Sliding-Mode Control of DC-DC Switching Converters


Departament d’Enginyeria Electrònica, Electrètica i Automàtica, Universitat Rovira i Virgili (URV), Campus Sescelades, Avinguda Països Catalans 26, 43007 Tarragona, Spain, e-mail: luis.martinez@urv.cat

Abstract: The sliding-mode control of DC-DC switching converters is presented in contrast to the traditional pulse width modulation approach. The method of the equivalent control is illustrated in the design of a cascade control for the voltage regulation of a non-minimum phase converter. The theoretical predictions are validated by means of PSIM simulations. Prospective applications of the method reported are also discussed.

Keywords: switching dc-dc converters, power supplies, sliding-mode control, variable structure systems.

1. INTRODUCTION

DC-DC switching converters are the keystones of the so-called power processing systems, which can be described in contrast to signal processing systems. While in the former case, the energy guarantees the transformation of input into output information, in the power processing systems the information is used to control the power flow from the input to output with the maximum efficiency. If we observe the elements employed in the analogue implementation of a signal processing system, we will find out that it is composed of resistors, capacitors, and semiconductor devices operating in linear region, the inductor being the element to be excluded in all designs. Performing the same observation in a power processing system would reveal that this one is made of inductors, capacitors and semiconductor devices operating in on-off zone, the resistor being the excluded elements in all configurations.

The basic electronic features of a power processing system are: 1) voltage regulation 2) maximum power transfer 3) capability of association with other power processors 4) power signal generation from time-varying reference signal tracking.

In this paper, we will focus on the voltage regulation function by reviewing the state of the art in power processing systems based on pulse width modulation (PWM) and by comparing these systems with those employing sliding-mode control.

2. PWM-BASED VOLTAGE REGULATION

PWM-based voltage regulation is performed by means of the block diagram depicted in Fig. 1. The DC-DC switching converter constitutes the power stage and performs the transformation of the input energy under the form of unregulated voltage $V_g>0$ into output energy under the form of regulated voltage $V_C$. The regulation is obtained by processing the output voltage error $v_{err}=a(V_{ref}-v_C)=V_r-av_C$ by means of a controller whose output $v_{ctrl}$ is the input of a PWM. When $v_{ctrl}$ equals the saw-tooth signal $v_{ramp}$, the modulator establishes the ending instant for the subinterval of energy absorption in the switching converter. While $v_{ctrl}$ is still smaller than $v_{ramp}$, the modulator output signal $u(t)$ has a high level and the converter stores the input energy in the magnetic field of its inductors ($T_{ON}$ subinterval). When $v_{ctrl}$ equals $v_{ramp}$, the signal $u(t)$ takes its low value and the switching converter transfers the energy stored during the $T_{ON}$ subinterval to the output load ($T_{OFF}$ subinterval). It has to be pointed out that the modulator inputs $v_{ctrl}$ and $v_{ramp}$ are analogue while its output $u(t)$ is a binary digital signal. Also, the sum of subinterval durations $T_{ON}$ and $T_{OFF}$ is $T$, which is a constant value given by the external signal $v_{ramp}(t)$. Both subintervals are usually expressed in terms of its relative duration with respect to the switching period $T$, namely, $d=T_{ON}/T$, which is called the duty cycle, and $d^*=T_{OFF}/T$, which is complementary to $d$ ($d^*=1-d$). Signal $v_{ctrl}(t)$ has a slow variation in comparison with $v_{ramp}(t)$ to allow the modulation of the pulses duration. Regardless of the type of converter, the increase of the duty cycle implies a higher output voltage in absolute value. The periodic repetition of
the process of absorbing-transferring energy with a high frequency (very small period T) allows the obtaining of a constant DC voltage at the output if the switching frequency components are properly filtered.

DC-DC converters constitute a particular class of nonlinear, time-varying systems. The periodic repetition of the sequence absorption-transfer results in a cyclical variation of the converter topological configuration, which leads to a repetitive sequence of two or three circuit structures whose time durations are not constant due to the modulation of $T_{ON}$ and/or $T_{OFF}$. Therefore, DC-DC switching converters can be considered as variable structure systems and the voltage regulation has been traditionally undertaken by means of linear control based on a small signal model of the converter in the frequency domain.

2.1 Small signal low-frequency model

There are three basic switching converters so that all other converters can be described by comparison with any of these canonical elements which are depicted in Fig. 2.

![Fig. 2. The three basic DC-DC switching converters: a) buck b) boost c) buck-boost.](image)

The switch in Fig. 2 has two possible positions which correspond to the $T_{ON}$ subinterval ($u=1$) or $T_{OFF}$ ($u=0$). A common switch implementation is unidirectional in current and it is carried out by a combination of a MOSFET and a diode. The DC voltage converter ratios are the following

- **Buck converter:**
  \[ V_i/V_o = D \]  
  (1)

- **Boost converter:**
  \[ V_i/V_o = 1/(1-D) \]  
  (2)

- **Buck-Boost converter:**
  \[ V_i/V_o = -D/(1-D) \]  
  (3)

where $V_o$ is the steady-state output voltage, $V_i$ is the nominal input voltage and $D$ is the steady-state duty cycle. Note that the buck converter steps down the input voltage, the boost converter steps up and the buck-boost can perform both functions depending on the value of the duty cycle and reverses the polarity with respect to the input voltage.

If the inductor current in any of the three basic converters is different from zero all along the switching period $T$, the converter is said to operate in continuous conduction mode (CCM). If there is a subinterval within $T_{OFF}$ during which the inductor current is zero, the converter will operate in discontinuous conduction mode (DCM) because of the turn-off of the switch diode. For example, the boost converter in CCM exhibits the repetitive sequence of the two circuit configurations shown in Fig. 3a. Similarly, Fig. 3b shows the sequence corresponding to DCM.

![Fig. 3. Circuit sequence of the boost converter: a) CCM b) DCM.](image)

Assuming CCM operation, the dynamic model of the converter during the $T_{ON}$ and $T_{OFF}$ subintervals are given by

\[ \dot{x} = A_1 x + B_1 \quad t_{on} < t < t_{off} + T \]  
(4)

\[ \dot{x} = A_2 x + B_2 \quad t_{off} + T < t < t_{on} + T \]  
(5)

where $x = [i_L, v_C]^T$ is the state vector.

The averaged dynamic model can be expressed as

\[ \dot{x} = (A_d + A_d')x + (B_d + B_d') \quad t_{on} < t < t_{off} + T \]  
(6)

where $x$ is now the averaged state variable.

Defining

\[ A_d = A_d + A_d' \quad B_d = B_d + B_d' \]  
(7)

Equation (6) can be written as follows

\[ \dot{x} = A_d x + B_d \quad t_{on} < t < t_{off} + T \]  
(8)

Equation (8) shows that only one system described by the pair $(A_d, B_d)$ leads the state vector from the same initial value $x(t_{on})$ to the same final value $x(t_{off} + T)$ than the combination of the two dynamic systems associated respectively to pairs $(A_1, B_1)$ and $(A_2, B_2)$. Note that in (7) matrices $A_d$ and $B_d$ are the result of weighting $A_1$ and $B_1$ by $d$ and $A_2$ and $B_2$ by $(1-d)$ respectively, i.e., by the fraction of the switching period during which the system is described by $(A_1, B_1)$ and by the fraction corresponding to $(A_2, B_2)$. Therefore, we substitute a switched model given by (4)-(5) by an averaged model given by (8) (Middlebrook and Cuk, 1976). It has to be pointed out that (8) is bilinear since $A_d$ is an affine function of the duty cycle $d$ which eventually results in a product of the control $d$ and the state $x$.

Impose $\dot{x} = 0$ in (8) yields the steady-state value of the state vector given by

\[ X_{SS} = -A_{SS}^{-1}B_{SS} \]  
(9)

where $A_{SS} = A_d + A_d'$ and $B_{SS} = B_d + B_d'$, are the values of $A_d$ and $B_d$ in steady state respectively obtained by substituting $d$ by $D$.

Linearising (8) around the equilibrium point given by (9) yields the following small signal dynamic model of the DC-DC switching converter.
\[ \dot{x} = A_{SS} x + B_{SS} \frac{1}{g} \dot{v}_g + [(A_1 - A_2) X_{SS} + B_{1SS} - B_{2SS}] \dot{u} + B_p \dot{i}_d \]  
(10)

where \( \dot{x} \) and \( \dot{d} \) are the incremental variations superposed to the equilibrium point and steady state duty cycle respectively.

On the other hand, \( \dot{v}_g \) and \( \dot{i}_d \) represent the perturbations of the input voltage and load current respectively, and \( B_p \) is a matrix given by \([0 \ 1/C']\), where \( C \) is the converter output capacitance shown in Fig. 2.

Fig. 4 shows the dynamic model of the switching regulator in the \( s \) domain derived from (10). The transfer functions depicted in the block diagram are defined in Table 1.

### Table 1. Transfer functions

| \( G_s(s) = \frac{\dot{V}_s(s)}{\dot{V}_g(s)} \) | Converter input to output transfer function |
| \( G(s) = \frac{\dot{V}_s(s)}{D(s)} \) | Converter control to output transfer function |
| \( Z_o(s) = \frac{\dot{V}_o(s)}{I_o(s)} \) | Converter output impedance |
| \( M(s) = \frac{\dot{D}(s)}{\dot{V}_{rad}(s)} \) | Descriptive function of the PWM (Middlebrook, 1981) |
| \( G_c(s) = \frac{\dot{V}_{rad}(s)}{E(s)} \) | Controller transfer function |

![Fig. 4. Dynamic model of a PWM DC-DC switching regulator](image-url)

It has to be remarked that modelling the PWM by means of a descriptive function implies to assume that the duration of the its output pulses is a continuous-time signal, actually being a discrete-time signal. Assuming that the highest frequency component of \( v_{rad} \) is considerably smaller than \( 1/T \) (switching frequency), the pulse duration can be expressed as a continuous-time function and the modulator can be modelled by its descriptive function. For that reason, the dynamic model of the regulator is only valid for low modulation frequencies (Middlebrook and Cuk, 1976).

### 2.2 Linear control

In the design of the controller transfer function \( G_c(s) \) in Fig. 4 two cases have to be distinguished: a) \( G_c(s) \) is a minimum-phase function; b) \( G_c(s) \) is a non minimum-phase function. The first case is called voltage-mode control and it is employed with converters derived from the buck power stage.

This type of control has reached a high degree of standardisation and there are specific analogue chips that ensure a good phase margin with sufficient bandwidth and zero error steady-state tracking to a reference voltage of step type by using classical design techniques based on Bode or Nyquist methods (Erickson and Maksimovic, 2001). The second case is called current-mode control and corresponds to the rest of converter families wherein boost and buck-boost are the most representative. The most extended solution in this case consists in a cascade control, in which an inner loop controls the dynamics of the fast variable, i.e., the inductor current, and an external loop establishes the reference for the internal loop and regulates the output voltage. In this type of control it is often inserted in the inner loop an external signal of ramp type in order to stabilise this loop because it exhibits an intrinsically unstable behaviour for duty cycles bigger than 50 %. The PWM cascade control of switching converters was originally developed by the European Space Agency and there are currently commercial chips like UC1842 of Texas Instruments that implement this type of control.

It is worth noting that the classic controller is designed by using linear transfer functions obtained after the linearization of a real nonlinear system with discontinuous control input. For this kind of systems the natural way to deal with the controller design is using the sliding mode theory due to their switching nature (Utkin, 1978).

### 3. SLIDING-MODE CONTROL

The existence of sliding motions in a DC-DC switching converter can be investigated by means of Filippov’s method (Filippov, 1964) and its immediate corollary proposed by Utkin (Utkin, 1978), (Venkataraman, 1986), (Sira-Ramirez, 1987), which is known as the equivalent control method. The substitution of the control variable in the switched model of the converter by the analytical expression of the equivalent control leads directly to the sliding ideal dynamics, i.e., to the averaged dynamic behaviour of the system evolving over the discontinuity surface (Utkin, 1978).

Equations (4) and (5) can be rewritten as follows

\[ \dot{x} = A_1 x + B_1 \text{ for } u = 1 \]  
(10)

\[ \dot{x} = A_2 x + B_2 \text{ for } u = 0 \]  
(11)

In compact form

\[ \dot{x} = A x + B \mu \]  
(12)

Defining

\[ A = A_1, \quad \delta = B_2, \quad B = A_1 - A_2, \quad \gamma = B_1 - B_2 \]  
(13)
the following bilinear description can be obtained
\[ \dot{x} = (Ax + \delta) + (Bx + \gamma)u = f(t, x, u) \] (14)

On the other hand, we call \( f^+ = f(t, x, u^+) \) to the vector field corresponding to \( u^+ = u \), i.e., the control applied to the converter when the trajectory of the state vector is above the sliding surface. Similarly, \( f^- = f(t, x, u^-) \) will correspond to the trajectory below the surface. Note that \( u^+ = 1 \) or 0 and that \( u^- \) will be complementarily 0 or 1.

Assuming that \( \forall s \) is the gradient of the sliding surface, there will be a sliding-mode if the projections of the field vectors \( f^+ \) and \( f^- \) over the gradient surface are of opposite sign and point towards the surface \( S \) defined by \( S = \{ x : s(x) = 0 \} \) (Fig.5). Equivalently,

\[
\begin{align*}
\lim_{t \to 0} \langle \nabla s, f^+ \rangle &< 0 \\
\lim_{t \to 0} \langle \nabla s, f^- \rangle &> 0
\end{align*}
\]

where the notation \( \langle a, b \rangle \) represents the scalar product of vectors \( a \) and \( b \).

From (20) it is concluded that the equivalent control will exist provided that
\[ \langle \nabla s, Bx + \gamma \rangle \neq 0 \] (transversality condition) (21)

Note that this condition implies that \( Bx + \gamma \) cannot be tangent to the sliding surface.

The use of sliding surfaces made up of linear combinations of state variables have been successfully investigated in the generation of sliding motions in DC-DC switching converters following the sequence: 1) Obtaining the circuit differential equations; 2) Establishing the sliding region; 3) Obtaining the ideal sliding dynamics by substituting the equivalent control (20) in the system state equations (14) and performing an order reduction by imposing (17); 4) Calculating the equilibrium points of the ideal sliding dynamics; 5) Analyzing the stability of the equilibrium points; 6) Designing the compensating networks; and finally 7) Implementing the controller.

We will illustrate this procedure by designing a cascade control of a boost converter based on sliding-mode and discuss the results in the conclusions of the paper in comparison with a PWM approach.

### 3.1 Sliding mode-based cascade control

The sliding-mode analysis of a boost converter (Fig. 2b) will illustrate the procedure showing the well-known facts: the equilibrium point is unstable for voltage control and stable for current control.

In CCM, from (14) the converter state equations for a constant resistive load \( R \) are the following

\[
\begin{align*}
\frac{di_L}{dt} &= -\frac{v_C}{L} (1-u) + \frac{V_L}{L} \\
\frac{dv_C}{dt} &= \frac{i_L}{C} (1-u) - \frac{v_C}{RC}
\end{align*}
\]

where \( u = 1 \) during \( T_{ON} \) and \( u = 0 \) during \( T_{OFF} \).

If a sliding mode is enforced on the switching line \( s(x) = V_{ref} - v_C = 0 \), then in sliding mode \( v_C = V_{ref} > 0 \) and, according to the equivalent control method, the transversality condition results in

\[
\begin{align*}
\frac{ds}{dt} &= -\frac{i_L}{C} (1-u) + \frac{v_C}{RC} \\
\frac{\partial}{\partial u} \frac{ds}{dt} &= \frac{i_L}{C} \neq 0
\end{align*}
\]

and in

\[ u_{eq} = 1 - \frac{v_C}{Ri_L} \] (24)

The corresponding ideal sliding dynamics will be given by the inductor current equation

\[ \frac{di_L}{dt} = \frac{V_L}{L} - \frac{V_{ref}^2}{LRi_L} \] (25)
which is unstable since the inductor current deviation, from the equilibrium point \( I_L = \frac{V_{ref}^2}{V_gR} \), and its time derivative have the same sign.

Similarly, for the switching line \( s(x) = I_{ref} - i_L = 0 \) (\( I_{ref} > 0 \)), sliding motions exist in the region \( V_g < v_C \) since

\[
\begin{align*}
\frac{ds}{dt} &= \frac{v_C}{L} (1-u) - \frac{V_g}{L} \\
\frac{\partial}{\partial u} \frac{ds}{dt} &= -\frac{v_C}{L} < 0
\end{align*}
\]

(26)

and the equivalent control is

\[
0 < \left( u_{ref} = 1 - \frac{V_g}{v_C} \right) < 1
\]

(27)

The equilibrium point in sliding-mode \( V_C = \sqrt{V_gR}I_{ref} \) of the voltage equation

\[
\frac{dv_C}{dt} = \frac{v_C}{RC} + \frac{V_g}{v_C}I_{ref}
\]

(28)

is stable, because in this case the voltage deviation and its derivative have opposite signs.

In order to regulate the output voltage \( v_C \), a compensating network is introduced in cascade with the sliding mode current controller as shown in Fig. 6, whereas Fig. 7 depicts the block diagram corresponding to the dynamical model of the switching regulator around the equilibrium point. The superscript (\(^{\delta}\)) represents the perturbations superimposed to the equilibrium values which are represented by capital letters \( V_C = V_{ref}, I_{ref}, I_L, \) and \( V_g \).

It can be observed that the switching surface is implemented by means of a hysteresis comparator acting as internal current control loop. The reference to the current loop \( I_{ref} \) is given by an external loop that regulates the output voltage using linear techniques. Also, it has to be pointed out that the external voltage loop reference is \( V_s = \alpha V_{ref} \) with \( \alpha < 1 \). On the other hand, the system has been linearized around the equilibrium point of the ideal sliding dynamics as

\[
\begin{align*}
\frac{d\hat{v}_C}{dt} &= a\hat{v}_C + b_1\hat{i}_{ref} + b_2\frac{d\hat{i}_{ref}}{dt} + b_3\hat{\dot{g}}_g \\
\frac{a}{RC} &= \frac{V_g}{V_gRC}, \quad b_1 = \frac{V_{ref}L}{V_gRC}, \quad b_2 = \frac{V_{ref}L}{V_gRC}
\end{align*}
\]

(29)

with \( a < 0, \quad b_1 > 0, \quad b_2 < 0, \quad b_3 > 0 \), this resulting in a transfer function \( G(s) \) that relates the capacitor voltage variations to the variations of \( i_{ref} \) given by

\[
G(s) = \frac{\hat{V}_C(s)}{\hat{i}_{ref}(s)} = \frac{b_2s + b_1}{s - a}
\]

(30)

For the particular choice depicted in Fig. 6 of a compensating network of PI type with an additional pole \( p \), the transfer function \( G_c(s) \) is given by

\[
G_c(s) = K_C \left( \frac{s + z}{s} \right) \left( \frac{p}{s + p} \right)
\]

(31)

where from Fig. 6, \( K_C = \frac{R_sC_2}{R_4(C_1 + C_2)}, \quad z = \frac{1}{R_4C_2}, \quad p = \frac{C_1 + C_2}{R_4C_1C_2} \), and \( \alpha = \frac{R_3}{R_4 + R_5} \).

The controller design can be simplified if we choose the controller zero to cancel the converter pole and the controller high frequency pole to mirror the right half plane converter zero, i.e., \( z = -a \), and \( p = -\frac{b_2}{b_1} \). With these values for the controller zero and high frequency pole, the system will be stable if \( K_c \) satisfies the inequality

\[
0 < K_c < \frac{1}{b_2}
\]

(32)

The previous analysis is correct as far as the system remains in sliding regime and the ideal sliding dynamics can be considered as a good approximation of the real dynamics. For the first condition, it has to be ensured that either the converter initial conditions are on the sliding surface or the system trajectory reaches the surface from zero initial conditions. For the second condition, the hysteresis width \( H \) (depending on \( R_1, R_2 \) and the comparator voltage supply) must be sufficiently small to assure high switching frequency.
4. SIMULATION RESULTS

Figs. 8 and 9 show the PSIM simulation of the converter start-up in the time-domain and phase-plane respectively for the specifications: $V_g = 10$ V, desired output voltage $V_C = 30$ V, nominal power of 90 W, switching frequency of 50 kHz, and current and voltage peak-to-peak ripples of 4.5 A and 350 mV, respectively. The resulting converter nominal parameters have been approximated to $L = 29 \mu$H, $C = 100 \mu$F and $R = 10 \Omega$. The behavior in Figs. 8 and 9 corresponds to the response of the converter controlled by the current inner loop ($I_{\text{ref}} = 9$ A) so that no voltage regulation has been considered. Also, it has to be pointed out that the state vector reaches the discontinuity surface and then slides to the equilibrium point.

![Fig. 8. Start-up of a boost converter in the time-domain under a constant current sliding mode control.](image1)

![Fig. 9. Start-up of the sliding-mode boost converter simulated in Fig. 8 in the phase-plane.](image2)

Fig. 8. Start-up of a boost converter in the time-domain under a constant current sliding mode control. $v_C(0)=V_g, i_L(0)=0$ A.

Observe that the capacitor voltage recovers the desired output voltage of 30 V in less than 2 ms and exhibits 1 V overshoots during the transient states. The inductor current, in turn, absorbs the change imposed by the new load requirement. Thus, the state-vector has changed from an initial equilibrium point of $I_L = 9$ A, $V_C = 30$ V to $I_L = 4.5$ A, $V_C = 30$ V, hence preserving the value of the output voltage.

5. EXPERIMENTAL RESULTS

To validate the previous simulations some experimental measurements have been carried out in a boost prototype. The converter and control nominal parameters are the same as in the previous simulations. However, some parameters, i.e. the inductance value, are operation-point dependent.

The start-up from an initial condition of $v_C=10$ V is depicted in Fig. 11, which is in good agreement with Fig. 8.

On the other hand, the transient response due to the load changes is shown in Fig. 12, which is also in good concordance with Fig. 10, in spite of losses, switching delays and other experimental imperfections.

![Fig. 10. Response of the sliding-mode controlled boost converter of Fig. 6 to a load perturbation.](image3)

Gain margin of 13.2 dB. If an input voltage perturbation or a load change is introduced in the system when the converter is at the equilibrium point, the state vector will evolve to a new equilibrium point. As an example, Fig. 10 shows the response of the inductor current and output voltage when a current sink connected in parallel to the nominal resistive load introduces 50% current load perturbations. The current sink changes from 0 A to 1.5 A at $t = 4$ ms and returns to 0 A at $t = 7$ ms.
Therefore, the sliding-mode cascade control is a competitive solution for voltage regulation in DC-DC switching converters and can be an alternative to both PWM voltage-mode and cascade controllers.

There are also other voltage regulation functions that can benefit from the sliding-mode approach. This is mainly the case of many hysteresis control schemes that could be analyzed using the methodology reported in this paper. Their analyses under the equivalent control perspective could lead to optimum designs based on analytical conclusions.

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6. CONCLUSIONS AND PROSPECTIVE APPLICATIONS

The principles of voltage regulation of PWM DC–DC switching converters have been reviewed in this paper. It has been shown that the controller design is carried out in the frequency domain and that it is based on an averaged small signal model of the converter, which is valid for low modulation frequencies. In contrast to the PWM small signal frequency domain approach, the sliding-mode control is a time domain method, which can be indistinctly used to compensate small and large signal perturbations.

The sliding-mode approach based on the equivalent control method is particularly appropriate for designing the regulation loop of a DC-DC switching converter because it offers analytical insight on the averaged dynamics of the converter. This technique has been illustrated in the design of a cascade control of a boost converter following a systematic procedure, which can be easily reproduced in any converter. In a clear-cut contrast with the PWM approach, no compensating external ramp has to be added to the internal current loop.