CONTROL OF HYDRO-ELECTROMECHANICAL SYSTEM USING THE GENERALIZED PID AND THE CRONE CONTROLLERS

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Abstract: This article deals with a comparative study between two fractional controllers: the generalized PID and the CRONE. A frequency-domain design methodology is developed to compare the two controllers. So the performance specifications are firstly translated into open-loop constraints (used for the CRONE controller), and then translated into controller constraints (used for the generalized PID controller). A method with two steps is proposed to determine the optimal parameters of the generalized PID controller. This frequency-domain design methodology is applied to a nonlinear hydro-electromechanical plant. From a grey-box approach, the modelling of the plant is presented, showing a nonlinear behaviour which is discussed around three operating points considered as reference input for the control loop. Then, a linear uncertain model is deduced in order to design the two controllers. Finally, results obtained in simulation and with the test bench are presented.

Keywords: Grey-box modelling approach, Generalized PID and CRONE controllers, Fractional calculus.

1. INTRODUCTION

The generalized PID controller is one of the mostly used controllers in the industry due to its easy implementation. The synthesis of this regulator started with Ziegler and Nichols in 1942 [1]. Numerous methods and algorithms were developed since that time in order to improve the behaviour of this controller [2], [3]. Moreover, the application of the fractional derivation in the physics domain is sharply increasing. Even though the fractional calculus was introduced in the 17th century by Leibniz [4], its applications had started in the second half of the 20th century and the beginning of the 21st century [5] [6] [7] [8] [9] [10]. Regarding the control systems, the fractional calculation was introduced by Manabe in 1961 [11] then by Oustaloup [12] in 1975 when he introduced the control of laser by a regulator of order 3/2. This experiment was the base of the CRONE command development. Recently, Podlubny [13] has proposed a generalized PID controller which transfer function $C(s)$ is given by:

$$C(s) = k_p + k_i s^\lambda + k_d s^\mu,$$

where $\lambda$ and $\mu$ represent the differentiation and integration orders which can be real. Numerous algorithms were then introduced in order to tune these two parameters to get a more reliable and robust controller [14] [15] [16] [17] [18].

In this article, a comparative study between a generalized PID controller and a CRONE controller is presented. From methodology point of view, the main difference between the two controllers is the structure definition:
- the structure of the generalized PID is $a$ priori fixed from traditional PID in cascade form;
- the structure of the CRONE is $a$ posteriori deduced from the loop shaping.

This main difference is discussed in the following sections with the frequency-domain design methodology used to the synthesis of the two controllers. Then, this design methodology is applied to a test bench consisting on a double-direction electropump that fills and empty a main tank equipped with a level sensor. A level disturbance is introduced by a second electropump. In order to design the two controllers, a grey-box approach [19] is used to derive a plant model. This approach combines physical knowledge and information embedded in data. Once the plant transfer function is defined, both controllers are calculated with respect to some performance specifications.

In more details, this article is composed as follow: in section 2, we present the synthesis method for both controllers. We start by introducing the specifications then we show the algorithms to calculate the generalized PID controller and the CRONE controller. In section 3, we present the hydro-electromechanical test bench. The identification method is firstly introduced then the plant transfer function is derived. Then, the PID and the CRONE controllers are designed with respect to the specifications proposed by the user. The simulation and the test bench results are presented. Finally, section 4 presents a conclusion.

2. SYNTHESIS METHODS

A. Performance specifications

The performance specifications concern:
- the stability degree;
- the bandwidth;
- the precision in steady state;
- the rejection level of the measured noise;
- the rejection level of the output perturbation;
- the plant input sensitivity.

In the following, the study is limited in the frequency domain.
So, the **stability degree** can be specified with respect to the module margin $M_\Phi$, the gain margin $M_G$ or the phase margin $M_\Phi$. The last one will be used in this article. Thus,

$$M_\Phi = \pi + \arg \beta(j\omega_b),$$

(2)

where $\beta(s)$ represents the open-loop transfer function and $\omega_b$ the crossover frequency in the open loop defined such as:

$$\left|\beta(j\omega_b)\right| = 1.$$  

(3)

The stability degree specification can be presented as the following constraint:

$$M_\Phi \geq M_{\Phi_{\text{min}}},$$

(4)

where $M_{\Phi_{\text{min}}}$ represents the minimal acceptable value of the phase margin. In substituting relation (4) in equation (2), the constraint of the open-loop argument for the crossover frequency $\omega_b$ is easily computed as follow:

$$\arg \beta(j\omega_b) \geq -\pi + M_{\Phi_{\text{min}}}.$$  

(5)

The argument of the controller can be deduced for equation (5) in respect to equation (6)

$$\arg \beta(j\omega) = \arg C(j\omega) + \arg G(j\omega),$$

(6)

where $G(s)$ represents the plant transfer function. Hence, the reduced form of the controller argument constraint at the crossover frequency is:

$$\arg C(j\omega_b) \geq -\pi + M_{\Phi_{\text{min}}} - \arg G(j\omega_b).$$  

(7)

The bandwidth specification is also computed at the crossover frequency $\omega_b$. The objective of this constraint is to fix the closed-loop dynamics speed. Hence, the frequency range that is concerned by this condition is

$$\omega_m \geq \omega_{b_{\text{min}}},$$

(8)

where $\omega_{b_{\text{min}}}$ represents the minimal acceptable value of $\omega_b$. Referring to equation (3) and knowing that:

$$\left|\beta(j\omega)\right| = \left|C(j\omega)\right| \left|G(j\omega)\right|,$$

(9)

a controller gain constraint at the frequency $\omega_b$ can be deduced from relation (8) as follow:

$$\left|C(j\omega_b)\right| \geq \left|G(j\omega_{b_{\text{min}}})\right|^{-1}.$$  

(10)

Concerning the **rejection level of the measured noise**, it is calculated using a specification applied to the complementary sensitivity function module as follow:

$$\forall \omega \geq \omega_T, \quad \left|\beta(j\omega)\right| = \left|\beta(j\omega)\right| + \beta(j\omega) \leq A,$$

(11)

where $A$ shows the desired rejected noise level for the given frequency $\omega_T$:

$$\left|\beta(j\omega_T)\right| = A.$$  

(12)

If the value of $\omega_T$ is chosen to be much bigger than $\omega_B$, relation (12) can be rewritten as:

$$\forall \omega \geq \omega_T, \quad \left|\beta(j\omega)\right| \geq \beta(j\omega) \leq A.$$  

(13)

Hence, a new controller gain constraint around the frequency $\omega_T$ is deduced from relation (11) as follow:

$$\forall \omega \geq \omega_T, \quad \left|C(j\omega)\right| \leq A \left|G(j\omega_T)\right|^{-1}.$$  

(14)

The **rejection level of the output disturbance** is used for low frequencies ($\omega_b < \omega_0$) and is computed using a specification of the sensitivity module function as follow:

$$\forall \omega \leq \omega_s, \quad \left|S(j\omega)\right| = \left|1 + \beta(j\omega)\right|^{-1} \leq B,$$

(15)

where $B$ represents the desired rejected output disturbance level for the frequency $\omega_b$:

$$\left|S(j\omega_b)\right| = B.$$  

(16)

When choosing $\omega_b < \omega_0$, relation (15) can be written as follow:

$$\forall \omega \leq \omega_s, \quad \left|S(j\omega)\right| = \left|\beta(j\omega)\right|^{-1} \leq B.$$  

(17)

Hence, a new controller gain constraint around the frequency $\omega_b$ is deduced from relation (15) as follow:

$$\forall \omega \leq \omega_s, \quad \left|C(j\omega)\right| = \left|B \left|G(j\omega_b)\right|\right|^{-1}.$$  

(18)

The plant input sensitivity is computed using a specification of the form:

$$\forall \omega \geq \omega_R, \quad \left|\beta(j\omega)\right| = \left|\beta(j\omega)\right| - D,$$

(19)

where $D$ represents the maximal value at the frequency $\omega_R$:

$$\left|\beta(j\omega_R)\right| = D.$$  

(20)

As for the third condition, if the frequency $\omega_b$ is much bigger than $\omega_b$, the relation (19) can be rewritten as:

$$\forall \omega \gg \omega_R, \quad \left|C(j\omega)\right| \leq D.$$  

(21)

The constraints (14) and (21) represent almost the same aspect which is the controller gain at high frequencies. Hence, they can be reduced to one constraint by choosing the lowest value of these two relations. Thus,

$$\forall \omega \geq \omega_s, \quad \left|C(j\omega)\right| \leq \text{Min}\left\{A \left|G(j\omega_T)\right|^{-1}, D\right\}.$$  

(22)

**B. Generalized PID controller**

Two forms are the most used for representing the generalized PID controller: the parallel form and the cascade form. This second one is used in this article. The transfer function can be presented as follow:

$$C(s) = C_0 C_I(s) C_D(s),$$

(23)

with

$$C_I(s) = \frac{\left(1 + \frac{\lambda}{\omega_b}\right)}{\left(1 + \frac{\lambda}{\omega_a}\right)},$$

(24)

and

$$C_D(s) = \frac{\left(1 + \frac{\lambda}{\omega_b}\right)}{\left(1 + \frac{\lambda}{\omega_a}\right)} \omega_b \omega_a,$$

(25)

where $\omega_b, \omega_a$ et $\omega_0$ are the transitional frequencies, $C_0$ a constant, $\lambda$ the integration and $\mu$ the differentiation real order.

As previously mentioned, the computation of the PID transfer function is done directly according to the specifications. In order to have a more reduced form of the PID transfer function, let’s introduce some new variables as follow:

$$a = \omega_b / \omega_b, \quad b = \omega_a / \omega_b, \quad \omega_0 = \sqrt{\omega_b \omega_a},$$

(26)

where $\omega_0$ represents the frequency for which the controller phase is maximal (e.g., $\phi_{\omega_0} = \text{Max}[\arg C(j\omega)] = \arg C(j\omega_0)$). Inserting $\omega_0$ in equation (23) leads to below PID function:
\[ C(s) = C_0 \left( \frac{1+s/s_{\omega_m}}{b/s_{\omega_m}} \right)^n \frac{1 + \sqrt{\alpha}/s_{\omega_m}}{1 + \sqrt{\alpha}/s_{\omega_m}}. \]  

(27)

Hence, the PID module and phase are given by:

\[
\begin{align*}
C(j\omega) &= C_0 \left[ 1 + (b\omega_i/\omega_n)^{1/2} \right] - \lambda (\arctan(b\omega_i/\omega_n) - \pi/2) \\
&+ \mu (\arctan(\sqrt{\alpha}/\omega_n) - \arctan(\sqrt{\alpha}/\omega_n))
\end{align*}
\]

(28)

At the open-loop crossover frequency \( \omega_n \), relation (28) can be rewritten as follow:

\[
\begin{align*}
\varphi_n &= \arg C(j\omega_n) = \lambda (\arctan(b) - \pi/2) \\
&+ \mu (\arctan(\sqrt{\alpha}/\omega_n) - \arctan(\sqrt{\alpha}/\omega_n))
\end{align*}
\]

If \( b \gg 1 \) (minimization of the noise effect caused by the integration due to the phase delay of \( \omega_n \)), system (29) becomes:

\[
\begin{align*}
\varphi_n &= \arg C(j\omega_n) = C_0 a^{\mu/2} \\
&+ \mu (\arctan(\sqrt{\alpha}/\omega_n) - \arctan(1/\sqrt{\alpha}))
\end{align*}
\]

(30)

where \( \varphi_m = \mu \arcsin((a-1)/(a+1)) \).

After recalculating the controller module and phase at central frequency \( \omega_n \), we obtain for low frequencies (\( \omega \ll \omega_n \)):

\[
\begin{align*}
\varphi_n &= \arg C(j\omega_n) = C_0 a^{\mu/2} \\
&+ \mu (\arctan(\sqrt{\alpha}/\omega_n) - \arctan(1/\sqrt{\alpha}))
\end{align*}
\]

(31)

and for high frequencies (\( \omega \gg \omega_n \)):

\[
\begin{align*}
\varphi_n &= \arg C(j\omega_n) = C_0 a^{\mu} \\
&+ \mu (\arctan(\sqrt{\alpha}/\omega_n) - \arctan(1/\sqrt{\alpha}))
\end{align*}
\]

(32)

Considering the system (28), the constraints of relations (7), (10), (14), (18) and (21), valid independently of the controller nature, can be rewritten to suit best the generalized PID controller as follow:

\[
\begin{align*}
\lambda (\arctan(b) - \pi/2) + \varphi_n &\geq -\pi + M_{\Phi_{\omega_n}} - \arg G(j\omega_n), \\
C_0 (1 + b)^{1/2} b^{-\lambda} a^{\mu/2} &\geq \left| C(j\omega_{\omega_{\min}}) \right|^{1/n}, \\
\forall \omega \geq \omega_n, \quad C_0 a^{\mu} &\leq \left| G(j\omega_{\Phi_T}) \right|^{1/n} \\
\forall \omega \leq \omega_S, \quad C_0 (\omega_n/\omega_n)^{1/2} &\geq \left| B \left| G(j\omega_n) \right|^{1/n}
\end{align*}
\]

(33)

and

\[
\begin{align*}
\forall \omega \gg \omega_n, \quad C_0 a^{\mu} &\leq D.
\end{align*}
\]

(34)

In order to determine the optimal values of the parameters vector \( \mathbf{\theta} = [C_0, a, \lambda, \mu] \), two phases are required. The first one depends on the initial values of the vector \( \mathbf{\theta} \) and the second phase involves the search for the optimal values of this vector. In order to accomplish the first phase, the five following steps must be achieved.

1. Let’s \( \lambda = 1, \mu = 1, b = 10, \omega_n = \omega_{\omega_{\min}} \) and \( M_{\Phi} = M_{\Phi_{\omega_n}}; \)
2. One calculates \( \rho_0 = \left| C_0(j\omega_n) \right| \left| G(j\omega_n) \right| \) and \( \Phi_0 = \arg C_0(j\omega_n) + \arg G(j\omega_n); \)
3. One deduces \( \varphi_n = M_{\Phi} - \pi - \Phi_0; \)
4. One calculates the value of \( a \) with respect to the relation (31):
   \[ a = (1 + \sin \varphi_n)/(1 - \sin \varphi_n); \]
5. Knowing that \( |\beta(j\omega_n)| = 1 \), one deduces the value of \( C_0 \) as follow:

\[
C_0 = \left( \frac{\rho_0}{\rho_0^2} \right)_{n_{\omega_n} = n_{\omega_{\max}}}. \]

The second phase consists on determining the optimal values of the vector \( \mathbf{\theta} \). Two methods can be used in this case: the genetic algorithms or the optimization toolbox of Matlab and its function \texttt{fmincon}. In this article, the second method is used. This function requires a large number of inputs and returns the five values of the vector \( \mathbf{\theta} \) once all relations ((34) to (38)) are valid. The initial vector and the lower and upper bound vectors have a big impact on the output values. Note that, in order to emphasize the fractional behaviour of the generalized PID controller, the value of the integrator and differentiator orders, \( \lambda \) and \( \eta \), should vary between 0.1 and 0.9.

\section*{C. CRONE Controller}

The second approach used to design a controller, presented in this article, is based on defining this regulator with respect to the open-loop constraints (robust loop shaping). The CRONE controller is designed using this method.

Hence, the first step consists on defining the necessary specifications for the synthesis of the nominal plant transfer function. The frequency approach is also used in this part as the calibration of the sensitivity functions is easier.

The second step consists on reassigning the frequency closed-loop specifications into open-loop frequency specifications for the nominal plant. These new conditions take into account the plant behaviour at:

- low frequencies in order to have good accuracy in the steady state;
- middle frequencies, especially around the frequency \( \omega_n \), to get the stability degree robustness;
- high frequencies to have good input plant sensitivity.

Once the behaviour of the system in the open-loop is defined, a decision should be made concerning the CRONE generation that will be used. This decision depends on the plant uncertainties. If these uncertainties lead to the gain while the phase is constant, the second generation CRONE control is sufficient [9]. If both the gain and the phase vary, the third generation CRONE control must be used. In this paper, we are interested in the second generation as the third one presents complex differentiation and integration orders, and the use of special programs is a must to solve such equations [9].

Hence, the open-loop behaviour due to the plant gain uncertainties leads to the following transfer function:

\[
\beta(s) = \beta_0 \left( \frac{1 + s/\omega_0}{s/\omega_0} \right)^{n_{\omega_0}} \left( \frac{1 + s/\omega_0}{s/\omega_0} \right)^{n_{\omega_0}}, \]

(39)

where \( \omega_0 \) and \( \omega_0 \) represent the low and high transitional frequencies, \( n \) is the fractional order varying between 1 and 2, \( n_{\omega_0} \) and \( n_{\omega_0} \) are the asymptotic order behaviours for low and high frequencies and \( \beta_0 \) is a constant that assure a unit gain at the frequency \( \omega_0 \). This constant is calculated as follow:

\[
\beta_0 = \left( \omega_0/\omega_0 \right)^{n_{\omega_0}} \left( 1 + \omega_0/\omega_0 \right)^{n_{\omega_0}} \left( 1 + \omega_0/\omega_0 \right)^{n_{\omega_0}}. \]

(40)

Knowing that \( \beta(s) = C_{crown}(s) G(s) \), the CRONE transfer function \( C_{crown}(s) \) is deduced for the
nominal plant value, which is to say:

\[ C_{crone}(s) = \beta(s)/G(s). \]  

(42)

3. TEST BENCH APPLICATION

A. Presentation of the test bench

The test bench, used to test the behaviour of both controllers, is a hydro-electromechanical system constituted of:
- a double-direction electropump that can fill or empty the main tank,
- a secondary tank,
- a second electropump used to introduce a level disturbance in the main tank,
- a level sensor that measures the water level in the main tank,
- and a data acquisition board that enables the communication between the hydro-electromechanical system and the LabVIEW© program.

Figure 1 shows the physical test bench. Figure 2 represents the closed-loop functional block diagram of the hydro-electromechanical system.

![Fig. 1 – Hydro-electromechanical test bench](image)

![Fig. 2 – Functional block diagram of the hydro-electromechanical system](image)

B. Plant model

Advanced control design requires a model that describes process behaviour adequately. Such a model can be constructed using physical modelling (white-box model) or statistical identification techniques (black-box model). In this study, a grey-box approach [19] is used to derive a model of the plant. More precisely, the model structure is deduced from physical principles and the values of the unknown parameters of the model structure are estimated from measured data series around three operating points. The structure model is given by

\[ H(s) = k / (s (1 + s / \omega_0)) , \]  

(43)

where \( k \) is a constant and \( \omega_0 \) a transitional frequency.

The parameters \( k \) and \( \omega_0 \) are estimated from “System Identification Toolbox” of Matlab (output error estimation method). A PRBS (Pseudo Random Binary Signal) is used as the input signal. Its amplitude varies between \( \pm 2.5V \) whereas the minimal allowed period is 2 seconds and the number of samples 16. This input signal is applied to the electropump1 around three operating points (three levels in the tank: 2/4/6 cm) in order to respect small variations (±2cm) of level.

The values of \( k \) and \( \omega_0 \) for each functional point are given in table 1.

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>Level (cm)</th>
<th>( k ) (V/V)</th>
<th>( \omega_0 ) (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2</td>
<td>( 8.30 \times 10^{-4} )</td>
<td>( 4.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>2nd</td>
<td>4</td>
<td>( 5.90 \times 10^{-4} )</td>
<td>( 3.8 \times 10^{-3} )</td>
</tr>
<tr>
<td>3rd</td>
<td>6</td>
<td>( 2.24 \times 10^{-4} )</td>
<td>( 3.6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Max/Min

3.7 1.11

Table 1 – Values of \( k \) and \( \omega_0 \) for each functional point

As the transitional frequency values \( \omega_0 \) for each operating points are very close, the two controllers are designed from an uncertain model given by (43) with:

\[ k \in [2.24 \times 10^{-4}, 8.30 \times 10^{-4}], \omega_0 = 3.6 \times 10^{-3} \text{rad/s}. \]  

(44)

Figure 3.a presents the Bode plots of \( H(j\omega) \) obtained for the three operating points.

For the controller synthesis, the nominal model is arbitrary fixed by the third operating point (\( k_{nom} = 2.24 \times 10^{-4} \text{V/V} \)).

C. User constraints

Once the uncertain model transfer function is defined, the user constraints applied to synthesize both controllers are presented in this section:
- The stability degree: \( M_{\phi_{min}} = 45^\circ \leftrightarrow Q_{\max} = 3 \text{ dB} \), where \( Q_{\max} \) is the maximum of the complementary sensitivity function (resonance peak);
- The bandwidth : \( \omega_{\phi_{min}} = 0.0108 \ \text{rad} / \text{s} \);
- The rejection level of the measured noise:
  \[ \forall \omega \geq \omega_T = 0.108 \ \text{rad} / \text{s}, \ |\mathbf{T}(j\omega)| \leq 0.1 \ ; \]
- The rejection level of the output perturbation:
  \[ \forall \omega \leq \omega_T = 0.00108 \ \text{rad} / \text{s}, \ |\mathbf{S}(j\omega)| \leq 0.1 \ ; \]
- The plant input sensitivity:
  \[ \forall \omega \leq \omega_T = 10.8 \ \text{rad} / \text{s}, \ |\mathbf{R}(j\omega)| \leq 100 \ ; \]

D. Controllers synthesis

Considering the model transfer function and its uncertainties, the user specifications and the synthesis methods for the generalized PID and the CRONE controllers, the generalized PID transfer function is as follow:
\[ C_{\text{PID}}(s) = 50 \left( \frac{1 + s/3.1 \times 10^4}{s/3.1 \times 10^4} \right)^{0.2} \left( \frac{1 + s/3.95 \times 10^3}{1 + s/1.47 \times 10^3} \right)^{0.698}, \]  
whereas the CRONE transfer function is:
\[ C_{\text{CRONE}}(s) = 2.41 \times 10^5 \left( \frac{1 + s/3.6 \times 10^4}{s/4.87 \times 10^3 (1 + s/0.5014)} \right) \left( \frac{1 + s/6.98 \times 10^3}{1 + s/0.5014} \right)^{0.5455}. \]

Figure 3.b shows the Bode plots of both controllers.

E. Simulation results

Figures 4 present the open-loop Bode plots obtained with the generalized PID controller (Fig.4.a) and the CRONE controller (Fig.4.b) around the first (in green) and third (in blue) operating points.

Figures 5 also show the open-loop Nichols loci obtained with the generalized PID controller (Fig.5.a) and with the CRONE controller (Fig.5.b) around the first (in green) and third (in blue) operating points.

With the CRONE controller, the phase margin (45°) and the module margin (3 dB) remain constant for the extreme values of \( k \): illustration of the stability degree robustness versus gain uncertainties in frequency domain.

Figures 6, 7 and 8 present the different sensitivity functions obtained with the generalized PID controller (Fig.6.a, Fig.7.a, Fig.8.a) and with the CRONE controller (Fig.6.b, Fig.7.b, Fig.8.b) around the first (in green) and third (in blue) operating points.

For the nominal operating point (3rd point, in blue), all performance bounds are respected, except for the input sensitivity function obtained with the generalized PID controller (Fig.8.a). For the first operating point (in green), all performance bounds are also respected, except for the stability degree (Fig.7.a, in green) and for the input sensitivity function obtained with the generalized PID controller (Fig.8.a).

Figure 9 shows the closed-loop step responses obtained with the generalized PID controller (Fig.9.a) and with the CRONE controller (Fig.9.b) around the first (in green) and third (in blue) operating points. With the CRONE controller, the first overshoot and the damping remain constant for the extreme values of \( k \): illustration of the stability degree robustness versus gain uncertainties in time domain.

E. Test bench results

Figures 10 and 11 present the measured level in the main tank and the control signal in rejection mode obtained with the generalized PID controller (Fig.10.a, Fig.11.a) and with the CRONE controller (Fig.10.b, Fig.11.b) around the first (in red) and third (in blue) operating points. One notes that:
- the static error is nil with the CRONE controller, it is not the case with the generalized PID controller;
- the control signal remains smaller than the saturation value (±6V).

4. CONCLUSION

In this article, a frequency-domain design methodology has been used to compare the generalized PID and the CRONE controllers. From methodology point of view, the main difference between the two controllers is:
- the structure of the generalized PID is \textit{a priori} fixed;
- the structure of the CRONE is \textit{a posteriori} deduced from the loop shaping.

The performance specifications have been firstly translated into open-loop constraints (used for the CRONE controller), and then translated into controller constraints (used for the generalized PID controller). Then, to determine the optimal parameters of the generalized PID, a method with two steps has been proposed:
- in the first step, the initial parameters values are computed by putting the integrator and differentiator order equal to the unit, then by using a classical frequency-domain design method developed for a traditional PID;
- in the second step, from the initial parameters values, a constrained optimization is used with the optimization toolbox of MatLab to determine the final parameters values.

This comparative study has been applied to a nonlinear hydro-electromechanical plant. The results obtained in simulation and with the test bench have shown the interest of this study.
Fig. 6 – Sensitivity functions with: (a) the generalized PID controller, (b) the CRONE controller

Fig. 7 – Complementary sensitivity functions with: (a) the generalized PID controller, (b) the CRONE controller

Fig. 8 – Input sensitivity functions with: (a) the generalized PID controller, (b) the CRONE controller

Fig. 9 – Step responses with: (a) the generalized PID controller, (b) the CRONE controller

Fig. 10 – Output responses in rejection mode with: (a) the generalized PID controller, (b) the CRONE controller

Fig. 11 – Control signal responses in rejection mode with: (a) the generalized PID controller, (b) the CRONE controller

REFERENCES