Distributed MPC for Multi-zone Temperature Regulation with Coupled Constraints

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Abstract: Optimizing the operation of heating systems in buildings has become of high interest. The control objective of such systems is to minimize the heating energy bills while maintaining a certain indoor thermal comfort. Dealing with electric convectors, the heating price is proportional to the amount of heat delivered. Hence, using a linear model of the process in an MPC environment, the optimization problem can be stated in an LP form. In this paper we treat the case of multi-zone building temperature regulation, where the available electrical power is lower than the sum of the maximum powers of local heaters. This acts as a coupling constraint in our optimization problem. A distributed MPC (DMPC) algorithm based on Dantzig-Wolfe decomposition is proposed, considering also the thermal coupling between adjacent zones. Numerical results are presented in order to illustrate the effectiveness of the proposed control strategy.

Keywords: Distributed control, Predictive control, Decomposition methods, Temperature control in buildings

1. INTRODUCTION

The optimization of the systems running costs becomes of a high importance, and nowadays it is no longer possible to design a control system without considering it. In the European Union, more than 23% of the total energy consumption is due to the buildings heating and cooling systems. Knowing that the number of new low energy buildings is negligible compared to the old ones, in order to have a real impact on reducing the energy consumption, an optimal control for the existing buildings becomes imperative. Most of the residential and office buildings have an oversized heating/cooling capacity. In the case of electric heating system, a solution to reduce the user energy bill could be found by decreasing the maximum subscribed power. If the subscribed power is lower than the building heating system maximum power, the other electric appliances could be turned off during high heating demand periods. In an optimal control environment, this subscribed power acts as an input coupling constraint. The most popular controllers for the heating systems are PI and on/off, which cannot handle the subscribed power constraint other than dividing it into local constraints.

One of the major advantages of the model predictive control (MPC) is represented by the ability to handle constraints Mayne et al. [2000] in an optimal control environment. In MPC, an optimal control sequence is computed by minimizing a performance cost function. Only the first element of the open-loop command sequence is applied to the system. At the next instant, a new optimization is performed based on current measurements. Knowing the occupation profile of the rooms in advance, the anticipative effect of the MPC can be a major advantage over the common controllers. The use of the MPC techniques for the temperature control in buildings is not a novel idea. Different formulations of cost function and constraints have been analyzed in Freire et al. [2008]. A stochastic predictive control approach is proposed in Oldewurtel et al. [2008] handling with chance constraints.

For large-scale buildings, a centralized MPC (CMPC) is often undesirable and difficult to implement. Typically, the computational demand grows exponentially with the number of subsystems (rooms). An alternative of this control strategy is the decentralized one. A comparative study between a centralized and a decentralized GPC applied on a passive HVAC process can be found in Riadi et al. [2008]. In the decentralized case the control scheme is composed of independent controllers which take care of the local parameters. They cannot consider the coupling between subsystems, as well as the coupling constraint in an optimal way. Using a DMPC approach, the control structure is a decentralized one and the performance of the closed loop system improves throughout the iterations and information exchanges among the local agents. The controllers negotiate in order to converge to the optimal solution Venkat [2006] or to a Nash equilibrium point Li et al. [2005]. In this paper we propose a distributed control structure based on Dantzig-Wolfe decomposition in order
to regulate the air temperature in a multi-zone building with linear coupling input constraints.

The next section introduces the prediction model and formulates the MPC optimization problem, including the energy price, the occupation profile and the temperature comfort bounds. The main ideas of the Dantzig-Wolfe decomposition method are presented in Section 3. In order to consider the thermal coupling between adjacent zones, inner iterations are added to the decomposition procedure. The resulting algorithm constitutes the main contribution of the article. Section 4 illustrates the performances of the method through several simulation results, while Section 5 concludes the paper.

2. PROBLEM FORMULATION

Consider an LTI model for a zone \( i \in S = \{1, \ldots, s\} \) described by:

\[
\begin{align*}
\dot{x}_i(k+1) &= A_i x_i(k) + \sum_{j \in h_i \cup \{i\}} B_{i,j} u_j(k) \\
y_i(k) &= C_i x_i(k),
\end{align*}
\]

where the vector \( x_i \in \mathbb{R}^{n_i} \) is the local state, \( u_i, y_i \in \mathbb{R} \) are the local input (electrical heating power) and the local output (measured room temperature), respectively. \( h_i \) denotes the set of zones adjacent with the zone \( i \). Using the model (1), we can write the prediction equation for a control horizon \( N_u \) and a prediction horizon \( N_2 \) as:

\[
\hat{y}_i(k) = \left[ \hat{y}_i(k+1) \ldots \hat{y}_i(k+N_2(k)) \right]^T = \Psi_i \cdot x_i(k) + \sum_{j \in h_i \cup \{i\}} \Phi_{i,j} u_j(k),
\]

with

\[
\Psi_i = \left[ (C_i A_i)^T \ldots (C_i A_i^{N_2})^T \right]^T,
\]

\[
\Phi_{i,j} = \left[ \begin{array}{cccc}
\phi_{i,j}^0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
\phi_{i,j}^{N_2-1} & \cdots & \phi_{i,j}^{N_2-N_u} & \sum_{e=0}^{N_2-N_u} \phi_{i,j}^e \\
\end{array} \right],
\]

The control objective is to minimize the energy bill due to the indoor heating, which is usually a linear function of the consumed energy (our control inputs). The thermal comfort is defined locally by an upper and a lower bound \((\overline{p}_i(k+j) \text{ and } \underline{w}_i(k+j))\), respectively constraints of the predicted output, which are activated only during the occupation periods of the zone, see Fig. 1. Denote \( \delta_i(k) = [\delta_i(k+1) \ldots \delta_i(k+N_2)]^T \) as the future occupation profile over the prediction horizon for the room \( i \) at time step \( k \). Intuitively, each element of this vector is defined as:

\[
\delta_i(k+j) = \begin{cases} 
1, & \text{if } k+j \in \text{Occupation}_i \\
0, & \text{if } k+j \in \text{Inoccupation}_i.
\end{cases}
\]

To avoid infeasibility issues, the comfort constraints

\[
\begin{align*}
\mu_i(k+j) &= \delta_i(k+j)(\underline{w}_i(k+j) - \hat{y}_i(k+j)) \leq 0, \\
\overline{p}_i(k+j) &= \delta_i(k+j)(\overline{p}_i(k+j) - \hat{y}_i(k+j)) \leq 0
\end{align*}
\]

\( \forall i \in S, \forall j = 1, \ldots, N_2 \), are softened, adding a penalty, \( f_i \), to the cost criterion when they are not satisfied. Then, the optimization problem can be stated as:

\[
\min_{u_i(k), \forall i \in S} J(k) = \sum_{i \in S} \left( c_i^T(k) u_i(k) + \sum_{j=1}^{N_2} f_i(k+j) \right)
\]

subject to

\[
\begin{align*}
0 &\leq u_i(k+j) \leq \overline{u}_i, \forall j = 0, \ldots, N_u - 1, \forall i \in S, \\
\sum_{i \in S} u_i(k+j) &\leq \overline{u}_i, \forall j = 0, \ldots, N_u - 1,
\end{align*}
\]

where the comfort penalty function, \( f_i \), is defined as:

\[
f_i(k+j) = \begin{cases} 
0, & \text{if } \overline{p}_i(k+j) \leq 0 \text{ and } \underline{w}_i(k+j) \leq 0 \\
\lambda_i \overline{p}_i(k+j), & \text{if } \overline{p}_i(k+j) > 0 \\
\lambda_i \underline{w}_i(k+j), & \text{if } \underline{w}_i(k+j) > 0,
\end{cases}
\]

\( \lambda_i \) is the weighting factor that penalizes the criterion when the comfort constraints are not accomplished.

The linear programming problem (4) can be written in a standard form, dropping \( k \) for the simplicity of notations, as:

\[
\begin{align*}
\min_{z, w_i, u_i, \forall i \in S} & \sum_{i \in S} c_i^T u_i \\
\text{subject to} & \quad D_0 z + D_z u_i^i + \cdots + D_{z_2} u_i^i = f \\
& \quad E_{u_1} u_1^i + \cdots + E_{u_s} u_s^i = g_i \\
& \quad z, u_1^i, \ldots, u_s^i \geq 0,
\end{align*}
\]

where the matrices and the vectors composing this formulation are defined in Appendix. The optimization variables \( u_i^i \) and \( u_s^i \) are obtained by adding the required number of slack variables to the \( u_i \) and \( u_s \), respectively. These auxiliary variables (as well as \( z \)) are used in order to transform the inequality constraints into equality constraints and to measure the gap between the temperature bounds and the predicted output values. The next section will detail the distributed MPC algorithm based on the Dantzig-Wolfe decomposition, which will solve (5) at each sample time.

3. DANTZIG-WOLFE DECOMPOSITION

Dantzig-Wolfe decomposition Dantzig and Wolfe [1960] is very suitable when the constraint matrix of the LP
problem has a block-angular structure, in order to parallelize the computation of the optimization problem. This decomposition method splits a single large-scale linear programming problem into several independent problems which are coordinated by a single master problem (MP). The optimal solution of the original large-scale problem can be shown to be identical to the solution obtained after a finite number of iterations [Dantzig and Thapa, 2003], solving sequentially the MP and the subproblems.

3.1 Block-angular structured constraint matrices case

In this section we will briefly present the main points of the decomposition method for the block-angular constraint matrix optimization problem (the case where \( E_{i,j} = 0, \forall i \neq j \) in (5)). Thus, the first constraint is called the complicating constraint because it prevents obtaining the optimal solution by solving each subproblem independently. In the followings, we will refer to the relaxed problem, as the original problem without the complicating constraint. This decomposition technique is based on the theorem of convex combination and on column generation techniques Dantzig and Thapa [2003]. In the followings we will develop the statement using the convex combination theorem for bounded convex polyhedral. Hence, any point in \( X = \{ x | Ax = 0, x \geq 0 \} \) can be written as a convex combination of the extreme points, \( p_i, \) of \( X:\]

\[
x = \sum_{i=1}^{N} \lambda_i p_i, \quad \sum_{i=1}^{N} \lambda_i = 1, \quad \lambda_i \geq 0, \quad \forall i \in N.
\]

(6)

We can formulate now an equivalent LP problem of (5), called the master problem (MP) as:

\[
\min_{\bar{X},\lambda_{i,j}} \sum_{i \in S} \sum_{j=1}^{N_i} c_i^T p_{i,j} \lambda_{i,j} \quad \text{(7a)}
\]

subject to

\[
D_0 \bar{z} + \sum_{i \in S} \sum_{j=1}^{N_i} D_i p_{i,j} \lambda_{i,j} = f : \gamma, \quad \text{(7b)}
\]

\[
\sum_{j=1}^{N_i} \lambda_{i,j} = 1 : \sigma_i, \quad \lambda_{i,j} \geq 0, \quad \forall j = 1, ..., N_i, \quad \forall i \in S, \quad \text{(7c)}
\]

where \( N_i \) denotes the number of extreme points of the feasible region of the subproblem (8), while \( \gamma \in \mathbb{R}^{N_i} \) and \( \sigma_i \in \mathbb{R} \) are the corresponding dual variables of the constraints (7b) and (7c), respectively. Solving the MP (7) using the Simplex method, only a basic set of the extreme points is needed. Knowing this, solution of the original problem can be found by solving iteratively a dynamically constructed restricted master problem (RMP). A starting basic feasible solution can be obtained by solving (in parallel) all the subproblems i:

\[
\min_{u_i} c_i^T u_i, \quad \text{(8a)}
\]

subject to

\[
E_{i,i} u_i = g_i, \quad \text{(8b)}
\]

\[
u_i \geq 0, \quad \text{(8c)}
\]

but using a different (arbitrarily chosen) cost \( c_i^T \) in order to obtain different extreme points of the relaxed problem feasible region. Once we have a basic feasible solution, solving the RMP provides us the Simplex multipliers (the dual solutions \( \gamma \) and \( \sigma_i \)). The reduced cost associated with a new basic feasible solution can be computed as Conejo et al. [2006]:

\[
r = \sum_{i \in S} (c_i^T - \gamma D_i) p_{i,j} - \sigma_i, \quad \text{which should be negative and preferably a minimum. To find the minimum reduced cost, the subproblems are modified as followings:}
\]

\[
\min_{u_i} (c_i^T - \gamma D_i) u_i, \quad \text{(9)}
\]

subject to (8b) and (8c). The solution of the new relaxed problem composed by all \( i \) subproblems of the form (9) will be added to the current basis. The procedure presented above will be repeated until the minimum computed reduced cost \( r \) is nonnegative. This means that there is no basic feasible solution that can improve the current solution. An upper and a lower bound of the objective function value can be obtained at iteration \( f \) of the algorithm:

\[
J^{(f)}(l) = \sum_{i \in S} \sum_{j=1}^{N_i} c_i^T p_{i,j} \lambda^{(f)}_{i,j}, \quad \text{(10)}
\]

\[
J^{(f)}(l) = \sum_{i \in S} (c_i^T - \gamma D_i) u^{(f)}_{i} + \gamma^{(f)} f, \quad \text{(11)}
\]

where the superscript \( (l) \) indicates the iteration number. These bounds allow to stop the algorithm once a prespecified tolerance is satisfied.

3.2 Non-block-angular constraint matrices case

In the previous section we exposed the basic ideas of the Dantzig-Wolfe decomposition applied to a block-angular structured constraint matrix. For our temperature control problem, this situation corresponds to neglecting the thermal coupling between zones. If the thermal influences between adjacent zones are important (and considered in the prediction model), the constraint matrix loses its structure. For example if the rooms are disposed in a line series, the constraint matrix of the relaxed problem becomes tridiagonal. The appearance of the non-zero block matrices \( E_{i,j} \), \( j \in h_i \) prevents obtaining \( \gamma \) in a straightforward way the relaxed problem solution (solving the problem by blocks). Using the idea of Moroşan et al. [2010], we propose solving the relaxed problem by a communication-based distributed method. Consequently, we introduce inner iterations in order to solve the relaxed problems which consider the coupling terms (see Algorithm 1). The modifications (compared to the block-angular case) will affect only the subproblems. Thus, the constraint (8b) becomes: \( E_{i,i} u_i = g_i - \sum_{j \in h_i} E_{i,j} u_j^{(l-1)} \), where the superscript \( (l, l-1) \) denotes that the corresponding value was computed at the previous inner iteration.

The resulting distributed control scheme has the structure presented in Fig.2, where MPC acts like a coordinator for local controllers (MPCs), also testing the stop conditions for both inner and outer (D-W) iterations. While the subproblems are independent and should be computed in a parallel environment in order to reduce the computational demand, at each outer iteration the master problem and the subproblems are solved sequentially. Note that at the end of every outer iteration a global feasible solution is provided.
Algorithm 1 DMPC based on D-W decomposition for multi-zone temperature control with coupling constraints

Require: Global problem (5), $\epsilon_i$, $\epsilon_2$, $z_i$, $\forall i \in S$

Ensure: Control inputs $u^{(l)}_i$, $\forall i \in S$

1: Initialization: $l = 1$, $l_i = 1$, $p_{i,j}$, $i \in S$, $j = 1, ..., N_i$
2: MPC$_c$ solves the RMP, obtaining $X_{i,j}^{(l)}$, $j = 1, ..., N_i$

and the dual solutions $\gamma^{(l)}$ and $\sigma^{(l)}$.

3: MPC$_c$ broadcasts $\gamma^{(l)}$ to all local controllers MPC$_i$

4: All MPC$_i$ solve (in parallel) their subproblem $i$, obtaining $u^{(l,(i))}_i$.

5: if $\left| u^{(l,(i))}_i - u^{(l,1)}_i \right|_\infty < \epsilon_i$, $\forall i \in S$ then

6: All MPC$_i$ send their solution, $u^{(l)}_i$, to MPC$_c$

else

7: Update $l_i = l_i + 1$ and Goto step 4

end if

Scenario 2: We vary now the control horizon and we fix $s = 40$ and $\xi = 1$. $N_u$ influences the constraint.

To study the complexity of the distributed algorithm compared to the centralized solver, we decided to use a simple thermal model of a room, knowing that the dimensions of the optimization problems are independent of the system complexity (the number of states). Considering all the rooms identically, the matrices of model (1) are: $A_i = \begin{bmatrix} 0.9921 & 0.598 \end{bmatrix}$, $B_{i,1} = \begin{bmatrix} 0.2595 \end{bmatrix}$, $B_{i,j} = \begin{bmatrix} 0.022 \end{bmatrix}$, $C_i^T = [1]$. In order to compare the performances of the distributed algorithm with a centralized solver we will use the suboptimality factor defined as $S_f(\%,:) = 100(J_f - J_0)/J_0$ ($J_f$ and $J_0$ being the optimal costs in the distributed and the centralized case, respectively) and the computational time.

Computation time, which could be important in real time, is proportional with the number of iterations. Consequently, in the results presented below we will also show the number of outer iterations and the mean number of inner iterations required for each case. The parameters whose influence is studied are: the number of subsystems $s$, the control horizon $N_u$ and the coupling static gain $\xi$. In the simulation results presented below we used $N_2 = 30$, $N_u = 15$ (the number of occupation samples within the prediction horizon) and $\epsilon = \epsilon_i = 10^{-3}$. The solver used was the MATLAB Simplex (linprog function with simplex option). In order to have more consistent statistical results, five different values for the initial states of the subsystems were considered. This explains the multiple values of the parameters shown in the figures of each scenario.

Fig. 2. Distributed MPC control structure

4. SIMULATION RESULTS

4.1 Experimental study of algorithm efficiency

For efficiency analysis, the proposed DMPC algorithm was implemented on a sequential machine. Thus, the equivalent computational time using a distributed computing environment (ignoring the communication time required) can be computed as:

$$t_{distr} = \sum_{i=1}^{t_{DW}} \left( t_{MPC}(l_i) + \sum_{j \in S} \max t_{MPC_2}(i) \right),$$ (12)

where $t_{DW}$ is the number of outer iterations reached by the algorithm, $t_{MPC}(l)$ is the time required by MPC$_c$ to solve the restricted master problem at iteration $l$, while $t_{MPC_2}(i)$ is the computational time required to solve the subproblem $j$ at inner iteration $i$ within the D-W iteration $l$.

Scenario 1: The parameters $N_u = 15$ and $\xi = 1$ are fixed, while the number of subsystems is changed. Fig. 3 shows a very good scaling behavior of the algorithm in a distributed computing environment, compared to the centralized solver. However, lowering the computational burden by using the DMPC algorithm, is it possible only for a relative important number of zones composing the building (10, in this case). When the number of subsystems exceeds 150, the centralized solver fails to offer the solution within the sample time limit, represented by a line in Fig. 3 (using a 3GHz dual CPU machine). Regarding the suboptimality factor, the distributed solution is very close to the centralized optimum.

Scenario 2: We vary now the control horizon $s$ and fix $s = 40$ and $\xi = 1$. $N_u$ influences the constraint.

Fig. 3. The influence of the number of subsystems, $s$
matrix dimensions of both the master problem and the subproblems. The gap between the centralized and the distributed computational time becomes more important as \( N_u \) increases (Fig. 4), while the other performance indices are slightly influenced.

![Fig. 4. The influence of the control horizon, \( N_u \)](image)

**Scenario 3** Here we show the influence of the coupling static gain over the performances of the distributed algorithm, for \( s = 40 \) and \( N_u = 15 \). As we could anticipate, a greater coupling gain between subsystems implies more negotiation (inner) iterations in order to find a Nash equilibrium. In the mean time, the equilibrium point moves away from the optimal as the coupling factor becomes important and the suboptimality factors grows.

![Fig. 5. The influence of the coupling static gain, \( \xi \)](image)

### 4.2 Three-zone building simulation study

As the previous paragraph studied the efficiency of the distributed method from a numerical point of view, the aim of this section is to test the proposed algorithm on a virtual building, comparing the proposed control structure with other controllers in terms of comfort and energy cost. The simulation model is the same as the three-zone building used in Moroșan et al. [2010]. The control inputs of the system are the electrical power of the local convectors (of \( P_r = 1200W \) maximum power), while the outputs are the air temperatures measured in each zone. The local costs, \( c_r(k+j) \), can be expressed as \( c_r(k+j) = c_r(k+j)T_r(k)/360 \cdot 10^3 \text{€/W} \) in terms of electricity price, \( c_r \), which is in \( €/kWh \). The electricity tariff considered is \( c_r = 0.0742€/kWh \), while the upper bound total electrical power \( P_r = \alpha \sum_{i \in S} P_{ir} \), is varied with \( \alpha \in (0,1) \). In the simulations, the weather conditions used were those measured in Rennes on January 1st, 1998.

The prediction model of the three-zone building was obtained using the thermal dynamics equations, in a similar manner as in Liao and Dexter [2004]. We will not detail here the model for the reasons of brevity. We mention only that compared to the zone model (1) structure, our prediction model has two more inputs: the ground temperature (fixed at 10°C) and the external air temperature. We assume that no external air temperature predictions are availables, so within the predictor this value is considered constant for the entire prediction horizon and equal to the temperature measured at the current time step. The following values for the MPC parameters were used: \( N_u = N_2 = 30, \epsilon = \epsilon' = 10^{-3}, \lambda_i = 10^3, \forall i \in S, T_0 = 600s. \)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r ) [W]</td>
<td>19.5</td>
<td>20.5</td>
<td>21.5</td>
</tr>
<tr>
<td>( \overline{P}_r ) [W]</td>
<td>20.5</td>
<td>21.5</td>
<td>22.5</td>
</tr>
<tr>
<td>OP 1</td>
<td>08:00 - 12:00</td>
<td>13:00 - 17:00</td>
<td>17:00 - 20:00</td>
</tr>
<tr>
<td>OP 2</td>
<td>08:00 - 17:00</td>
<td>10:00 - 19:00</td>
<td>14:00 - 18:00</td>
</tr>
<tr>
<td>OP 3</td>
<td>08:00 - 17:00</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In order to have a better view over the control performances of the distributed scheme, we propose to compare the heating costs using three different occupation scenarios, resumed in Table 1. In Table 2, we compare in terms of heating cost the proposed DMPC structure, the equivalent CMPC controller and a decentralized PI control structure. We considered two values for the initial conditions (initial temperature of indoor air and walls), which corresponds to a daily occupation (16C) and to a long inoccupation period (10C), respectively. For each of the above profile, we reduced the subscribed power by 30% and by 50%, respectively, comparing to \( \sum_{r \in S} P_{ir} \). The setpoints of the local PIs, \( w_{opt}^i = (\overline{w}_{ri} + \overline{w}_{ri})/2 \), are anticipated with 3h when \( T_i = 10C \) and 5h when \( T_i = 16C \), in order to satisfy the comfort requirements. To make the PI a better opponent we choose to divide the total maximum heating power only among the occupied zones, by saturating the control inputs of the system are the electrical power of the local convectors (of \( P_r = 1200W \) maximum power), while the outputs are the air temperatures measured in each zone. The local costs, \( c_r(k+j) \), can be expressed as \( c_r(k+j) = c_r(k+j)T_r(k)/360 \cdot 10^3 \text{€/W} \) in terms of electricity price, \( c_r \), which is in \( €/kWh \). The electricity tariff considered is \( c_r = 0.0742€/kWh \), while the upper bound total electrical power \( P_r = \alpha \sum_{i \in S} P_{ir} \), is varied with \( \alpha \in (0,1) \). In the simulations, the weather conditions used were those measured in Rennes on January 1st, 1998.

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increase the computational demand and the distributed control approach could be imperative for medium-scale buildings also.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$T_i$ [C]</th>
<th>Control law</th>
<th>Cost/day [€]</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td>OP 1</td>
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<tr>
<td>0.5</td>
<td>10</td>
<td>CMPC</td>
<td>1.15</td>
</tr>
<tr>
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</tr>
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<td></td>
<td>PI</td>
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<td>CMPC</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>DMPC</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PI</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 2. Heating cost comparison

5. CONCLUSION

In this paper we focused on multi-zone building temperature control constrained by a maximum subscribed power. In order to reduce the heating costs, while preserving a certain thermal comfort for the occupants, the control problem was stated in a LP formulation. To reduce the computational time required to solve the optimization problem online, for large-scale buildings we proposed a distributed MPC algorithm based on Dantzig-Wolfe decomposition. In order to consider the thermal coupling between adjacent zones, we added inner negotiation iteration to the algorithm which originally intends to solve only block-angular constraint matrix LPs. The efficiency of the proposed algorithm was shown regarding the computational time, convergence speed and economical features.

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REFERENCES


APPENDIX

\[ c_i^T = [c_i^T \delta_{1,N_i} \delta_{1,N_i} \delta_{1,N_i} \lambda_{1,N_i} \lambda_{1,N_i} \lambda_{1,N_i}], \]

\[ D_0 = \begin{bmatrix} I_{N_i,N_i} & 0_{N_i,N_i} & 0_{N_i,N_i} & 0_{N_i,N_i} & 0_{N_i,N_i} & 0_{N_i,N_i} \end{bmatrix}, \]

\[ f = \Delta_1 \begin{bmatrix} \pi_1 \lambda_{1,N_i} \Delta_1 (\mathbf{x}_i - \mathbf{x}_i) \end{bmatrix} \]