An Optimal Servomechanism Controller for SISO Positive Systems

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Abstract: An optimal linear-time quadratic clamping regulator (LTQcR) for open-loop stable SISO LTI positive systems is introduced in this paper and tested on controlling an industrial hydraulic system. It is shown that excellent tracking, robustness and disturbance rejection of unmeasurable nonnegative constant disturbances is obtained.

Keywords: positive systems; experimental positive control; robust tracking and disturbance rejection; switching control, servomechanism problem

1. INTRODUCTION

This paper introduces an optimal linear-time-quadrate clamping regulator (LTQcR) to solve the robust servomechanism problem for SISO stable positive systems. In particular, the effectiveness and robustness of the controller for the tracking problem of nonnegative constant tracking signals with unmeasurable nonnegative constant disturbances will be illustrated by applying the controller to an experimental industrial hydraulic system.

We note that studies of controlling hydraulic systems have been previously carried out, e.g. Astrom & Östberg [1986], Dai & Astrom [1999], Johansson & Nørgaard [1998], Gatzke et al. [2000], Vadigepalli et al. [2001], and references therein. Although the latter citations considered control strategies ranging from PI control to “inner-outer” factorization-based multivariable internal model control to µ-analysis-based H∞ control and some decentralized methods, none of the studies have looked at the industrial hydraulic system from the view point of positive systems.

Positive systems carry the property of having all the state variables being nonnegative for all time. These systems are of great practical importance, as the nonnegative property occurs quite frequently in industrial applications and in nature, e.g. in a tank-like system one cannot imagine having a negative amount of liquid being present in the tank. Over several decades, positive systems have garnered great interest. Numerous results on positive systems can be found, e.g. on reachability and controllability, on realization of positive systems, on 2D positive linear systems, on stability control, pole-assignment, positive observer design and some general stability feedback control, and others. We refer the interested reader to Roszak & Davison [2010], which presents numerous citations on the latter topics.

The paper is organized as follows. Background and preliminaries are given first, where the terminology, and a brief description of positive systems and compartmental systems is introduced. The main results of the paper are then outlined; in particular, the system of interest is defined and the problem statement is given and studied. The main experimental results, simulations, and discussions of the paper are described in Section 4, while all concluding remarks complete the paper.

2. BACKGROUND AND PRELIMINARIES

Let the set \( \mathbb{R}_+ := \{ x \in \mathbb{R} \mid x \geq 0 \} \), the set \( \mathbb{R}^n_+ := \{ x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_i \in \mathbb{R}_+, \forall i = 1, \ldots, n \} \).

A matrix \( A \in \mathbb{R}^{n \times n} \) is Hurwitz or stable when all the eigenvalues \((\lambda)\) of \( A \) are in the open left half plane of the complex plane \( \mathbb{C} \), i.e. the real part of all eigenvalues is negative. A nonnegative matrix \( A \) has all of its entries greater or equal to 0, i.e. \( a_{ij} \in \mathbb{R}_+ \). A Metzler matrix \( A \) is a matrix for which all off-diagonal elements of \( A \) are nonnegative, i.e. \( a_{ij} \in \mathbb{R}_+ \) for all \( i \neq j \). A compartmental matrix \( A \) is a matrix that is Metzler, where the sum of the components within a column is less than or equal to zero, i.e. \( \sum_{i=1}^{n} a_{ij} \leq 0 \) for all \( j = 1, 2, \ldots, n \).

A positive linear system in the traditional sense Farina & Rinaldi [2000] is defined next.

Definition 1. A linear system

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

where \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( C \in \mathbb{R}^{r \times n} \), and \( D \in \mathbb{R}^{r \times m} \) is considered to be a positive linear system if for every nonnegative initial state and for every nonnegative input the state of the system and the output remain nonnegative.

Notice that Definition 1 states that the input to the system must be nonnegative, a restriction that we will abide to throughout this paper and illustrate via experimentation. It turns out that Definition 1 has a very nice interpretation in terms of the matrix quadruple \((A, B, C, D)\).
An interesting subset of positive systems is that of compartmental systems. The main mathematical distinction, for LTI systems, between a positive system and a compartmental system is that a positive system’s $A$ matrix is Metzler, while a compartmental system’s $A$ matrix is compartmental. The inclusion of compartmental systems is made because in general, compartmental systems are stable, a property of great significance throughout the paper and in our experimental setup. For a more complete study and interesting results on compartmental systems see Jacquez & Simon [1993] and references therein.

3. LQc-R CONTROL

In this section, we consider the servomechanism problem for positive LTI systems under nonnegative control. Previous work on this problem has been covered in Roszak & Davison [2010], Roszak & Davison [2009a] for multivariable LTI positive systems using “tuning regulators” and special linear quadratic controllers, respectively. In this paper we deal with strictly optimal state feedback servomechanism controllers for positive systems with restricted control bounds, which allow much faster and improved response.

Consider the following positive system plant:

$$
\begin{align*}
\dot{x} &= Ax + bu + e_\omega \omega \\
y &= cx + du + f_\omega \\
e &= y - y_{ref}
\end{align*}
$$

where $A$ is an $n \times n$ Metzler stable matrix, $b \in \mathbb{R}_+^n$, $c \in \mathbb{R}_+^{1 \times n}$, $d \in \mathbb{R}_+$, $e_\omega \in \mathbb{R}_+^n$, $f \in \mathbb{R}_+$; the signal $y_{ref} \in Y_{ref} \subset \mathbb{R}_+$ is a constant, as is $\omega \in \Omega \subset \mathbb{R}_+$. The sets $Y_{ref}$ and $\Omega$ are defined as in Assumption 1 below.

We now outline the problem of interest which we refer to as the positive servomechanism problem.

Find an LTI controller connected as in the diagram (Figure 1) such that the closed-loop system satisfies

(a) asymptotic stability in the sense of Lyapunov with respect to the origin, and for every $y_{ref} \in Y_{ref}$, $\omega \in \Omega$, with the initial condition of the controller $x_0(0)$ having the property $u(0) \in \mathbb{R}_+$

(b) the states $x(t) \geq 0$, output $y(t) \geq 0$, and initial condition $x(0) \in \mathbb{R}_+$

(c) tracking of the reference signal occurs, i.e. $e(t) = y(t) - y_{ref} \to 0$, as $t \to \infty$. In addition, (d) assume that a controller has been found so that conditions (a), (b), (c) are satisfied; then for all perturbations of the nominal plant model which maintain properties (a) and (b), it is desired that the controller can still achieve asymptotic tracking and regulation, i.e. the controller is robust and property (c) still holds.

Existence Results Roszak & Davison [2010] A necessary condition for a solution to exist to Problem 3 is that:

(i) $d - cA^{-1}b \neq 0$, and

(ii) for a given $y_{ref} \in Y_{ref}$ and given $\omega \in \Omega$ the following steady-state control signal condition should hold: $0 < u_{ss} < \bar{\pi}$, where

$$
\begin{align*}
u_{ss} := \frac{cA^{-1}e_\omega - f_\omega + y_{ref}}{d - cA^{-1}b}
\end{align*}
$$

Next, we provide a supplementary result 1.

Lemma 4. Consider the system matrices of the plant (2). If the necessary condition $d - cA^{-1}b \neq 0$ then this implies that:

$$
\begin{align*}
d - cA^{-1}b > 0.
\end{align*}
$$

Moreover,

$$
\begin{align*}
f - cA^{-1}e_\omega \geq 0.
\end{align*}
$$

Our key assumption, which is a necessary condition for a solution to exist to Problem 3 is presented next.

Assumption 1. Given the plant (2) assume that the necessary condition $d - cA^{-1}b \neq 0$ holds, which implies from Lemma 4 that $d - cA^{-1}b > 0$. Also, assume the sets $\Omega$ and $Y_{ref}$ are chosen such that for all $y_{ref} \in Y_{ref}$ and all $\omega \in \Omega$, the steady state of the input (3) has the property $0 < u_{ss} < \bar{\pi}$, where $\bar{\pi} \in \mathbb{R}_+ \setminus \{0\}$.

The latter assumption can also be written in the following way:

$$
\begin{align*}
0 < \frac{y_{ref} - (f - cA^{-1}e_\omega)\omega}{d - cA^{-1}b} < \bar{\pi}.
\end{align*}
$$

Remark 5. The last inequality (4) brings out numerous answers regarding the positive steady-state assumption on the input presented in Assumption 1; namely,

(a) if no disturbances are present then the steady-state $u_{ss}$ assumption holds for all $y_{ref}$ with the property that $0 < \frac{y_{ref}}{d - cA^{-1}b} < \bar{\pi}$.

(b) if the tracking signal is omitted and only positive disturbances are considered, then the assumption on $u_{ss}$ will not hold, i.e. if $y_{ref} = 0$ and $\omega \neq 0$, then $u_{ss} < 0$ and the steady-state assumption will not be satisfied.

(c) in the case of unmeasurable/measurable disturbances, we can also deduce that if $\bar{\pi} = \infty$ and the disturbances are small in comparison to the tracking signal, i.e. if $y_{ref} > (f - cA^{-1}e_\omega)\omega$, then the assumption on $u_{ss}$ will hold true.

With the above plant and assumption stated, we outline the problem of interest, which we refer to as the positive servomechanism problem.

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1. Note: due to space limitations, proofs have been removed from the paper.

2. We are of course assuming that $f - cA^{-1}e_\omega \neq 0$.
3.1 Servomechanism Problem: LTQR approach

In this section, the solution to Problem 3 under the LTQR controller is presented. The Linear Tuning Quadratic Regulator (LTQR) of interest in this section and throughout this paper is defined next.

The LTQR controller is given by:

\[
\dot{y} = y - y_{ref} \\
u = [K_x \ K_\eta] \begin{bmatrix} x \\ \eta \end{bmatrix}, \quad 0 < u(0) < \overline{u}
\]  

(5)

where \(u(0)\) is fixed and where \(K_x \in \mathbb{R}^{1 \times n}\) and \(K_\eta \in \mathbb{R}\) are found by solving the expensive control problem Davison & Scherzinger [1987], Davison & Davison [2002]:

\[
\int_0^\infty \eta^T \eta + \rho^2 u^T u \, d\tau
\]

where \(\rho > 0\), for the system:

\[
\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A & 0 \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} u
\]

(7)

\[
\eta = [0 \ 1] \begin{bmatrix} x \\ \eta \end{bmatrix}
\]

(8)

with \(y_{ref} = 0\) and \(\omega = 0\).

For convenience, since we will be interested in letting \(\rho \to \infty\), we re-write (6) as

\[
\int_0^\infty \epsilon^2 \eta^T \eta + u^T u \, d\tau
\]

where \(\epsilon > 0\). In this case, since it has been assumed that \(A\) is stable and that \(d - cA^{-1}b \neq 0\), it directly follows that the system (7) has the property that

\[
\text{rank}(\begin{bmatrix} A & b \\ c & d \end{bmatrix}) = n + 1
\]

which implies it is stabilizable, and by inspection, (7) is detectable. This implies that there exists a unique stabilizing optimal state feedback controller for (7) given by:

\[
u = [K_x \ K_\eta] \begin{bmatrix} x \\ \eta \end{bmatrix}.
\]

The latter control law can also be presented in a slightly different fashion, where \(y_{ref} \neq 0\) and \(\omega \neq 0\), which yields the same gain matrix as that of the control strategy above. Thus, we can replace the latter with

\[
\int_0^\infty \epsilon^T \epsilon + \rho^2 u^T u \, d\tau
\]

(10)

where \(\rho > 0\), or

\[
\int_0^\infty \epsilon^2 \epsilon^T + \hat{u}^T \hat{u} \, d\tau,
\]

(11)

where \(\epsilon > 0\) for the system:

\[
\begin{bmatrix} \dot{x} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} A & 0 \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ \epsilon \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \hat{u} \\
\epsilon = [0 \ 1] \begin{bmatrix} \dot{x} \\ \epsilon \end{bmatrix}
\]

and where

\[
\hat{u} = [K_x \ K_\eta] \begin{bmatrix} \dot{x} \\ \epsilon \end{bmatrix}.
\]

The above approach to the cheap control problem for LTI systems under constant tracking and disturbance signals has been justified in Davison & Davison [2002].

The main result under the LTQR is presented next.

**Theorem 6.** Consider system (2). Then for all \(x(0) \in \mathbb{R}_+^n\) there exists an \(\epsilon^*(x(0), u(0)) > 0\) such that for all \(\epsilon \in (0, \epsilon^*(x(0), u(0))]\) the controller (5) solves Problem 3.

Through the remainder of this section \(\epsilon^*(x(0), \eta(0))\) will be denoted by simply \(\epsilon^*\).

Problem 3 can be interpreted as the tracking and disturbance rejection problem under nonnegative LTQR control for positive LTI plants experiencing unmeasurable constant nonnegative disturbances and having constrained inputs from both above and below.

3.2 Implementation

In the remainder of this paper, practical implementation of the theory presented in the previous section is considered. In particular, we consider linear tuning quadratic clamping regulators (LTQcR) to prevent the input signals from going negative. These controllers will first be described and later, in Section 4, used within an experimental setup.

First, however, we consider two results. The first result pertains to Figure 2, while the second to Figure 3.

![Fig. 2. Implementation system 1.](image)

![Fig. 3. Implementation system 2.](image)

![Fig. 4. LTQ controller.](image)

**Theorem 7.** Consider the system of Figure 2 where the positive LTI system is represented as (2) and where the block diagram of Figure 4 represents the LTI controller. Assume \(y_{ref} \in \mathbb{R}_+, \omega \in \mathbb{R}_+\), and \(d - cA^{-1}b \neq 0\).
Then, for all \( \epsilon > 0 \), and all \( x(0) \in \mathbb{R}_+^n \) there exists a \( t^*(x(0), \epsilon) \geq 0 \) such that for all \( t \in [t^*(x(0), \epsilon), \infty) \)
\[
    u_{in}(t) > 0
\]
if and only if
\[
    u_{ss} > 0,
\]
where \( u_{ss} \) is given by (3).

We now come back to the system of Figure 3 and present a corollary to the latter theorem.

**Corollary 8.** Consider the setup of Figure 3 where the positive LTI system is represented as (2) and where the block diagram of Figure 4 represents the LTI controller. Assume \( y_{ref} \in \mathbb{R}_+, \omega \in \mathbb{R}_+, \) and \( d - cA^{-1}b \neq 0 \).

Then, for all \( \epsilon > 0 \), and all \( x(0) \in \mathbb{R}_+^n \) there exists a \( t^*(x(0), \epsilon) \geq 0 \) such that for all \( t \in [t^*(x(0), \epsilon), \infty) \)
\[
    u_{in}(t) < \overline{u}
\]
if and only if
\[
    u_{ss} < \overline{u}.
\]

**LTQcR Controller**

Next, from the results of Theorem 6, Theorem 7, and Corollary 8 we present a linear tuning quadratic clamping regulator (LTQcR) with “anti-reset” wind-up, which clamps all signals at zero or employs the LTQR whenever the input signal is positive. The LTQR is presented below.

\[
    \hat{\eta} = y - y_{ref} \quad (u = 0 \text{ and } \epsilon < 0) \quad \text{ or } \quad (u = \overline{u} \text{ and } \epsilon > 0)
\]
\[
    \hat{\eta} = 0 \quad \text{ else,}
\]

with
\[
    u = \begin{cases}
    0 & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    [K_x \ K_y] \begin{bmatrix} \eta \\ \eta \end{bmatrix} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    \overline{u} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    [K_x \ K_y] \begin{bmatrix} \eta \\ \eta \end{bmatrix} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    \overline{u} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    [K_x \ K_y] \begin{bmatrix} \eta \\ \eta \end{bmatrix} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    \overline{u} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    [K_x \ K_y] \begin{bmatrix} \eta \\ \eta \end{bmatrix} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    \overline{u} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    [K_x \ K_y] \begin{bmatrix} \eta \\ \eta \end{bmatrix} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \\
    \overline{u} & \text{ if } ((0 < u < \overline{u}) \text{ or } (u = 0 \text{ and } \epsilon < 0) \text{ or } (u = \overline{u} \text{ and } \epsilon > 0))
    \end{cases}
\]

with \( 0 < u(0) < \overline{u} \) and fixed and where the optimal control gains \( K_x \) and \( K_y \) are defined as in the LTQR (5).

The introduction of the LTQcR is necessary as in Theorem 6 we do not know how small the \( \epsilon^* \) should be due to unmeasurable disturbances, thus, the input \( u_{ss} \) is actually unknown. However, by Theorem 7 we can deduce that under the LTQcR controller if \( u_{ss} < 0 \), then our controller will “shut itself off” in finite time and remain shut off and if \( 0 < u_{ss} < \overline{u} \) we know that there exists a time \( t^*(x(0), \epsilon) \) such that the trajectory will return to the linear region.

The control strategy presented by (12) will play a crucial role in the experimental results presented in the next section.

A significant advantage of the LTQcR controller is that if the disturbances are too large such that the steady-state assumptions do not hold, the controller shuts itself off and stays off for all time. We are now ready to show how the LTQcR controller behaves when applied to an industrial experimental system.

**4. EXPERIMENT: INDUSTRIAL HYDRAULIC SYSTEM**

The purpose of this section is to validate and justify, experimentally, the results presented in the prequel via the use of an experimental hydraulic system setup. Here we concentrate on how well the LTQcR (12) performs and abides to robustness issues.

**4.1 Experimental apparatus**

The entire hydraulic system apparatus (see Figure 5) has been assembled from industrial components; this includes the actuators, sensors, valves, piping, and all digital communication. We note that the actuators (valves) are controlled by compressed air, and all signal communication between the actuators/sensors to the digital computer are obtained by commercial current variation (4mA to 20mA) techniques, and controlled within the loop by voltages (0V to 10V). Although all components used are industrial we have chosen to incorporate a standard personal computer running MATLAB Version 7.2.0.232 (R2006a) to carry out all real-time control. For a more in-depth summary and a complete listing of all industrial components used please refer to Davison & Fathi [1999].

The apparatus (Figure 5) consists of four water tanks, interconnected via numerous piping and valves, where the water circulates between the tanks and can be controlled via two digitally controlled valves (we will only be interested in using one valve due to the SISO constraint of our theory) that provide water inflow into two upper tanks. An overview diagram of the system is provided by Figure 6. The experimental apparatus has numerous valves which can be opened/closed to increase/decrease the water flow between respective tanks during the experimentation; thus, allowing for major perturbation of the system model. The only measurements taken during the experimentation are that of the height of the water in Tank 1 (Figure 6) via a sensor which provides a voltage level (varying from 1V corresponding to near empty, to 5V near full, with a 1V increase/decrease representing approximately 2.4L of water rise/drop) and the valve control voltage (0V corresponding to nearly closed and 10V corresponding to fully open) of the input into Tank 1 (Valve A in Figure 6).

Note: the compartmental setup of the hydraulic system is stable and its linearized model is a positive system.
Fig. 6. Diagram of the hydraulic system.

We now turn our focus to experimental results.

4.2 Experimental results

Throughout this subsection we refer to Figure 6 and Table 1. Our goal will be to illustrate the theory behind the control approach via the use of various perturbations and disturbances on the nominal plant, which is represented by Case 1 from Table 1. Before we begin the discussion on the experimental results we present a crude linearized model approximation of the hydraulic system obtained via experimental results. The model is:

\[
\dot{x} = -0.00891x + u \quad (13)
\]

\[y = x.\]

Table 1. Experimental Cases

<table>
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<th>Case</th>
<th>A</th>
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<th>C</th>
<th>D</th>
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</table>

4.3 Experiment I

In our first experiment we consider Case 1 under various initial conditions and \(x\) values, i.e. for various water levels of Tank 1 and for different types of tuning parameters. However, due to space limitations, we only illustrate the LTQcR (12) controller with \(x_0 = 4.4V\), \(\bar{\eta} = 10\), \(\epsilon = 50\), \(y_{ref} = 2V\) in Figure 7) with a resultant settling time of 2.5 minutes.

In general, we found that for various initial conditions and no changes to the nominal plant, the LTQcR works well using the crude mathematical model.

4.4 Experiment II

Next, we put the LTQcR controller to the test by applying numerous perturbations into the system. In particular we consider the case of \(\bar{\eta} = 10\), \(x(0) = 3.60V\), \(\epsilon = 50\), and \(y_{ref} = 2V\) under the transitions of Table 2. In this case, we have Case 1 of Table 1 changing to Case 2 in 8.5 minutes, then from Case 2 to Case 3 in 15 minutes, etc. finally ending in Case 6. Figure 8 illustrates the results of the LTQcR controller (12). Notice that saturation has played a key role in this set up.
4.5 Experiment III

In this experiment we repeat Experiment II of the previous subsection and then at approximately 38 minutes add a large disturbance coming into both Tank 1 and Tank 2, corresponding to Case 7 in Table 1. The point of this experiment is to show that if the steady state $u_{ss}$ does not abide to Assumption 1, then no solution to Problem 3 exists. Figure 9 illustrates the results. Notice that the controller shuts off by itself as discussed in the prequel.

Fig. 9. Experiment 3: LTQcR (12) under large disturbances and perturbations.

5. CONCLUSION

In this paper we have introduced an SISO optimal LQR servo called an LTQcR (12) controller to solve the servomechanism problem for positive systems, and have demonstrated some of its properties by applying the controller to an industrial experimental hydraulic system. The controller has successfully controlled the hydraulic system for the case of tracking disturbance rejection, and for the case of various plant perturbations.

REFERENCES


