squares sense to equation (1). However, a solution can be computed (Aster (2005); Lebedev (2002); Golub (1996)), when a norm objective is set to minimize \( \| Ax - y \|_2 \) and \( \| x \| \). Brand (2003) provided online singular value decomposition (SVD) revision for lightweight recommender systems. This idea is expanded to estimate the density function in this manuscript.

In this paper, Preisach hysteresis property is given, then its density function is identified in least squares sense using SVD,
moreover, online SVD updating is used to estimate the Preisach parameter, finally, feedforward compensation is presented to linearize the Preisach hysteresis.

2. PREISACH HYSTERESIS AND ITS PROPERTY

2.1 Preisach modeling

The basic element of hysteresis, i.e. hysteresis relay, is as shown in Figure 1(left), where the output operator \( \gamma_{\alpha\beta} \) can be assumed to be \( \theta_1 \) and \( \theta_2 \). The switching threshold values of the hysteresis operator are \( \alpha \) and \( \beta \). The ascending branch \( bce \) is given with monotonically increasing input, while the descending branch \( dfa \) is traced with monotonically decreasing input. The past and present evolution in the input signal can be captured in a limiting region on the Preisach plane, as shown in the Figure 1(right). Thus the Preisach plane can be used to graphically and efficiently yield the hysteresis output, i.e., the voltage-to-displacement relationship, which is given in equation (2)

\[
f(t) = \int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta}[u(t)] d\alpha d\beta,
\]

where \( \mu(\alpha, \beta) \) is the distributed density function and \( f(t) \) is the displacement of smart actuators. In the Preisach model, the input \( u(t) \) is applied to all the hysteresis operators \( \gamma_{\alpha\beta}[u(t)] \), then the output operators are weighted by density functions \( \mu(\alpha, \beta) \), and summed continuously over possible values of \( \alpha \) and \( \beta \). The hysteresis output can thus be considered as the superposition of a continuous set of relay operators over the input range. Let \( S \) represent the Preisach triangle formed by \( \alpha \geq \beta \) and the saturation value of input voltages. \( S \) can be divided to \( S^\pm \) with \( \gamma_{\alpha\beta}[u(t)] = \theta_1 \) and \( S^- \) with \( \gamma_{\alpha\beta}[u(t)] = \theta_2 \), as shown in Figure 2. Thus the equation (2) can be written as equation (3)

\[
f(t) = \int_{S^+} \mu(\alpha, \beta) \gamma_{\alpha\beta}[u(t)] d\alpha d\beta + \int_{S^-} \mu(\alpha, \beta) \gamma_{\alpha\beta}[u(t)] d\alpha d\beta
\]

\[
= \theta_1 \int_{S^+} \mu(\alpha, \beta) d\alpha d\beta + \theta_2 \int_{S^-} \mu(\alpha, \beta) d\alpha d\beta
\]

\[
= (\theta_1 - \theta_2) \int_{S^+} \mu(\alpha, \beta) d\alpha d\beta + c \theta_2
\]

where \( c = \int_{S^-} \mu(\alpha, \beta) d\alpha d\beta \) is a constant. Without loss of generality, assume \( \theta_1 = 1 \), \( \theta_2 = 0 \), thus the equation (3) can be simplified as equation (4), which is a weighted computation of the shadowing area in limiting region, where \( \theta_1 \) and \( \theta_2 \) can be absorbed in the density function \( \mu(\alpha, \beta) \).

\[
f(t) = \int_{S^+} \mu(\alpha, \beta) d\alpha d\beta
\]

2.2 Preisach hysteresis loop

Let \( u_{\min} \) and \( u_{\max} \) be the minimum and maximum values of the input to smart actuators. Then the double integration in equation (4) performed over the range \( u_{\min} \leq \beta \leq \alpha \leq u_{\max} \). The minimum and maximum input values, the switching condition \( \alpha \geq \beta \) geometrically forms a limiting triangle on the Preisach plane where each pair of \( (\alpha, \beta) \) on the plane defines a unique operator \( \gamma_{\alpha\beta}[u(t)] \). The limiting triangle corresponds to the hysteresis major loop, moreover, the Preisach density function \( \mu(\alpha, \beta) \) is defined to be zero outside the triangle. Each changing in the input signal will shape the limiting region in the Preisach plane accordingly.

One input signal is given, as in Figure 2(left). As the input signal is monotonically increased to a value \( M_1 \), all the hysteresis operators \( \gamma_{\alpha\beta}[u(t)] \) with switching values less than \( M_1 \) will be activated, i.e. their \( \gamma_{\alpha\beta}[u(t)] \) are equal to 1. Conversely, the input signal is monotonically decreased from \( M_1 \) to \( m_1 \), the \( \gamma_{\alpha\beta}[u(t)] \) with switching values larger than \( m_1 \) is equal to 0. Geometrically, this corresponds to a division of the limiting triangle into two regions: \( S^+(t) \) with \( \gamma_{\alpha\beta}[u(t)] = 1 \) wherein the hysteresis operators are activated and \( S^-(t) \) with \( \gamma_{\alpha\beta}[u(t)] = 0 \) wherein the hysteresis operators are deactivated. The interface link \( L(t) \) between the \( S^+(t) \) and \( S^-(t) \) is a horizontal line given by the line equation \( \alpha = M_1 \), as shown in Figure 2. Let \( \alpha_{\min} = 10 \), \( \alpha_{\max} = 0 \), \( \beta_{\max} = 10 \), \( \beta_{\min} = 0 \), \( \mu(\alpha, \beta) = 4 \), (i.e., uniform lattice density). Consider an input signal as shown in Figure 3(left), the formed Preisach loop is shown in Figure 3(right), where memory can be found, i.e. the hysteresis output depends on not only the current input, but also the past input.

Fig. 1. Preisach operator and plane

Fig. 2. Input signal and the corresponding limiting region in the Preisach plane

Fig. 3. Preisach hysteresis loop
3. IDENTIFICATION OF PREISACH HYSTERESIS

The identification of Preisach model essentially entails the estimation of density functions. In the continuous form, the density functions \( \mu(\alpha, \beta) \) are continuous over the limiting region and it is thus difficult to identify continuous \( \mu(\alpha, \beta) \). In this paper, the limiting region will be considered to be comprising of discrete lattices. Each lattice cell may have a weight assigned to it which is the discrete equivalence of a specific lattice density. The area of lattice \((i, j)\) is \( s_{ij}\), then the hysteresis output can be computed by transforming the double integral to a numerical summation as shown in equation (5) which is also linear-in-parameter and suitable for adaptive estimation and control

\[
\int s_{ij} \mu(\alpha, \beta) d\alpha d\beta = \sum \sum \mu_{ij}v_{ij}
\]

where \(\mu_{ij}\) and \(v_{ij}\) are the density value and area of the lattice \((i, j)\), respectively.

3.1 Least squares estimation via SVD

The Preisach plane can be discretized into \( L \times L \) lattices. Thus, there are \( L^2/2 \) lattices to be identified in the Preisach triangle. \( f(i) \) can be rewritten as equation (6)

\[
\Sigma \mu_{ij}v_{ij} = R^TV,
\]

where regression vector \( R^T = [y_{11} \quad y_{12} \ldots \quad y_{1L} \quad y_{22} \ldots \quad y_{LL} \ldots \ldots] \).

Then, a measure of estimation error of the densities is defined as shown in equation (7). However, equation (8) represent the norm of \( x \) which is a vector formed by all the density values in lattice \((i, j)\), and the relative estimation error can be represented as \(\|x - \hat{x}\|/\|x\|\).

\[
\|x - \hat{x}\| = \sqrt{(\mu_{11} - \hat{\mu}_{11})^2 + (\mu_{21} - \hat{\mu}_{21})^2 + \cdots + (\mu_{L(L+1)/2} - \hat{\mu}_{L(L+1)/2})^2}
\]

\[
\|x\| = \sqrt{\mu_{11}^2 + \mu_{21}^2 + \cdots + \mu_{L(L+1)/2}^2}
\]

Uniform convergence may not be observed in the parameters due to the PE problem associated with the large number of parameters and the limited frequencies in the input signal. This PE condition is mathematically equivalent to an ill-conditioned matrix which can be addressed by incorporating singular value decomposition (SVD) in least squares estimation. Over a time range \( t_0 < t < t_N \), data samples are collected at time instances \( t_0, t_1, \ldots, t_n, \ldots, t_N \). With \( N \) samplings, equation (6) can be produced and rearranged in the following matrix equation (9).

\[
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1L} & \cdots & Y_{LL} \\
Y_{12} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Y_{1L} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
Y_{LL} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}\begin{bmatrix}
X_{11} \\
X_{12} \\
\vdots \\
X_{1L} \\
\vdots \\
X_{LL} \\
\end{bmatrix} = \begin{bmatrix}
F_{0} \\
F_{1} \\
\vdots \\
F_{N} \\
\vdots \\
F_{N'} \\
\end{bmatrix}
\]

Let

\[
A = \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1L} & \cdots & Y_{LL} \\
Y_{12} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Y_{1L} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
Y_{LL} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

Equation (9) can be reduced to equation (1), and its dimension is \( L(L + 1)/2 \times (N + 1) \). Let \( \hat{x} \) be the estimate of \( x \) and \( E \) be the error between \( A\hat{x} \) and \( y \). If \( M = A^TA \) is non-singular, the solution of equation (1) is unique. It is shown in theorem 1.

Theorem 1. (Least squares estimation) Astrom (1994): The norm of error \( \|A\hat{x} - y\| \) can be minimal for parameter \( x \), also the minimum is unique and given by equation (10)

\[
\hat{x} = (A^TA)^{-1}A^Ty
\]

If \( A^TA \) is singular, i.e. \( \det(A^TA) = 0 \), the PE condition is not satisfied which is likely to occur in this application of Preisach modeling. Moreover, noise can exist in the data, which can lead to small nonzero singular values. In this case, SVD can be used to obtain the pseudo-inverse of \( A^TA \). The SVD of \( A^TA \) is shown in equation (11).

\[
M = USV^T
\]

where \( U = [u_1 \quad u_2 \ldots \quad u_n] \), \( V = [v_1 \quad v_2 \cdots \quad v_n] \), \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r) \), \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \). In this case, SVD can be used to obtain the pseudo-inverse of \( A^TA \). The SVD of \( A^TA \) is shown in equation (11).

\[
M = USV^T
\]

Small singular values can be truncated in order to yield improved least squares estimation in an ill-condition situation. However, too large truncation value can increase the estimation error. The approximation of pseudo inverse of \( M \) is given by equation (14).

\[
M = \sum_{r=1}^{N} \sigma_{r}u_{r}v_{r}^{T}
\]

Small singular values can be truncated in order to yield improved least squares estimation in an ill-condition situation. However, too large truncation value can increase the estimation error. The approximation of pseudo inverse of \( M \) is given by equation (14).

\[
M^+ = \sum_{r=1}^{N} \frac{1}{\sigma r} u_{r}v_{r}^{T}
\]

Then, we can obtain the estimation of \( x \) in least squares sense

\[
\hat{x} = M^+A^Ty
\]
Estimation by singular value decomposition in least squares sense is time-consuming, for example the SVD of matrix $M$ is about 20 seconds in above section. Bunch (1978) provided some methods of SVD revision and updating. Brand (2003) applied rank-1 modifications to the movie recommender systems. In this section, the identification of Preisach hysteresis is obtained using SVD updating, based on the initial values $A_0$, $y_0$ in section 3.1. The corresponding equation is shown in (17).

$$A_0 x_0 = y_0$$

If new data is added to $A_0$ and $y_0$, the SVD can be modified step by step by rotating $U_r$ and $V_r$, which is a method to revise SVD online. Let $M_0 = A_0 \Sigma_0 A_0^T$, $y_0 = A_0^T y_0$, then equation (18) can be given.

$$M_0 x_0 = y_0$$

Similarly as above section, the least squares estimation by SVD can obtained in equation (19).

$$x_0 = M_0^+ y_0$$

When a new vector $a_1$ and $y_1$ corresponding output value added to the matrix, this can be represented in equation (20).

$$\begin{bmatrix} A_0^T & 0 \\ A_1^T \\ \end{bmatrix} \begin{bmatrix} A_0 \\ a_1 \\ \end{bmatrix} x_1 = \begin{bmatrix} A_0^T y_0 \\ a_1^T y_1 \\ \end{bmatrix}$$

where $y_0$ is a vector, $y_1$ is a scalar.

Equation (20) can be rewritten as equation (21).

$$(A_0^T A_0 + a_1^T a_1) x_1 = A_0^T b_0 + a_1^T y_1$$

Let $M_1$ represent $A_0^T A_0 + a_1^T a_1$, equation (21) can be transformed to equation (22).

$$M_1 x_1 = B_0 + a_1^T y_1$$

Then, the parameter estimation with new data is represented in equation (23).

$$x_1 = M_1^+ (B_0 + a_1 y_1)$$

The approximation of $M_1^+$ is shown below. Assume the rank of matrix $M_1$ is $k$. Using the analysis in section 3.1, the initial estimation obtained offline is shown in equation (24).

$$M_0 = U_0 \Sigma_0 V_0^T = U_0 \Sigma_0 V_0^T + \varepsilon_r$$

where $\varepsilon_r$ is the residual error.

Then $M_1$ can be written in equation (25).

$$M_1 = M_0 + a_1 a_1^T$$

Neglecting $\varepsilon_r$, $M_1$ can be reformulated as equation (26).

$$M_1 = \begin{bmatrix} U_r & a_1 \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} V_r & a_1 \end{bmatrix}^T$$

The component of $a_1$ orthogonal to the space spanned by $U_r$ and $V_r$ is shown in equation (27) and (28), respectively.

$$p = a_1 - U_r U_r^T a_1$$

$$q = a_1 - V_r V_r^T a_1$$

Let $R_U = \|p\|$ and $R_V = \|q\|$, then unit vector $u_p$ and $v_q$ can be shown in equation (29) and (30).

$$u_p = p / R_U$$

$$v_q = q / R_V$$

According to the method provided by Brand (2003), $[ U_r \ a_1 ]$ and $[ V_r \ a_1 ]$ can be written in equation (31) and (32).

$$[ U_r \ a_1 ] = \begin{bmatrix} U_r & u_p \\ 0 & R_U \end{bmatrix}$$

$$[ V_r \ a_1 ] = \begin{bmatrix} V_r & v_q \\ 0 & R_V \end{bmatrix}$$

Then $M_1$ can be written in equation (33).

$$M_1 = \begin{bmatrix} U_r & u_p \\ 0 & R_U \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & R_U \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I & V_r \\ 0 & R_V \end{bmatrix} \begin{bmatrix} U_r & v_q \\ 0 & R_V \end{bmatrix}$$

Let $K = \begin{bmatrix} I & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I & V_r \\ 0 & R \end{bmatrix}$. Then $M_1$ can be given as equation (34).

$$M_1 = \begin{bmatrix} U_r & u_p \end{bmatrix} K \begin{bmatrix} V_r & v_q \end{bmatrix}$$

Singular value decomposition of $M_1^+$ is transformed to the SVD of $K$ which is $(r+1) \times (r+1)$, as is shown in equation (35). SVD of small matrix $K$ can save computing time compared with SVD of large matrix $M_1$.

$$K = U_k \Sigma_k V_k^T$$

where $U_k V_k^T = U_k^T U_k = I$, and $V_k V_k^T = V_k^T V_k = I$

Then, the approximation SVD of $M_1$ can be shown in equation (36).

$$M_1 = U_k \Sigma_k \begin{bmatrix} V_r & v_q \end{bmatrix} \begin{bmatrix} V_k \end{bmatrix}$$

Let $U_1$ be the $1 : r$ columns of $U_r u_p$ and $U_k$. $V_1$ is the $1 : r$ columns of $V_r v_q$. $\Sigma_1 = \Sigma_k (1 : r, 1 : r)$. This is the SVD updating with fixed rank.
With the identification result obtained via the possible approaches presented in above section, a hysteresis compensator based on inverse Preisach can be designed as shown in Figure 7, where the estimator is used to identify the Preisach density functions. Using the input and output data, the estimator will carry out the parameter identification and feed the estimate result as well as the Preisach memory curve. The feedforward compensator will compute the inverse Preisach based on these information and enforce the plant output to track the desired trajectory, thereby compensating the effects of the hysteresis.

**Algorithm 1.** Feedforward compensation algorithm by inverse Preisach

If \( f(k) < f_i(k) \)

\[
\begin{align*}
&u(k+1) = u(k) + i\Delta, \quad i = 1, \ldots, m \\
&f(k+1) = f(u(k+1))
\end{align*}
\]

break;

end

else if \( f(k) > f_i(k) \)

\[
\begin{align*}
&u(k+1) = u(k) - j\Delta, \quad j = 1, \ldots, n \\
&f(k+1) = f(u(k+1))
\end{align*}
\]

break;

end

else \( u(k+1) = u(k) \)

end

At time instant \( k \), we can define the reference input as \( f_i(k) \), hysteresis output as \( f(k) \), control action as \( u(k) \), and relay output as \( \gamma(k) \). The inverse Preisach works to obtain \( u(k+1) \) based on identified Preisach model \( \Psi \) and estimated parameters of \( \psi(k) \), enforcing \( f(k+1) \) to track \( f_i(k) \). The process can be represented using the following pseudo code algorithm 1.

The iteration step \( \Delta \) is given in equation (39).

\[
\Delta = \frac{\lambda u_{\text{max}}}{L}
\]  

(39)

where \( \lambda \) is the factor to regulate the step which is less than 1, and \( L \) is the discretization level of the Preisach plane. The control action \( u(k+1) \) is computed to enforce \( f(k+1) \) to track the reference signal \( f_i(k) \).

According to the identification result in above section, the feedforward compensation is presented to the Preisach hysteresis using inverse Preisach as shown in algorithm 1, where \( m=40, n=40, L = 50, \lambda = 1, u_{\text{max}} = 10 \). The performance is shown in Figure 8. It can be seen that the hysteresis output can track reference signal with feedforward compensation.

## 5. CONCLUSION

Hysteresis in smart actuators can be described using Preisach model which shows memory characteristics. The Preisach plane is discretized and SVD method is used to estimate the parameter in least squares sense. In order to fast and online compute the SVD with new input data, SVD updating is given. The SVD of large dimension matrix is transformed to the SVD of small dimension matrix, which saves computing time. Finally, one feedforward compensation is provided, and the hysteresis output is enforced to track the reference signal.

**REFERENCES**

Fig. 8. Feedforward compensation result


