Comparison of Static and Dynamic Control Allocation Techniques for Integrated Vehicle Control

Ali Tavasoli* Mahyar Naraghi**

* Sama Technical and Vocational Training College, Islamic Azad University, Karaj Branch, Karaj, Iran (e-mail: ali.tavassoli@Kiau.ac.ir)
** Department of Mechanical Engineering, Amirkabir University of Technology, Tehran, Iran (e-mail: naraghi@aut.ac.ir)

Abstract: Comparison of static and dynamic control allocation techniques for nonlinear constrained optimal distribution of tire forces in a vehicle control system is presented. The total body forces and moments, obtained from a high level controller, are distributed among tire forces, which are constrained to nonlinear constraint of saturation, through two approaches. For the static control allocation technique the interior-point method is employed to perform the nonlinear constrained optimization problem. Also, by the dynamic control allocation technique a dynamic update law is derived for desired forces of each tire.

Keywords: Integrated Vehicle Dynamics Control, Static Control Allocation, Dynamic Control Allocation.

1. INTRODUCTION

In spite of many researches done for maintaining vehicle stability in critical conditions through two last decades, it is yet a challenging area due to saturation of tire forces in such conditions. The most recent approach for vehicle stability enhancement includes Optimal Distribution of tire Forces (ODF), also Control Allocation (CA), where to achieve control objectives as much as possible total body forces/moments are distributed among the longitudinal and lateral force of each tire in an optimal manner. Two general approaches are developed for CA problem which are static control allocation and dynamic control allocation techniques.

By the static control allocation method the total body forces/moments which are obtained from a high level controller are allocated to available actuators by optimizing a suitable cost function. Mokhyamar and Abe introduced an ODF method in order to optimize entire tires work-load usage (Mokhyamar & Abe, 2004). Plumlee et.al examined the use of quadratic programming for CA problem of a ground vehicle (Plumlee, Bevly, & Hodell, 2004). The problem of control allocation in automotive vehicles using multi-parametric nonlinear programming has been subject of another study (Tøndel & Johansen, 2005). Wang et.al in (Wang & Longoria, 2009) used fixed point method for control allocation in a coordinated vehicle control scheme. A new method for adaptive optimal distribution of braking and lateral tire forces was employed by (Naraghi, Roshanbin, & Tavasoli, 2009) where a constrained adaptive optimization problem was solved analytically.

The main problem in static control allocation is its computational burden, due to numerical solution of the constrained optimization problem at each sampling instant, for practical applications. To deal with this difficulty Johansen developed dynamic control allocation method in (Johansen, 2004) for a class of nonlinear systems where a dynamic update law for system inputs was derived so that instead of solving an optimization problem at each instant the control allocation problem was solved dynamically. This method was extended for systems with unknown parameters in (Tjønnås & Johansen, 2008). Also, dynamic control allocation was utilized for vehicle yaw control in (Tjønnås & Johansen, 2009).

In the present work nonlinear constrained optimal distribution of tire forces using both static and dynamic control allocation approaches is considered for integrated vehicle control system. The vehicle motion is governed by tires forces and these forces are constrained due to tire saturation, and, thus, the redundant set of actuators cannot be allocated in any order and the saturation constraints must be taken into account to ensure that the commanded forces can be actually yielded. Considering tires saturation induces nonlinear constraints in control allocation problem. To handle this problem, in this paper, static and dynamic control allocation methods are considered for vehicle control. For static allocation purpose the interior-point is formulated and employed to optimize nonlinear constrained tire forces. To formulate dynamic allocation control for our problem a dynamic update law based on (Tjønnås & Johansen, 2008) is derived. Simulation results are conducted to evaluate the effectiveness of each method. To show how each method enhances vehicle handling and stability in maneuvers with critical conditions, it is compared with previous work in literature, i.e. (Mokhyamar & Abe, 2004), where tires saturation was ignored in ODF. Also to investigate the differences between two approaches the vehicle behavior under two methods is compared and conclusions are derived.
2. HIGH LEVEL SLIDING MODE CONTROLLER

The main purpose of high level control is to design a control law for total lateral force and yaw moment to make the actual yaw rate and side-slip angle follow their desired values. Here the control design is based on a 2DOF linear vehicle model, with constant speed (Ellis, 1994) where the equations are:

\[ mV(\dot{\beta} + r) = F_y \]  
\[ I \ddot{\beta} = M \]

in which \( m \) is the total mass of the vehicle, \( I \) the yaw moment of inertia, and \( V \) the vehicle velocity. \( \dot{\beta} \) and \( r \) are the actual vehicle side-slip angle and the yaw rate, respectively. \( M \) and \( F_Y \) are sum of external moments in the yaw direction and lateral forces acting on the vehicle. Here a sliding mode control system for vehicle system dynamics which was considered in (Naraghi, Roshanbin, & Tavasoli, 2009) is used as the high level controller. Therefore, the total lateral force and yaw moment are obtained as:

\[ F_Y = mV[\dot{\beta}_d + r + (\lambda + k)(\beta_d - \beta) + \lambda k \int (\beta_d - \beta) dt] \]  
\[ M = I[\ddot{\beta}_d + (\lambda + k) (r_d - r) + \lambda k \int (r_d - r) dt], \lambda, k > 0 \]

where the desired values of yaw rate and side-slip angle, \( r_d \) and \( \beta_d \), are calculated on the basis of the driver’s steering input and the vehicle forward speed using a linear model, known as bicycle model, as given by (Wong, 1993).

3. CONTROL ALLOCATION FORMULATION

The motion of a vehicle is governed by the tire-road friction forces and these forces are bounded as described by friction circle concept. To construct the total lateral force and yaw moment demanded by high level controller in Equations (3) and (4), and also to satisfy the braking acceleration command by driver, longitudinal and lateral force of each tire must be determined. In this paper a vehicle system in which each tire can be braked/derived and steered independently is considered. Thus the overall control system contains eight actuators and only three control objectives, making the IVDC an over-actuated control system. A general approach to resolve redundancy is to define an optimization problem where a cost function, for specific performance, is considered and actuator limitations are treated as inequality constraints.

The well accepted cost function for ODF in IVDC systems is the sum of work-load of four wheels which is written as:

\[ f = \sum_{i=1}^{4} A_i \frac{x_i^2 + y_i^2}{z_i} \]

where \( i \) denotes wheel number, \( x_i \) and \( y_i \) stand for desired longitudinal and lateral tire forces, \( z_i \) is the vertical load, all defined in the vehicle body fixed coordinate system, as shown in Figure 1, \( A_i \) is weighting coefficient and \( \mu_i \) is tire friction coefficient of the \( i \)th tire. Defining the actuators vector, \( u \), a 8×1 vector which contains tire forces, as:

\[ u = [X_1 \ X_2 \ X_3 \ X_4 \ Y_1 \ Y_2 \ Y_3 \ Y_4]^T \]

the cost function in matrix form is written as:

\[ f(u) = u^T W u \]

in which \( W_{8 \times 8} \) is a diagonal matrix known as weighting matrix for control allocation problem.

Fig. 1. Schematic view of forces acting on the vehicle

The sum of the lateral forces and yaw moments acting on the vehicle by each tire force should equal to the required total lateral force, \( F_Y \), and yaw moment, \( M \), computed based on high level sliding mode control. Thus as depicted in Figure 1, all of the variables in objective function must satisfy the two equality constraints given by:

\[ F_Y = \sum_{i=1}^{4} Y_i \]  
\[ M = \sum_{i=1}^{2} (L_i Y_i - L_r Y_{i+2}) + \frac{d}{2} \sum_{i=1}^{2} (X_{2i} - X_{2i-1}) \]

Also the longitudinal forces, \( X_i \), should satisfy the demanded total longitudinal acceleration, \( a_v \), by driver, i.e.

\[ F_x = m a_v = \sum_{i=1}^{4} X_i \]

As seen by Equations (8)-(10) these equality constraints are linear in terms of actuator inputs and can be expressed in matrix form as:

\[ Au = v \]

where \( A \in R^{3 \times 8} \) is a constant matrix and the vector of generalized forces/moment, \( v \), is given by,

\[ v = [F_x \ F_y \ M]^T \]

On the other hand the resultant force of each tire is constrained due to frictional properties of tire forces. According to friction circle concept the nonlinear saturation constraint of \( i \)th tire is expressed by:

\[ (\mu_i Z_i^2 - X_i^2 - Y_i^2) \geq 0, \]  \( i = 1, 2, 3, 4 \)

Using (9), Equation (13) can be written as

\[ C_i(u) \geq 0 \]

where \( C_i \) is a 4×1 vector whose \( i \)th component is:

\[ C_i(u_{i}, u_{i+4}) = (\mu_i Z_i^2 - (u_x^2 + u_{y+4}^2), \ i = 1, 2, 3, 4 \]

The control allocation problem is defined as the minimization of cost function (7) constrained to three equality constraints (11) and four nonlinear inequality constraints (14).
4. STATIC CONTROL ALLOCATION: APPLICATION OF INTERIOR-POINT METHOD

Static control allocation includes solving of the proposed nonlinear constrained optimization problem at each sampling instant. A powerful class of algorithms for solving nonlinear constrained optimization problems is the set of Interior-Point (IP) methods. Interior-point methods have been effectively applied to formulate linear and quadratic programming control allocation problems of flight control (Petersen and Bodson 2005, 2006). In (Petersen and Bodson 2005) various types of IP methods was studied for solving control allocation problem and, among the investigated algorithms, primal-dual path-following was shown to be the superior one. In this paper, in a similar approach the IP method has been applied to static control allocation problem of the integrated vehicle control system. In the following we formulate primal-dual path-following for our system. A detailed approach to theory of interior-point methods is found in related literature (Wright, 1997; Nocedal & Wright, 2006).

At the outset, to avoid infeasible solutions, the equality constraints are embedded in cost function and the new objective is considered as:

\[ J = \rho u^T W u + (A u - v)^T (A u - v) = \frac{1}{2} u^T G u + c^T u + d, \quad \rho > 0 \]

\[ G = 2(\rho W + A^T A), \quad c = -2A^T v, \quad d = v^T v \]  \hspace{1cm} (16)

One of the basic ideas behind the IP methods is to use barrier functions to satisfy the inequality constraints. In this regard the optimization problem is transformed into

\[ \min_{u,s} L = \frac{1}{2} u^T G u + c^T u + d - \mu \sigma_k \sum_{i=1}^4 \log(s_i) \]

subject to \[ C_j(u) - s = 0, \quad s \geq 0 \] \hspace{1cm} (17)

where \( \mu \) is a positive parameter and \( \log(\cdot) \) denotes the natural logarithm function. Since with no difficulty it can be shown that the objective and the constraint functions are convex the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for a global minimum (Boyd & Vandenberghe, 2009). Applying KKT formulation to (17), the optimality conditions can be expressed as,

\[ Gu + c + A_i^T \lambda = 0, \quad A_i(u) = -\nabla C_i(u), \quad C_j(u) - s = 0, \quad S \lambda - u = 0, \quad e = [1 \ 1 \ 1 \ 1]^T \]

\[ s > 0, \quad \lambda > 0 \]  \hspace{1cm} (18)

where \( \lambda \) denotes a 4-component vector containing Lagrange multipliers corresponding to four inequality constraints and \( S \) is a \( 4 \times 4 \) diagonal matrix whose diagonal elements are the components of vector \( s \). Primal-dual interior-point methods apply the Newton’s method to solve the set of nonlinear equations (18). The steps of primal-dual path-following interior-point method for our system are as follows:

1) Newton’s Step Direction: In each iteration, the step direction is obtained according to Newton’s method for nonlinear equations by replacing \( \{s, \lambda, u, \Delta u\} \) in KKT conditions (18) with \( \{s + \Delta s, u + \Delta u, \lambda + \Delta \lambda\} \) and taking only the first-order terms to get

\[ r_c = (G + A_2) \Delta u + A_i^T \Delta \lambda = 0 \]

\[ r_b = A_i \Delta u - \Delta s = 0 \]

\[ r_s = S \lambda + A \Delta s = 0 \]  \hspace{1cm} (19)

where

\[ r_c = Gu + c + A_i^T \lambda, \quad r_b = C_i(u) - s, \quad r_s = S \lambda - u \]

\[ A = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4), \quad A_2 = 2 \text{diag}(A, A) \]  \hspace{1cm} (20)

Equations (19) are linear in terms of step directions and can be solved to achieve step directions given by

\[ \Delta u = (G + A_2 + A_i \Delta \lambda - 1)(A_i S^{-1} (r_s + \Delta r_b) - r_c) \]

\[ \Delta \lambda = -S^{-1}(r_s + \Delta r_b) + S^{-1} \Delta A \Delta u \]

\[ \Delta s = r_b - A_i \Delta u \]  \hspace{1cm} (21)

2) Step length Calculation: Usually, a full step along the direction of (21) would violate the bound \( \{s, \lambda\} \geq 0 \), so a line search along the Newton direction is performed and the new iterate \( \{u^+, s^+, \lambda^+\} \) is calculated as

\[ u^+ = u + \alpha_{s} \Delta u, \quad s^+ = s + \alpha_{s} \Delta s, \quad \lambda^+ = \lambda + \alpha_{\lambda} \Delta \lambda \]

where \( \alpha_{s} \) and \( \alpha_{\lambda} \) are chosen so that the longest step lengths is obtained without violating nonnegativity condition.

3) Updating the Barrier Parameter \( \mu \): The sequence of barrier parameters \( \{\mu_k\} \) must converge to zero so that, in the limit, the solution of the nonlinear programming problem (17) is achieved. Hence the barrier parameter is updated as:

\[ \mu_{k+1} = \sigma_k \frac{s_k}{\lambda_k}, \quad \sigma_k \in [0,1] \]  \hspace{1cm} (23)

where the centering parameter \( \sigma_k \) is updated as follows:

\[ \sigma_k = \left( \frac{\mu_{aff}}{s_k^2 \lambda_k / \mu} \right)^3 \]  \hspace{1cm} (24)

with

\[ \mu_{aff} = (s_k + \alpha_{s} \Delta s_{aff}) (\lambda_k + \alpha_{\lambda} \Delta \lambda_{aff}) / 4 \]  \hspace{1cm} (25)

where \( \alpha_{s} \) and \( \alpha_{\lambda} \) are the corresponding longest step lengths.

4) Stopping Criteria: To identify a criterion for stopping iterations, the following error function, which is based on KKT conditions (18), is defined

\[ E(u,s,\lambda,\mu) = \max \{||Gu + c + A_i \lambda||, ||C_i(u) - s||, ||S \lambda - u||\} \]  \hspace{1cm} (26)

When \( E(u,s,\lambda,\mu) \) has converged sufficiently close to zero, the algorithm is terminated.

All of these steps are summarized in Table 1.

Table 1. Pseudocode for interior-point algorithm

<table>
<thead>
<tr>
<th>Given A, W, and v</th>
</tr>
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<tbody>
<tr>
<td>Compute G, c, and d using (16).</td>
</tr>
<tr>
<td>Choose initial value for control input u_0 and error tolerance ( \varepsilon ), set ( k \leftarrow 0 ), and compute initial values for ( s_0, \lambda_0, ) and ( \mu ).</td>
</tr>
<tr>
<td>Compute residuals ( r_c, r_b, ) and ( r_s ) given in equation (20).</td>
</tr>
<tr>
<td>Repeat until ( E(u,s,\lambda,\mu) \leq \varepsilon )</td>
</tr>
<tr>
<td>Compute step direction from equation (21),</td>
</tr>
<tr>
<td>Compute step sizes ( \alpha_{s} ) and ( \alpha_{\lambda} ) in (22),</td>
</tr>
<tr>
<td>Update the variables ( u, s, ) and ( \lambda ) using (23),</td>
</tr>
<tr>
<td>Compute ( \mu_{aff} ) from (25) and then ( \sigma_k ) from (24),</td>
</tr>
<tr>
<td>Update the barrier parameter ( \mu_{k+1} ) based on equation (23),</td>
</tr>
<tr>
<td>Compute residuals ( r_c, r_b, ) and ( r_s ) from (20),</td>
</tr>
<tr>
<td>Set ( k \leftarrow k + 1 )</td>
</tr>
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End
5. Dynamic Control Allocation

In the dynamic control allocation approach an update law is used for system inputs. As mentioned in (Johansen, 2004), since it is not required to solve the nonlinear constrained optimization problem at each sampling time, the main advantage of dynamic control allocation is its computational efficiency. In what follows, the introduced algorithm in (Johansen, 2004) is formulated for the proposed integrated vehicle control scheme. However, the interested reader might refer to (Johansen, 2004) for convergence analysis and more technical details of the considered method.

The problem is formulated by introducing the Lagrangian
\[
\ell = u^T W u + (v - Au)^T \lambda - \omega \sum_{i=1}^{4} \log \left( C_{i1}(u_i, u_{i+1}) \right)
\]
with \(\omega > 0\) and \(\lambda\) being a 3-component vector of Lagrange multipliers corresponding to equality constraint (11) and, hence, different form the similar symbol in (18).

By dynamic control allocation method \(u\) and \(\lambda\) are computed in the form of the Newton-like update law
\[
\begin{bmatrix}
\dot{u} \\
\dot{\lambda}
\end{bmatrix} = -\gamma (\mathbf{H}^T \mathbf{H} + \varepsilon I_p q)^{-1} \mathbf{H}^T \dot{\phi} + \frac{\partial F_z}{\partial u} + \frac{\partial F_z}{\partial \lambda}
\]
where \(\gamma > 0\) and \(\varepsilon \geq 0\), \(p\) and \(q\) are the dimensions of \(u\) and \(\lambda\) respectively, and for our problem \(\mathbf{H}\) is written as:
\[
\mathbf{H} = \begin{bmatrix}
\frac{\partial^2 \ell}{\partial u^2} & \frac{\partial (Au)^T}{\partial u} \\
\frac{\partial (Au)}{\partial u} & 0
\end{bmatrix}
\]
for (28) the feedforward-like terms \(\phi\) and \(\delta\) are chosen so that the following scalar algebraic equation holds:
\[
\alpha^T \phi + \beta^T \phi + \delta = 0
\]
with
\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \mathbf{H} \begin{bmatrix}
\frac{\partial \delta}{\partial u} \\
\frac{\partial \delta}{\partial \lambda}
\end{bmatrix}, \quad \delta = -(Au - v)^T v
\]
Assume that the stability of the system obtained by equations (1)-(4), with \(\beta\) and \(r\) as outputs and \(M\) and \(F_r\) as inputs, under high level controller could be shown through a Lyapunov function \(V_0(t, r, \beta)\), then, by the Lyapunov function
\[
V(t, r, \beta, u, \lambda) = \sigma V_0(t, r, \beta) + \frac{1}{2} \left( \frac{\partial \ell^T}{\partial u} \frac{\partial \ell}{\partial u} + \frac{\partial \ell^T}{\partial \lambda} \frac{\partial \ell}{\partial \lambda} \right)
\]
with \(\sigma > 0\) global exponential convergence to optimality conditions is shown in (Johansen, 2004).

It can be explained, under some considerations, that the right of equation (28) with \(\varepsilon = 0\) is equivalent to the solution of the set of linear equations in (19), i.e. one Newton’s step for solving KKT conditions. So, equation (28) can be interpreted as one Newton’s step from the solution of the control allocation problem at the current sampling time for the solution at the next instant. Equation (28) differs from (19) primarily in terms \(\phi\) and \(\delta\) which are due to \(\delta\) and hence \(v\). As described in (Johansen, 2004) these terms provide a feedforward-like compensation in the update law of (28) to maintain the time-varying optimum. However, the feedforward terms don’t appear in (19) and (21) since at each Newton’s step of a sampling time \(v\) is constant, and, when iterating the nonlinear equations of (18), no additional term arises due to variations of \(v\). Despite the log-barrier term in (27), depending on the size of \(\gamma\) and \(\omega\), there is no guarantee that this step would not violate the inequality constraints in (14). This can be resolved, however, by choosing \(\gamma\) in (28) through a line search approach, similar to the second step of the interior-point stages described in the previous section, or any other algorithm presented in (Nocedal & Wright, 2006). Then, this can require a barrier parameter adaptation for \(\omega\) in (28) to prevent the solution at each sampling time from approaching the boundaries of (15) prematurely so that a longer Newton’s step is possible from the current point, especially during the transient response. Also, equation (28) could induce a Newton’s step from an infeasible point, violating the inequality constraints (14), because of the time-varying nature of these constraints. This is due to the fact that the term \((\mu_i Z_i)^2\) in (15) is time-varying. Consequently, since the current inequality constraints are different than those of the succeeding sample time, it is possible for the feasible solution at the current time to violate the inequality constraints at the next sample time. Specifically, this can take place for example once the road conditions suddenly change to those of a slippery one, where a sudden slump occurs in the friction coefficient \(\mu_i\). Then, equation (28) needs to be modified for Newton’s steps from infeasible points.

After desired longitudinal and lateral forces of each tire are computed by each of the proposed control allocation algorithms the active steering angle, \(\delta_i\), at wheel \(i\) can be determined by using the inverse of simple tire model as explained in (Naraghi, Roshanbin, & Tavasoli, 2009). Also, by ignoring the rotational dynamics of wheels, it is assumed that the applied torque at each wheel equals to the wheel radius times the desired longitudinal force. Nevertheless, to avoid wheel lock, specifically in critical maneuvers where the demanded longitudinal forces by control allocation module may be high, an ABS for a brake by wire system is considered to compute the final torque, \(T_b\), at each wheel. The overall scheme of the proposed vehicle control system can be found in Figure 2.
6. SIMULATION RESULTS

In order to evaluate the effectiveness of the static and dynamic control allocation techniques to enhance the vehicle handling and stability, maneuvers in critical conditions of tire forces are investigated. A 9DOF nonlinear vehicle model is used for simulation. Dugoff’s tire model generates tire forces and the vehicle parameters used for simulation are those of a passenger car (Rajamani, 2006). Also a driver model, described and validated experimentally in (Abe, 2009), is used to simulate the driver’s behavior in the considered maneuvers. The results are, also, compared with those obtained by the proposed method in (Mokhyamar & Abe, 2004), where no saturation constraint is considered for ODF. For the first scenario, the vehicle behavior is tested in a standard double-lane change maneuver. The vehicle runs with an initial velocity of 110 km/h for .5 s followed by driver’s braking acceleration demand of -0.4g for 3 seconds, on a slippery road, where tire-road friction coefficient is considered to be 0.4. The results are shown in Figure 3. As seen by figure 3, due to critical conditions of maneuver the vehicle guided only by driver and without any active control has failed to track the desired yaw rate completely and, therefore the vehicle with no control is far from the driver’s desired path. It is seen that by active control system employing the unconstrained ODF method the performance of yaw tracking, and, thus, desired path following, have been increased significantly. On the other hand the system exploiting either static or dynamic control allocation has the best performance in tracking the desired yaw and, also, in reducing the side-slip, with a desired value of zero, and, as a result, as shown by Figure 3, path of the vehicle with each of these systems has neared the driver’s intent more effectively.

The reasons behind these observations can be described by referring to tire forces distribution plots. First it is seen that, since saturation constraints are ignored by ODF, this system has assigned large values of longitudinal and lateral forces, far from being actually achievable, to the front tires in parts of maneuvers. As Figure 3 shows, both static and dynamic control allocation methods, however, by considering tires saturation limits, have presented more proper and balanced force distribution and, as a result, the actual forces will be near the demanded ones. Also by dynamic control allocation the allocated forces to tires have appropriately converged to the real optimal values of static control allocation, and both methods have performed, with a minor difference, the same control allocation task. Consequently, the vehicle under static and dynamic control allocation systems has had almost the same operation in this case. As tires work-load plot shows, by unconstrained ODF system the demanded work-load of each wheel, in some parts of scenario, is more than its saturation limit, i.e. more than 1, and hence these demanded values by ODF are not actually achievable. At the same time, the other tires have yet limit to saturation, which could be utilized for vehicle control. On the other hand, by static control allocation
method all tires are saturated uniformly, for a period of time, to utilize the maximum capacity of integrated system for vehicle control. Also, dynamic control allocation has performed more balanced work-load distribution than ODF method, although it has failed to converge completely to the exact optimal load distribution, i.e. completely saturating all tires, in critical instants, as accomplished by static control allocation. Furthermore, both control allocation techniques enhance the vehicle control task not only through balancing the work-load distribution, but also through adjusting the contribution of longitudinal and lateral tire forces to control objectives more properly. For example, in this special case, dynamic and static control allocation approaches, by considering saturation constraints, have commanded lower level of longitudinal tire forces at some instants, compared to unconstrained ODF. Large value of longitudinal forces by ODF results in a detrimental reduction of the final generated lateral tire forces at the handling limit, due to tires saturation. This increases the error between the desired and really produced values of the total lateral force and yaw moment and, so, the performance of the vehicle control in terms of reducing the side-slip angle and tracking the desired yaw rate is lessened by unconstrained ODF also in this manner.

In the considered scenario, although both static and dynamic allocation methods offer almost the same performance of vehicle control, they have yet some minor differences. For example, in some points, oscillatory yaw rate response and also more side-slip angle by dynamic control allocation are evident in figure 3. These differences between two methods are in that the allocated forces by dynamic allocation have not completely converged to exact optimal set provided by static allocation. This is mainly because of transient response of dynamic control allocation, specifically to panic reactions of driver in both steering and braking at critical conditions, and also its relatively low robustness against highly nonlinear characteristics of vehicle dynamics and tire forces. The transient response by dynamic control allocation can be even more considerable once sudden changes occur in tire/road friction coefficient, e.g. when the vehicle suddenly enters a slippery or a split-μ road, or after an actuator fails in a specific maneuver where this method is used for actuator failure compensation, which is one of the main advantages of control allocation methods (Härkegard & Glad, 2005).

In order to evaluate the vehicle operation under considered methods in more critical conditions, we examine another double-lane change with the same conditions of the first scenario except for the vehicle initial velocity which is increased by 130 km/h. The results are shown in Figure 4. Figure 4 reveals that once again static and dynamic control allocation methods have the best properties in converging to the desired yaw rate and side-slip angle, and, thus, in following the desired path. However, oscillatory response of yaw rate and also growth of side-slip angle by dynamic control allocation are more considerable in this more critical maneuver. Accordingly, by static control allocation the vehicle path has better converged to the driver’s intent. To explore the causes of these differences between the static and dynamic allocation methods in this scenario, tire forces are plotted for both methods.

Fig. 4. Simulation results for the second scenario
As tire forces plots in Figure 4 reveal, in this more critical manoeuvre, where the nonlinear characteristics of vehicle dynamics and tire forces are more significant and, also, the driver exhibits more panic behaviour, both allocated lateral and longitudinal forces of tires by dynamic control allocation in some instants are far from those distributed by static control allocation, i.e. from optimal set. As a result, the lateral force of front-left tire by dynamic allocation is in some parts significantly far from the optimal one and the other lateral forces are oscillatory around the optimal set. The difference between forces distributed by dynamic allocation and those allocated by static allocation is even more considerable for longitudinal forces.

7. CONCLUSIONS

Nonlinear constrained optimal distribution of tire forces in an integrated vehicle dynamics control scheme was presented. Static and dynamic control allocation techniques were formulated and utilized for this purpose. Interior-point method was used to handle the nonlinear constrained optimization problem in static control allocation and also a dynamic update law was employed for dynamic control allocation task. Simulation results were conducted and vehicle operation under static and dynamic control allocation was evaluated. The results of both methods were compared with each other and also with the results of previous work in literature, i.e. unconstrained ODF. Since no optimization problem is required in dynamic allocation, the main advantage of this method over static allocation, where a nonlinear programming is required at each sampling time, is its more computational efficiency for real-time implementations. Also, it was seen that each method by presenting more-balanced work-load distribution and also by adjusting the contribution of longitudinal and lateral tire forces to control objectives more properly, can effectively implement more capacity of tire forces to enhance the performance of the vehicle control task, in comparison with unconstrained ODF. Furthermore, although by dynamic control allocation the set of distributed tire forces approximately approaches the optimal one, i.e. that of static control allocation, there is yet an amount of error, depending on severity of maneuver. This error of dynamic control allocation is mainly due to its transient response to panic behavior of driver and its relatively low robustness to highly nonlinear characteristics of vehicle dynamics and tire forces. Therefore, in maneuvers with more critical conditions, where the nonlinear characteristics of vehicle dynamics and tire forces are more substantial and, also, the driver might exhibit more panic reactions, the error between the allocated tire forces by dynamic control allocation and optimal solution of static control allocation is more significant. These errors will become even more noticeable, for example, when a sudden change in tire/road friction coefficient occurs, or when dynamic allocation is utilized to compensate for an actuator failure occurrence. Consequently, when compared to the results of static control allocation, the performance of the integrated vehicle control task through dynamic control allocation approach can be decreased in such maneuvers.

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