Extended SDRE control of 1-DOF robotic manipulator with nonlinearities

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Abstract: In this paper, the extended state dependent Riccati equation (ESDRE) method is applied to robotic joint control. The control scheme compares the predicted reference state and the actual state, thus rapidly reducing the steady-state error, even in the presence of significant model uncertainties. The state dependent coefficient (SDC) model and the corresponding control law are formulated for the discrete state-space by linear interpolation among operating points. The approach is validated on a 1-DOF rigid robotic manipulator which major nonlinearities are the velocity dependent Strubeck friction and the position dependent gravity. Considering the relative weighting matrices as parameters of the underlying optimization problem by control design the state dependent feedback function is determined. The developed ESDRE control proves superior compared to the PID reference controller with auxiliary gravity compensator. Considering the robust performance and ease of implementation the proposed ESDRE scheme constitutes a serious competitor to the established model-based robot control strategies.

Keywords: State feedback, Nonlinear control, State-space methods, Nonlinearity, Riccati equations, Robot control, Manipulators, Control oriented models

1. INTRODUCTION

State-dependent Riccati equation (SDRE) techniques provide a systematic and effective way to design the state-feedback control for nonlinear plants. The SDRE method employs the same quadratic cost function of system states and control efforts as the linear quadratic regulator (LQR) design. The major difference between the SDRE method and a linear state feedback optimal control is the explicit state dependency of the state-space model and thus the associated control law. An overview of SDRE techniques is provided by Cloutier [1997]. A recent review of the theory developed to solve SDRE regulation problem is presented by Cimen [2008]. SDRE techniques require a state-dependent model in terms of system and input matrices and an at least efficient, but preferably online operating method of solving the state-dependent algebraic Riccati equation. Menon et al. [2002] propose a direct solution using the Hamiltonian matrix (Schur algorithm) and the Kleinman recursive algorithm and evaluate their realtime properties in the context of missile flight control. Further realtime applications of the SDRE method are reported by Erdem and Alleyne [2001], for a 2-DOF underactuated robot, and by Erdem and Alleyne [2004] for a magnetic levitation setup. Haessig and Friedland [2002] propose an alternative method which considers differential instead of algebraic state-dependent Riccati equations, solving them in realtime by numerical integration. In order to avoid complex computation to solve the SDRE in realtime, the optional SDRE point design with corresponding gain scheduling (see e.g. by Cloutier [1997]) is recently pursued. Ruderman et al. [2008] show the efficiency of the state-feedback control applied to an electro-mechanically actuated motion of the drives, subject to a homogenous load without substantial state nonlinearities. The recent work develops a novel extended scheme for state-feedback control of a robotic manipulator motion. The single joint manipulator subject to gravity and friction can be considered as a subsystem of the more complex robot kinematic chain with multiple degrees of freedom.

The proposed extended state dependent Riccati equation (ESDRE) method is motivated by an alternative linear state-control strategy recently published by Roppenecker [2009]. Here, the conventional structure of a state-feedback control is extended by a feed-forward compensator which computes the reference state trajectories which are thereafter compared with the actual system state. Thus, the feed-forward branch becomes independent of the state feedback and provides the steady-state accuracy after a transient response. The novelty of proposed approach is the combination of the extended linear state-control structure with the SDRE method applicable to nonlinear plants. The paper is organized as follows. The next section summarizes the main principles of state-dependent coefficient model and SDRE design method. The last part of Section 2 introduces and proves the feed-forward SDRE extension. In Section 3, the 1-DOF robotic manipulator with nonlinearities, its model and identification are described in more detail. Section 4 shows the experimental control evaluation including the tuning and the comparison of the developed ESDRE control with the reference PID control augmented by the gravity compensator. At the end, the main conclusions are summarized in Section 5.

* D. Weigel completed his bachelor thesis at RST, TU-Dortmund
2. EXTENDED SDRE FORMULATION

2.1 State-dependent coefficient model

Consider a general nonlinear autonomous system
\[
\dot{x} = f(x) + g(x)u, \tag{1}
\]
with the state vector \( x \) and the control vector \( u \). The system is transformed into the state dependent coefficient (SDC) form
\[
\dot{x} = A(x)x + B(x)u, \tag{2}
\]
with a system matrix \( A \) and an input matrix \( B \) in order to apply the SDRE control design approach. According to Cloutier [1997], there is an infinite number of ways to transform the multivariable nonlinear system into SDC form, under the assumption that the state nonlinearity \( f(x) \) is continuously differentiable and \( f(0) = 0 \). This requires that the system matrix is pointwise controllable and observable in the linear sense, in order to obtain a controllable and correspondingly observable SDC parametrization of the nonlinear system (2).

2.2 SDRE feedback control

Similar to LQR control design, the generalized SDRE problem is to minimize the quadratic cost function
\[
J = \frac{1}{2} \int_0^\infty \left( x^T(t)Q(x)x(t) + u^T(t)R(x)u(t) \right) dt, \tag{3}
\]
with the main difference that the weighting matrices \( Q \) and \( R \) become the functions of the state. \( Q(x) \) is required to be positive semi-definite and \( R(x) \), positive definite \( \forall x \), in order to ensure the local stability. For the infinite-horizon nonlinear regulator problem with a state-dependent feedback
\[
u(x) = -K(x)x = -R^{-1}(x)B^T(x)P(x)x, \tag{4}
\]
the weighting matrices penalize the transient state response and the control effort. \( P(x) \) denotes the unique, symmetric, positive-definite solution of the state-dependent algebraic Riccati equation
\[
A^T(x)P(x) + P(x)A(x) - P(x)B(x)R(x)^{-1}B(x)^T P(x) + Q(x) = 0. \tag{5}
\]
The obtained feedback gain constitutes a solution of the SDRE optimization problem
\[
\min_{K(x)} J, \quad \forall x. \tag{6}
\]
A potential drawback of the SDRE approach is that the designed control is not necessarily optimal, but only "suboptimal" (see e.g. Erdem and Alleyne [2004]), due to the ambiguity of the representation of \( A(x) \).

2.3 Feed-forward SDRE extension

Similar to other state-feedback control scheme the SDRE method does not ensure the steady-state accuracy without an additional feedforward filter, or a combination with the standard linear output feedback control, e.g. P or PI one. The algebraic computed feedforward filter
\[
V(x) = -\left( C^T( A(x) - B(x)K(x))^{-1}B(x) \right)^{-1} \tag{7}
\]
is directly obtained once the state-feedback control part \( K(x) \) is designed. \( C \) denotes the output matrix with constant coefficients. However, the pre-filtering method does not imply any comparison between the reference and actual state or output value, such that the steady-state accuracy rather depends on the fidelity of the SDC model. To ensure zero steady state error an alternative control structure, recently proposed by Roppenecker [2009], is combined with the SDRE method as depicted in Fig. 1. The extended SDRE control (ESDRE) with the reference signal \( r \) and the output signal \( y \) consists of the state-feedback part and the reference feedforward branch. The structure of the state-feedback control implies a comparison of the computed reference state \( x_r \) with the actual system state \( x \). The feedforward control part provides the steady-state accuracy and only depends on the plant description but not on the feedback gain \( K(x) \) as shown in the following.

The design of the state-dependent \( V(x) \) and \( W(x) \) matrices is performed independent of each other so that the ESDRE structure resembles Two Degrees-of-Freedom control strategies. In the following, we omit the explicit state dependency of the matrices \( A \) and \( B \) in the formulation, keeping in mind that the same calculus remains valid at each particular state \( x \). In order to design the feedforward control part, consider the system at steady-state, further denoted by the index \( \infty \). The prerequisite for applied feedforwarding is that the state-feedback stabilizes the system, and that the reference signal attains its steady state value \( r_\infty \). Thus, it is valid that
\[
x_\infty = Ax_\infty + Bu_\infty = 0 \quad \Rightarrow \quad y_\infty = C^T x_\infty = r_\infty . \tag{8}
\]

Summarizing this relation in a matrix form
\[
\begin{pmatrix} A & B \\ C^T & 0 \end{pmatrix} \begin{pmatrix} x_\infty \\ u_\infty \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} r_\infty, \tag{9}
\]
and inverting the matrix on the left hand side of (9) one obtains
\[
\begin{pmatrix} x_\infty \\ u_\infty \end{pmatrix} = \begin{pmatrix} A & B \\ C^T & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} r_\infty. \tag{10}
\]

Note, that the inversion of the matrix requires the plant to be free of invariant zeros. Upon applying the steady-state
control $u_\infty$ according to (10), the output $y_\infty$ attains the reference value $r_\infty$ after transient response of the system. At this, the system state converges to the steady-state $x_\infty$. Since the feedforward control part has to provide the steady-state independent on the feedback control part, the computed reference state has to fulfil $e_\infty = x_\infty - x_\infty = 0$ for any constant reference value $r_\infty$. At the same time, the steady-state control value $u_\infty(r) = u_\infty$ must be provided to the system input. In due consideration of requirements listed above and comparing the relation (10) with the control structure in Fig. 1, it becomes evident that the feedforward control part is determined by

$$
\begin{pmatrix}
V \\
W
\end{pmatrix} = \begin{pmatrix}
A & B \\
C^T & 0
\end{pmatrix}^{-1} \begin{pmatrix}
0 \\
1
\end{pmatrix}.
$$

(11)

For any appropriate SDC model the state-dependent feedforward matrices $V(x)$ and $W(x)$ are easily obtained for $\forall x$ independent of the designed state-feedback control.

3. 1-DOF ROBOTIC MANIPULATOR WITH NONLINEARITIES

3.1 Experimental testbed

The experimental testbed of the 1-DOF robotic manipulator is depicted in Fig. 2. The system is actuated by a brushless direct current (BLDC) motor with a rated power of 400 W and an idling speed of 3100 rpm. Both, the angular position and angular velocity of the motor shaft are captured by means of an embedded resolver with 15 Bit resolution. The gear unit with a nominal transmission ratio 1:160 transfers the generated electro-mechanical torque which is assumed to be linear related to the active motor current. The gear unit with an input and an output coupling exhibits a low elasticity and backlash which are negligible due to a high structural damping of the joint. A rigid link with a fixed payload is connected to the output shaft without additional elasticities. The control signal is the active motor current $i$ which is regulated by a low-level embedded control with a sample rate of 11 kHz. In order to account for the limited rated torque of the gear unit, and thus to preserve the mechanical structure the control signal is saturated at 4 A, which corresponds to an output torque of about 94 Nm. The measured angular position, the velocity, and the control input are provided via xPC Target realtime platform from MathWorks Inc. at a sampling rate of 2 kHz. The overall position operation range $0 < q < 360$ deg depicted in Fig. 2 is taken into consideration, whereas zero deg corresponds to the lower vertical position with no gravity torque.

3.2 Nonlinear system dynamics

The second-order nonlinear system dynamics is described by the standard 1-DOF manipulator equation

$$
J \ddot{q} + S(\dot{q}) + \sigma \dot{q} + G(q) = \tau,
$$

(12)

with the input driving torque $\tau$ and the output angular position $q$. Under assumption of rigid body dynamics the lumped manipulator inertia is summarized into $J$. $\sigma$ denotes the linear viscous damping factor. The position dependent gravity

$$
G(q) = mg l \sin(q),
$$

(13)

and the velocity dependent Striebeck friction term

$$
S(\dot{q}) = \text{sgn}(\dot{q}) \left( F_s + (F_s - F_c) \exp(-|\dot{q}| \delta) \right)
$$

(14)

constitute the most significant nonlinearities acting on the driving system. The overall moving mass with the lever arm $l$ is denoted by $m$, and $g$ is the gravitational force constant. The Striebeck friction map is upper bounded by the static friction (also called stiction) force $F_s$ and lower bounded by the Coulomb friction force $F_c$. The exponential factors $V_s$ and $\delta$ denote the Striebeck velocity and the Striebeck shape factor correspondingly.

![Fig. 2. Test bed of 1-DOF robotic manipulator](image)

![Fig. 3. Identified nonlinear system dynamics; measured and modeled velocity response and prediction error at 2 Hz excitation (a) and (d), 0.2 Hz excitation (b) and (e), 0.02 Hz excitation (c) and (f)](image)

The nonlinear system dynamics is identified from experiments with three periodic excitation profiles at 2 Hz, 0.2 Hz, and 0.02 Hz, each one with a linearly increasing amplitude as depicted in Fig. 3. The applied excitations cover a large operation range of angular velocities, and at the same time provide the cyclic angular displacements with maximal amplitudes of about 1.5 deg for 2 Hz, about
30 deg for 0.2 Hz, and about 100 deg for 0.02 Hz. Hence, the overall collected system response envelops a signature of the entire linear and nonlinear terms of the modeled dynamics essential for the identification. An equal number of data points subsampled from each excitation profile is taken into the identification data set. The applied subsampling guarantees that the data from the three scenarios contribute equally to the parameter solution. The standard nonlinear least-squares identification method is applied to determine the seven free system parameters while excluding $V_x$ and $\delta$ which were determined beforehand by Ruderman and Bertram [2010]. The velocity response of the identified model is compared with the experimental data in Fig. 3. The identified and the measured system responses coincide well, even as the model uncertainties become more distinct at low excitation frequencies (compare Fig. 3 (d), (e), and (f)). This can be explained by an increasing interplay between the gravity and friction nonlinearities at low velocities characteristic for the motion profile 0.02 Hz. The determined Striebeck parameters conform to the identified quasi-static friction of the plant reported by Ruderman and Bertram [2010]. The parameter deviation of 2%, 36%, and 14% for $\sigma$, $F_c$, and $F_s$ correspondingly compared to previous experiments are plausibly explained by changes in the friction behavior, as the bearing friction increases due to the increase in normal force caused by the applied additional payload.

3.3 SDC modeling

The nonlinear system dynamics described by (12) is transformed into the SDC form using the state vector $x = (q_1, q_2)^T$ and the input $u = i$. The SDC model possesses a constant input matrix

$$B = \begin{pmatrix} 0 & T_m N \\ J & -J \end{pmatrix},$$

(15)

and a state-dependent system matrix

$$A(x) = \begin{pmatrix} 0 & 1 \\ -\frac{\tan(\varphi_1(x_1))}{J} -\frac{\tan(\varphi_2(x_2))}{J} + \sigma \end{pmatrix},$$

(16)

in which $\varphi_1(x_1)$ and $\varphi_2(x_2)$ denote the angular coordinates of the gravity and Striebeck friction nonlinearities. The motor torque constant $T_m$ and the gear transmission ratio $N$ constitute the overall input gain. As both nonlinearities are symmetric to zero, the model states involve only the positive definition ranges. The positive state-subspace is discretized into 6697 states, 37 on the angular velocity axis, and 181 on the angular position axis. The operating points are obtained by linear interpolation between the two nearest neighbors.

4. CONTROL EVALUATION

4.1 Designed ESDRE control

The feed-forwarding matrices $V(x)$ and $W(x)$ are designed first on the basis of the SDC model as they are independent of the state-feedback control. From an optimization perspective each solution of the SRE problem represents an optimal state feedback in consideration of the weighted state trajectories and control efforts. Thus, for a favored control behavior an appropriate selection of the weighting matrices $Q$ and $R$ constitutes the main task in the design of the state-feedback controller. Using a software-in-the-loop (SIL) optimization framework the best combination of weighting coefficients is determined while minimizing the cost function

$$\min_{Q,R} J = \sum_i w_r (\dot{q}_{r,i} - \dot{q}_i)^2 + \sum_i w_u |u_i - u_{sat}|^2,$$

(17)

with the control weight $w_u = 1$ and the actuator boundary $u_{sat}$. In order to achieve a low velocity rising time and thus to enforce the maximal control dynamics the velocity error weighting function is defined as

$$w_r = \begin{cases} 10 & \text{if } |\dot{q}_r - \dot{q}| > 2 \text{ deg/s}, \\ 1 & \text{else}. \end{cases}$$

(18)

Considering the control penalty $R = 1$ as a basis of the weighting solution, two optimization parameters $\log(Q_{11}/R)$ and $\log(Q_{22}/R)$ are determined by means of a standard least-squares optimization. In the recent case, a simplified consideration of the state independent weighting matrices is pursued. Fig. 4 shows the evolution of the parameters (a), and of the penalty function (b). Already after nearly 40 iterations the weighting parameters as well as the penalty function converge towards their final values. Note, that the design of an appropriate reference trajectory significantly effects the performance of the ESDRE control.

![Fig. 4. SIL control design: (a) weighting parameters, (b) convergence of the penalty function](image)

The specially designed reference trajectory depicted in Fig. 5 (a) and (b) covers a large angular velocity and angular position range. The observed trajectories of the controlled motion coincide well with their simulated counterparts, thus confirming a high performance of the ESDRE approach, even in case of a relative simple single system model that exhibits distinct uncertainties as apparent from Fig. 3. The observed control effort depicted in Fig. 5 (c) exhibits a substantial chattering characteristic which is explained by a too coarse discretization of the fed back states. However, the SDRE control performs robustly as the state dynamics tracks the desired reference trajectories. A close-up in Fig. 5 (a) reveals a low steady-state position error of about 0.005 deg.

4.2 Reference PID control with gravity compensator

The modified PID control
1.5 Evaluation of ESDRE versus Reference control

Two different motion trajectories are experimentally evaluated on the testbed provided in 3.1 in order to compare the performance of the ESDRE control and the reference PID control with gravity compensation (further denoted as reference control). The first trajectory (I) is that one depicted in Fig. 5 (a), used for the SIL control tuning. The temporal evolution of the error of both control strategies is depicted in Fig. 7. It is evident, that the position error of the ESDRE control provides a steady-state characteristic which only depends on the angular velocity and not on the angular position, across a wide operating range.

Fig. 7. Control error of the evaluated first trajectory

The error dynamics exhibits an almost ideal first-order time delay behavior which is unavoidable due to a final system acceleration. The evaluated reference controller exhibits a substantially higher error than the ESDRE one. The error dynamics appear to be dependent on both, angular position and angular velocity. Note, that the contouring error of the ESDRE control is close to zero during zero-velocity ranges, at time intervals 7 – 8 s, 10 – 12 s, and 14 – 17 s. In contrast, the reference control exhibits a relatively high residual error at zero reference velocity, which in addition is more pronounced at the positions at which the arm is subject to stronger gravity.

The second (II) trajectory

\[ q_r(t) = \alpha_1 \sin(2\pi f_1 t) \sin(2\pi f_2 t) + \alpha_2 t \]  

(21)

is the product of two harmonic oscillations with a linearly increasing bias. This type of the reference signal induces a high dynamical manipulator motion with a wide varying velocity over the semicircle angular range as depicted in Fig. 8 (a) and (b) correspondingly.

The experimental control error is depicted in Fig. 9. At this, the reference controller performs the manipulator...
Fig. 8. Second (multiplicative) periodic motion trajectory motion up to the time about 17 s only, nearby the oscillating motion around the operation point of 50 deg. At this point, the low-level motor control consistently aborted, conditioned by the internal current monitoring, due to the motor overload, at which the motor stands still while been supplied with high currents, however below the actuator limits. The control termination of the reference controller argues for its weak control dynamics which does not allow to exploit the total range of the control value. On the contrary, the ESDRE control completes the entire motion without any overload of the motor part, and provides the equally good error dynamics within total position and herewith gravity range. The maximal control error is quite similar to that one of the first trajectory, and leads back to the motion lag which appears during acceleration and deceleration of the moving mass. The maximum absolute error, the mean error, and the error standard deviation are summarized in Table 1 for both controllers and both evaluated trajectories.

Table 1. Evaluated control errors

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>max [error]</th>
<th>mean error</th>
<th>stand. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>14.9</td>
<td>0.05</td>
<td>5.21</td>
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<tr>
<td>ESDRE</td>
<td>4.43</td>
<td>-0.07</td>
<td>2.61</td>
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<table>
<thead>
<tr>
<th>Trajectory</th>
<th>max [error]</th>
<th>mean error</th>
<th>stand. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>13.84</td>
<td>-0.01</td>
<td>6.64</td>
</tr>
<tr>
<td>ESDRE</td>
<td>5.0</td>
<td>0.16</td>
<td>2.62</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

This paper describes an extended state-dependent Riccati equation (ESDRE) design method, motivated by an alternative linear state-control strategy published by Roppenecker [2009], and the high dynamic requirements imposed on manipulators with nonlinearities across large operating ranges. The SDC model and the corresponding state-dependent control law are derived for the discretized state-space, while applying the linear interpolation between operating points. The optimal weighting matrices are determined within software-in-the-loop tuning framework applied with an appropriate reference trajectory and a composite penalty function. The proposed ESDRE control scheme provides a comparison between the computed reference state and the actual system state, thus rapidly reducing the steady-state error, even in the presence of substantial model uncertainties. The practical feasibility of the approach is demonstrated on a 1-DOF rigid robotic manipulator whose main nonlinearities are the velocity dependent Stribeck friction and the position dependent gravity. The analysis shows a remarkable agreement between the simulated and experimentally evaluated responses of the closed control loop. The developed ESDRE control, evaluated across the large position and velocity range proves superior in comparison to the PID control augmented by a standard gravity compensator. Since a multiple DOF manipulator dynamics can be transformed into the SDC form, the proposed ESDRE approach can be extended to the full state control of a common multi-joint robotic system. Considering the robust performance and ease of implementation the proposed ESDRE approach constitutes a serious competitor to state-of-the-art model-based robot control strategies.

REFERENCES


