A New Flux Observer for Induction Motors with On-Line Identification of Load Torque and Resistances

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Abstract:
Under the assumption that the stator currents and voltages are available from measurements along with the rotor speed, a novel local adaptive flux observer for induction motors is presented which, under persistency of excitation, estimates the motor fluxes and identifies on-line the rotor resistance. A sufficiently slow adaptation for the stator resistance is performed in order to isolate its estimation from the estimation of motor fluxes and rotor resistance. A second-order load torque identifier is also designed to be used in conjunction with the adaptive flux observer and the stator resistance identifier. The advantages of the proposed observer with respect to previous work are: the assumption of bounded stator currents integrals is no longer required; the dynamic order of the adaptive flux observer is reduced to six; a clear physical interpretation of the persistency of excitation condition is given. On the other hand the convergence analysis of the error dynamics holds only locally under suitable identifiability conditions while globally convergent algorithms are available in the literature.

1. INTRODUCTION

Direct current motors are extensively used in variable speed applications since their flux and torque are independently controlled by the field and the armature current. However, they have disadvantages due to the mechanical commutator and the brushes so that they are limited in high-speed, high-voltage operating conditions. Induction motors, on the other hand, are much more difficult to control but have definite advantages since they: i) have no commutator, no brushes, no rotor windings in squirrel cage motors; ii) have a simple rugged structure; iii) can tolerate heavy overloading; iv) can produce higher torques with a lower weight, smaller size, and lower rotating mass than direct current motors. Recent developments in vector control and direct torque control techniques have made induction motors suitable for high performance servo applications such as automated production, transportation or rail traction systems, provided that accurate torque or flux estimation and critical parameter identification are achieved to guarantee high efficiency and reliability.

Nonlinear rotor flux observers for induction motors, which were called bilinear observers, were first designed in 1978 (see [4, 9] and also [10] for a reduced order nonlinear observer); a complete theory for rotor flux observers, including observers with arbitrary rate of convergence, was successively developed in [5, 36]. Adaptive flux observers have also been designed (see for instance [20, 29]) which can be made adaptive with respect to rotor and stator resistances (see [16, 21]). In fact, flux observers were found to be very sensitive with respect to motor resistance variations typically occurring during operations due to motor heating. The influence of parameter uncertainty on the performance of flux observers are investigated in [15]; Kalman-like and adaptive observers are presented in [32] and in [6], respectively, to estimate on-line and simultaneously the unmeasured states and the parameters while least square techniques are used in [8, 27, 30]. Gradient techniques are used in [11] for parameter estimation in induction motors at standstill while sliding mode observers which are adaptive with respect to motor uncertain parameters are presented in [2, 12, 18]; discrete time observers are proposed in [1, 26, 38, 39]. Rotor resistance and mutual inductance are estimated in [31] while rotor resistance identification at steady-state conditions is discussed in [7, 13] using the gradient technique. The rotor time constant is estimated in [17, 33] by simple on-line identification schemes while a current perturbation signal is injected in [25] for the estimation of rotor and stator resistances. Off-line parameter estimations are treated in [3, 19, 35, 37] while flux and speed observers for vehicular applications are designed in [28]. A review on induction motor parameter estimation techniques can be found in [34].

In this paper, under the assumption that the stator currents and voltages are available from measurements along with the rotor speed, a novel nonlinear adaptive observer for induction motors is presented which is constituted by: i) a local adaptive flux observer which, under persistency of excitation conditions with a clear physical interpretation (compare with [6, 32]), is able to estimate the motor fluxes and to identify the rotor resistance; ii) a stator resistance identifier whose design, via two-time-scale arguments (see [14, 24] for a similar approach to parameter estimation in induction motors), allows us to isolate its estimation from the estimation of motor fluxes and rotor resistance; iii) a second-order load torque identifier to be used in conjunction with the adaptive flux observer and the stator...
resistance identifier. The assumption of bounded stator currents integrals (which is required in [16, 21] and may be unrealistic in the case of biased current measurements) is removed. The dynamic order of the adaptive flux observer is reduced to six (it is higher in [16, 21]) while the derived persistency of excitation inequality is guaranteed to be satisfied in the typical operating condition of constant rotor speed and flux modulus and non-zero electromagnetic torque. On the other hand, the convergence analysis of the error dynamics holds only locally under sufficient conditions for the identifiability of the stator resistance at steady-state while the algorithms proposed in [6, 16, 21, 32] are globally convergent.

2. PHYSICAL MODELING AND ASSUMPTIONS

Assuming linear magnetic circuits, the dynamics of a balanced non-saturated induction motor with one pole pair in a fixed reference frame attached to the stator are given by the well known fifth-order model (see for instance [23] and references therein)

\[
\frac{d\omega}{dt} = \mu(\psi_{ra}i_{rb} - \psi_{rb}i_{sa}) - \frac{T_L}{J} \\
\frac{d\psi_{ra}}{dt} = -\alpha\psi_{ra} - \alpha\psi_{rb} + \beta M i_{sa} \\
\frac{d\psi_{rb}}{dt} = -\alpha\psi_{rb} + \alpha\psi_{ra} + \alpha M i_{sb} \\
\frac{d\psi_{sa}}{dt} = -\left(\frac{R_s + \alpha M}{\sigma}\right)i_{sa} + \frac{1}{\sigma}u_{sa} - \frac{1}{\sigma}u_{sb} + \alpha\psi_{ra} - \beta\psi_{rb} \\
\frac{d\psi_{sb}}{dt} = -\left(\frac{R_s + \alpha M}{\sigma}\right)i_{sb} + \frac{1}{\sigma}u_{sa} + \alpha\psi_{rb} - \beta\psi_{ra}
\]

(1)

in which: \(\omega\) is the rotor speed, \((\psi_{ra}, \psi_{rb})\) are the rotor fluxes, \((i_{sa}, i_{sb})\) are the stator currents, \((u_{sa}, u_{sb})\) are the stator voltages, and \((i_{ra}, i_{rb})\) are the rotor currents that are available while the parameters \(T_L, \alpha,\) and \(R_s\) are typically uncertain owing to load torque dependence on applications and resistance variations which is due to motor heating. We will only invoke, in the following, the boundedness of state and input variables, which is the a standard assumption in the design of induction motor observers. Any restriction concerning the boundedness of stator currents integrals, which has been proposed for the design of flux observers in [16, 21], is no longer required.

3. OBSERVER DESIGN AND STABILITY ANALYSIS

We first introduce the variables \(z_a = i_{sa} + \beta\psi_{ra}, z_b = i_{sb} + \beta\psi_{rb}\) so that the motor electromagnetic equations in (1) become

\[
\frac{dzi_a}{dt} = \frac{R_s}{\sigma}i_{sa} - \frac{1}{\sigma}u_{sa} - \left(\frac{1 + \beta M}{\sigma}\right)i_{sa} - \omega^2 i_{sa} + \alpha z_a \\
\frac{dzi_b}{dt} = \frac{R_s}{\sigma}i_{sb} - \frac{1}{\sigma}u_{sa} - \left(\frac{1 + \beta M}{\sigma}\right)i_{sb} - \omega^2 i_{sb} + \alpha z_b
\]

in which \(\alpha\) and \(\hat{R}_s\) are the estimates for the uncertain parameters \(\alpha\) and \(R_s\), whose estimation laws are yet to be designed along with the terms \(v_a\) and \(v_b\). We shall apply the two-time-scale arguments which have been succesfully used in [24] (see also [14]) and which will allow us to isolate the estimation of the stator resistance from the estimation of motor fluxes and rotor resistance so that a persistency of excitation condition with a clear interpretation is obtained. We first design a flux observer and rotor resistance estimator in the case of known stator resistance (that is \(R_s = \hat{R}_s\)). This can simply be done by introducing the estimation errors \(\hat{i}_{sa} = i_{sa} - i_{sa}, \hat{i}_{sb} = i_{sb} - i_{sb}, \hat{z}_a = z_a - \hat{z}_a, \hat{z}_b = z_b - \hat{z}_b, \hat{\alpha} = \alpha - \hat{\alpha}\) and by considering the quadratic function \((k_z, k_\alpha)\) are positive design parameters

\[
V = \frac{1}{2}\left(\hat{i}_{sa}^2 + \hat{i}_{sb}^2\right) + \frac{1}{2k_z}(\hat{z}_a^2 + \hat{z}_b^2) + \frac{1}{2k_\alpha}z_a^2
\]

whose time derivative along the trajectories of the estimation error system satisfies

\[
\dot{V} = -k_z(\hat{i}_{sa}^2 + \hat{i}_{sb}^2) + \hat{\alpha}z_a \hat{i}_{sa} + \hat{\alpha}z_b \hat{i}_{sb}
\]

provided that

\[
v_a = -k_z\left(\omega\hat{i}_{sa} - \hat{\alpha}\hat{z}_a\right) \\
v_b = k_z\left(\omega\hat{i}_{sa} + \hat{\alpha}\hat{z}_b\right) \\
\hat{\alpha} = -k_\alpha\left((1 + \beta M)i_{sa} - \hat{z}_a\right)\hat{i}_{sa} + \left((1 + \beta M)i_{sb} - \hat{z}_b\right)\hat{i}_{sb}
\]

By virtue of a straightforward modification of the Persistency of Excitation Lemma A.1 in [22], we can establish that if there exist two positive real constants \(l_p\) and \(k_p\) such that the...
preliminary of excitation condition \((I_3\) is the \(3 \times 3\) identity
matrix)

\[
\int_{t}^{t+p} \Gamma^T(\tau)\Gamma(\tau)d\tau \geq k_p I_3, \quad \forall \ t \geq 0 \quad (5)
\]

holds with

\[
\Gamma = \begin{bmatrix}
\alpha & \omega & \beta (\psi_{ra} - M i_{sa}) \\
-\omega & \alpha & \beta (\psi_{rb} - M i_{sb})
\end{bmatrix}
\]

then the origin of the \((\hat{i}_{sa}, \hat{i}_{sb}, \hat{z}_a, \hat{z}_b, \hat{\alpha})\)-system is locally exponentially stable. Exponential rotor flux and rotor resistance estimates are thus obtained by

\[
\begin{bmatrix}
\dot{\hat{\psi}}_{ra} \\
\dot{\hat{\psi}}_{rb}
\end{bmatrix} = -\frac{1}{\beta} \begin{bmatrix}
\hat{i}_{sa} - \hat{z}_a \\
\hat{i}_{sb} - \hat{z}_b
\end{bmatrix}
\]

\[
\hat{R}_r = L_r \hat{\alpha}.
\]

Note that when the rotor speed and the rotor flux modulus are constant and the load torque is zero, so that \(\psi_{ra} = M i_{sa}\) and \(\psi_{rb} = M i_{sb}\), the persistency of excitation (5) cannot be satisfied. Recall that in these operating conditions the rotor resistance cannot be identified by stator currents and rotor speed measurements since the motor equations (1) become

\[
\dot{\omega} = 0, \quad \dot{\psi}_{ra} = -\omega \dot{\psi}_{rb}, \quad \dot{\psi}_{rb} = \omega \psi_{ra}
\]

\[
\frac{d i_{sa}}{dt} = -\omega i_{sb}, \quad \frac{d i_{sb}}{dt} = \omega i_{sa}
\]

and do not depend on the rotor resistance \(R_r\). On the other hand, it suffices that the rotor speed and the rotor flux modulus are constant (with \(\psi_{ra}^0 + \psi_{rb}^0 = c_\psi > 0\) and the load torque is non-zero to satisfy inequality (5) with \(t_p = \frac{2\pi}{\gamma}\), where \(\gamma\) is the rotor flux vector angle. Recall that in the conditions above,

\[
i_{sa}^2 + i_{sb}^2 = \frac{c_\psi}{M^2} + \frac{T_L^2}{\mu^2 c_\psi J^2}
\]

\[
i_{sa} \psi_{ra} + i_{sb} \psi_{rb} = \frac{c_\psi}{M}
\]

\[
\psi_{ra} i_{sa} - \psi_{rb} i_{sb} = \frac{T_L}{J \mu}
\]

\[
\gamma \dot{\omega} = \omega + \frac{R_r T_L}{c_\psi}
\]

so that the system (1) is locally observable at the extended state \((i_{sa}, i_{sb}, \omega, \psi_{ra}, \psi_{rb}, \alpha)\) from \((i_{sa}, i_{sb}, \omega)\) measurements (see [23] for details).

We now apply the two-time-scale arguments proposed in [24] (see also [14]) by invoking a similar assumption to that used in [24], that is the existence of a steady-state solution to the estimation error system (when a stator resistance estimation error \(\hat{R}_r = R_s - \hat{R}_r\) appears): the measurable component \(v_a i_{sa} + v_b i_{sb}\), at steady-state and in first approximation, according to

\[
\begin{align*}
\frac{\dot{i}_{sa}}{\sigma} &= -\frac{R_s}{\sigma} i_{sa} - \alpha (1 + \beta M) i_{sa} - \omega \hat{i}_{sb} + \hat{\alpha} \hat{z}_a \\
\frac{\dot{i}_{sb}}{\sigma} &= -\frac{R_s}{\sigma} i_{sb} - \alpha (1 + \beta M) i_{sb} + \omega \hat{i}_{sa} + \hat{\alpha} \hat{z}_b \\
\frac{\dot{z}_a}{\sigma} &= \frac{\hat{R}_s}{\sigma} i_{sa} + \frac{u_{sa}}{\sigma} \\
\frac{\dot{z}_b}{\sigma} &= \frac{\hat{R}_s}{\sigma} i_{sb} + \frac{u_{sb}}{\sigma}
\end{align*}
\]

is equal to

\[
-\frac{1}{\sigma} \left[ (i_{sa}^2(t) + i_{sb}^2(t)) + \varpi(t) \right] \\
\hat{R}_s(t) \hat{R}_s(t) = -\lambda(t) \hat{R}_s(t)
\]

with \(\lambda(t)\) a suitable time function.

The function \(\varpi(t)\) appearing in \(\lambda(t)\) depends on the steady-state solution to the estimation error system and is clearly unknown; we however introduce, as in [24], the following sufficient condition for the stator resistance identifiability at steady-state from the signal \(v_a i_{sa} + v_b i_{sb}\):

\[
\lambda(t) \geq c_\lambda > 0, \quad \forall \ t \geq 0
\]

with \(c_\lambda\) a positive real, that is the assumption that the influence of \(\varpi(t)\) does not affect the positive sign of \(\lambda(t)\) (note that if \(\varpi(t) = 0\) the positiveness of \(\lambda(t)\) is clearly related to stator resistance identifiability conditions). We thus obtain

\[
\dot{\hat{R}}_s = -k_R (v_a i_{sa} + v_b i_{sb})
\]

\[
= k_R \left[ k_\omega (i_{sb} - i_{sa}) - k_\omega (i_{sa} - i_{sb}) i_{sa} \right. \\
\left. - k_\omega (i_{sa} - i_{sb}) i_{sa} - k_\omega (i_{sa} - i_{sb}) i_{sb} \right]
\]

in which \(k_\omega\) is a sufficiently small positive design parameter.

We finally design a second-order load torque identifier which can be used in conjunction with the adaptive flux observer \((k_\omega\) and \(k_T\) are positive design parameters)

\[
\ddot{\omega} = \mu (\psi_{ra} i_{sa} - \psi_{rb} i_{sb}) - \frac{\dot{T}_L}{J} + k_\omega (\omega - \hat{\omega})
\]

\[
\ddot{T}_L = -k_T (\omega - \hat{\omega})
\]

to provide an on-line estimate of the load torque along with an estimate of the measured rotor speed. The proof is reported in [23] and is based on the quadratic function

\[
V_T = \frac{1}{2k_T} \ddot{T}_L^2 + \frac{1}{2} \omega^2 + \epsilon \ddot{\omega} T_L
\]

in which \(\ddot{\omega} = \omega - \hat{\omega}, \ddot{T}_L = T_L - \hat{T}_L\) and \(\epsilon > 0\) is a sufficiently small positive real.

The overall eighth-order dynamic nonlinear adaptive observer can be thus summarized as

\[
\frac{\dot{i}_{sa}}{\sigma} = -\frac{R_s}{\sigma} i_{sa} - \alpha (1 + \beta M) i_{sa} - \omega \hat{i}_{sb} + \hat{\alpha} \hat{z}_a \\
+ \omega \hat{z}_b + \frac{u_{sa}}{\sigma} + k_1 (i_{sa} - \hat{i}_{sa})
\]

\[
\frac{\dot{i}_{sb}}{\sigma} = -\frac{R_s}{\sigma} i_{sb} - \alpha (1 + \beta M) i_{sb} + \omega \hat{i}_{sa} + \hat{\alpha} \hat{z}_b \\
- \omega \hat{z}_a + \frac{u_{sb}}{\sigma} + k_1 (i_{sb} - \hat{i}_{sb})
\]

\[
\frac{\dot{z}_a}{\sigma} = \frac{\hat{R}_s}{\sigma} i_{sa} + \frac{u_{sa}}{\sigma}
\]

\[
\frac{\dot{z}_b}{\sigma} = \frac{\hat{R}_s}{\sigma} i_{sb} + \frac{u_{sb}}{\sigma}
\]
\[ -k_z \omega (i_{sb} - \hat{i}_{sb}) + k_z \hat{\alpha} (i_{sa} - \hat{i}_{sa}) \]
\[ \dot{\hat{z}}_b = -\frac{\hat{R}_s}{\sigma} i_{sb} + u_{sb} \sigma \]
\[ + k_z \hat{\omega} (i_{sa} - \hat{i}_{sa}) + k_z \hat{\alpha} (i_{sb} - \hat{i}_{sb}) \]
\[ \dot{\hat{\alpha}} = -k_\alpha \left[ (1 + \beta M) i_{sa} - \hat{z}_a \right] (i_{sa} - \hat{i}_{sa}) \]
\[ + \left[ (1 + \beta M) i_{sb} - \hat{z}_b \right] (i_{sb} - \hat{i}_{sb}) \]

\[
\begin{bmatrix}
\hat{\psi}_{ra} \\
\hat{\psi}_{rb}
\end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} i_{sa} - \hat{z}_a \\
 i_{sb} - \hat{z}_b
\end{bmatrix}
\]
\[ \hat{R}_s = L_r \hat{\alpha} \]
\[ \hat{\dot{R}}_s = k_R \left[ k_z \omega (i_{sb} - \hat{i}_{sb}) i_{sa} - k_z \omega (i_{sa} - \hat{i}_{sa}) i_{sb} \right. \]
\[ - k_z \hat{\alpha} (i_{sa} - \hat{i}_{sa}) i_{sa} - k_z \hat{\alpha} (i_{sb} - \hat{i}_{sb}) i_{sb} \]
\[ \dot{\hat{\omega}} = \mu (\hat{\psi}_{ra} i_{sb} - \hat{\psi}_{rb} i_{sa}) - \frac{\hat{T}_L}{J} + k_\omega (\omega - \hat{\omega}) \]
\[ \hat{T}_L = -k_T (\omega - \hat{\omega}) \]

in which: \( \hat{\psi}_{ra}, \hat{\psi}_{rb}, \hat{\omega}, \hat{T}_L, \hat{R}_s, \hat{\dot{R}}_s \) are the estimates for the unmeasured states \( \psi_{ra}, \psi_{rb} \) and for the uncertain parameters \( T_L, R_s, \dot{R}_s \), respectively; \( \hat{i}_{sa}, \hat{i}_{sb}, \hat{z}_a, \hat{z}_b, \hat{\alpha}, \hat{\omega} \) are auxiliary dynamic variables. It depends on: the available (\( i_{sa}, i_{sb}, \omega \)) measurements; the known motor parameters \( J, L_r, L_s, M \); the positive design parameters \( k_\alpha, k_z, k_R, \mu \); the initial conditions \( k_\alpha, k_z, k_R, \mu \) (\( k_R \) is sufficiently small).

### 4. SIMULATION RESULTS

We tested the nonlinear adaptive observer (9) by simulations for the three-phase single pole pair 0.6-kW induction motor OE-MER 7-80/C whose parameters are: \( J = 0.0075 \) Kgm², \( R_s = 5.3 \) Ohm, \( R_c = 3.3 \) Ohm, \( L_s = 0.365 \) H, \( L_r = 0.375 \) H, \( M = 0.34 \) H. The motor (with initial conditions \( \psi_{ra}(0) = 0 \) and \( \psi_{rb}(0) = 0.1 \) Wb) is controlled by the input-output feedback linearizing control (which relies on the perfect knowledge of all motor parameters) reported in Section 2.4 of [23]. The rotor speed and the flux modulus are reported in Figure 1. The design parameters are chosen as (all the values are in SI units): \( k_\alpha = 120, k_z = 3, k_R = 450, k_T = 0.1, k_\omega = 200, k_T = 100 \) s⁻¹. All the observer initial conditions are set to zero excepting for \( \hat{\alpha}(0) = 9 \) s⁻¹ and \( \hat{R}_s(0) = 5.4 \) Ohm. The rotor fluxes, the rotor and the stator resistances along with the corresponding estimates are reported in Figures 2-6: satisfactory performance are obtained even in spite of time-varying perturbations of the motor resistances and step-wise variations of the load torque. Note that, even though a steady-state error for the resistances estimation errors appears in the time interval [2.5, 4.5] s due to a constant rotor speed and flux modulus and a zero load torque, rotor fluxes (and therefore the load torque) are nevertheless correctly estimated.

### 5. CONCLUSIONS

On the basis of rotor speed and stator currents and voltages measurements, a novel induction motor nonlinear adaptive observer is designed. The convergence analysis for the error dynamics holds locally, that is for sufficiently small initial errors. Key features of the proposed solution with respect to previous ones are: i) no need for the assumption of bounded stator currents integrals; ii) a reduced dynamic order of the adaptive flux observer; iii) a persistency of excitation condition which is guaranteed to hold in the typical case of constant rotor speed and flux modulus and non-zero electromagnetic torque.

### REFERENCES


Fig. 4. Load torque $T_L$ and its estimate $\hat{T}_L$.

Fig. 5. Rotor resistance $R_r$ and its estimate $\hat{R}_r$.

Fig. 6. Stator resistance $R_s$ and its estimate $\hat{R}_s$.


