Position Control of Flexible Joint Robots by Adapting Methods for Rigid Robots

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Abstract: In this paper, it is demonstrated that many position control laws developed for rigid joint robots can also be used for flexible joint robots after some modifications. The main assumptions are smoothness of the control law and the availability of a Lyapunov function. The adaptation is based on a simple control principle: With the help of a position control method known from rigid joint robots, desired values for the motor positions are determined. If the motor positions and their desired values are identical, the link positions converge to their desired trajectories. Afterwards, the system inputs are specified such that the motor positions converge to their desired values. The method can also be applied under system uncertainties.

1. INTRODUCTION

Numerous robot manipulators, in particular those which are equipped with harmonic drive gears, possess a significant joint flexibility. Ignoring joint flexibility during modeling and control of manipulators may cause serious performance losses (Good et al. [1985], Sweet et al. [1984]). On the other hand, consideration of joint flexibility induces a more complicated control problem. The main reason for this is that in contrast to rigid robots, flexible joint robots are under-actuated. The motor positions at the joints double the degree of freedom as additional generalized coordinates besides the link positions. In the case of perfect system knowledge, feedback linearization of flexible joint robots is possible. However, the link accelerations and jerks, which are difficult to measure, become the new states of the linear model (Spong [1987]).

The position control problem for rigid robots has intensively been studied especially in the 1980s. As a result of these efforts, several control strategies have been proposed so far (e.g. Kumagai et al. [2009], Slotine et al. [1987], Spong [1992], Tang [1998]). In this paper, an interesting connection between rigid and flexible joint robots is introduced: If a position control law developed for rigid joint robots is of class $C^2$ and possesses a known Lyapunov function, it can also be used for the position control of flexible joint robots after certain modifications.

In a certain sense, the position control methods for flexible joint robots which are based on the singular perturbation theory also build a connection between rigid and flexible joint robots (Ge [1996], Spong et al. [1987], Subudhi et al. [2003], Taghirad et al. [2006]). However, those methods work only with the assumption that the stiffness coefficients are sufficiently large. A robot with highly stiff joints can approximately be split into a slow subsystem and a fast one. The slow subsystem is represented by the dynamics of a rigid joint robot, and thus, for this part of the system a control algorithm known from rigid robots can be used. In the present work, the magnitude of the stiffness does not play any role, therefore, the proposed algorithm is more generic.

Since the early 1990s, various control algorithms for flexible joint robots have been proposed. They differ from each other in certain respects: The adaptive control algorithms of Benallegue [1995], Chang et al. [1992], Chien et al. [2007], Ge [1996], and Yuan et al. [1993] are able to compensate the undesirable effects of unknown system parameters. Benallegue [1995], and Yuan et al. [1993] require acceleration and jerk measurements, which may hinder their applications. The robust control algorithms of Ailon [1996], Patel et al. [2006], and Yeon et al. [2008] consider a more generic system class, where neglected system dynamics as well as parameter uncertainties are permissible. In addition to these works, the approaches of variable structure control (Hsu et al. [1993]), neural networks (Chatlatanagulchai et al. [2005]), optimal control (Consolini et al. [2007]), $H^\infty$ (Taghirad et al. [2002]), and PID (Malki et al. [1997]) have been applied to the control of flexible joint robots to date.

The works in the literature differ from the present work fundamentally: They develop specific control methods, as the work here describes a way to adapt the methods from rigid robots to flexible ones. The idea of the adaptation is rather simple: The link positions will converge to their desired trajectories, if the motor positions track some appropriate trajectories. If we regulate the motor positions to these appropriate trajectories, the link positions must converge to their desired trajectories autonomously.

After giving the control problem in Section 2, and the preliminaries in Section 3, the effectiveness of this idea is first demonstrated for systems without uncertainties in Section 4. A small number of formulas suffices to present the main idea. In Section 5, system uncertainties are considered in such a manner that the adaptation of a certain subset of robust and/or adaptive control algorithms known from rigid robots is made possible. In Section 6, the well-known adaptive control algorithm of Slotine and Li [1987] is treated as an example, and
afterwards, this specific algorithm is simulated on a single link robot in Section 7.

2. SYSTEM CLASS AND PROBLEM STATEMENT

Flexible joint robots of the form (Spong [1987])

\[ M(q)\ddot{q} + h(q, \dot{q}) + K(q - \theta) = 0 \]  
\[ T\dot{\theta} + B\dot{\theta} - K(q - \theta) = \tau \]

are considered, where the vectors \( q, \theta \) and \( \tau \) are in \( \mathbb{R}^n \), and denote the link positions, the motor positions, and the motor torques at the joints, respectively. (Thus, \( q - \theta \) corresponds to the joint deformation). The matrix \( M(q) \in \mathbb{R}^{n \times n} \), which is symmetric and positive definite, denotes the mass matrix of rigid links. The vector \( h(q, \dot{q}) \in \mathbb{R}^n \) consists of the centrifugal, Coriolis, and gravitational forces. The matrices \( K, T \) and \( B \), which are diagonal and positive definite, are in \( \mathbb{R}^{n \times n} \), and denote the stiffness, inertia, and viscous friction matrices of the joints, respectively.

In the following, the arguments of the vector and matrix functions will often be neglected for the sake of brevity, if they are obvious from the context.

**Assumption 1:** The left side of Eq. (1) contain only smooth terms.

Let the function \( q_d : [t_0, \infty) \rightarrow \mathbb{R}^n \) denote the desired trajectory for the link positions \( q \), and be of class \( C^4 \), where \( t_0 \) denotes the initial time of the system (1)-(2). The control problem is formulated as follows: Specify the system inputs \( \tau \) such that the system and controller variables remain bounded, and the link positions \( q \) converge to their desired trajectories \( q_d \).

3. PRELIMINARIES

Multiplying Eq. (1) by \( K^{-1} \) from the left converts Eq. (1) into

\[ K^{-1}M(q)\ddot{q} + K^{-1}h(q, \dot{q}) + q = \theta . \]  

The structure of Eq. (3) is very similar to equations of motion of rigid joint robots, for whose position control there have been numerous methods in the literature (Spong et al. [2009]). The main difference between them is that the term \( \theta \) in (3) is replaced by the system inputs (motor torques at the corresponding joints) in equations of motion of rigid joint robots. This similarity leads us to the following simple consideration: With the help of the well-known position control methods belonging to rigid joint robots, one can determine desired values \( \theta_d \) for the motor positions \( \theta \) such that the link positions \( q \) converge to their desired trajectories \( q_d \), if \( \theta = \theta_d \) for all \( t \in [t_0, \infty) \) holds. After that, the system inputs \( \tau \) can be determined such that the motor positions \( \theta \) converge to their desired values \( \theta_d \).

In general, position control laws for rigid robots are generated by means of dynamical controllers. First step of an adaptation is to decide for a specific control law known from rigid robots, and replace the respective control law with \( \theta_d \). This will result in desired motor position values of the form

\[ \dot{x} = f(x, q, \dot{q}, t) \]  
\[ \theta_d = g(x, q, \dot{q}, t) \]

where \( x \) is the vector of controller states, and the vector fields \( f \) and \( g \) describe the state and output flows of (4)-(5), respectively. \( f \) and \( g \) explicitly depend on \( t \) since they usually include \( q_d \) and its first and second time derivatives.

**Assumption 2:** \( \theta_d \) in (5) is of class \( C^2 \).

**Assumption 3:** The original control law corresponding to Eq. (4)-(5) has been derived by means of a Lyapunov based control strategy, which stabilizes rigid robot dynamics globally.

Let the used Lyapunov function in the original control algorithm be \( V_0(e_x, e_1, e_2) \), where the error variables \( e_x \), \( e_1 \) and \( e_2 \) are defined by

\[ e_x = x - x_d \]  
\[ e_1 = q - q_d \]  
\[ e_2 = \dot{q} - \dot{q}_d \]

and the vector \( x_d \) denotes the desired values for the controller states \( x \). The time derivative of \( V_0(e_x, e_1, e_2) \) along the trajectories of (3) and (4) is

\[ \dot{V}_0(e_x, e_1, e_2) = \frac{\partial V_0}{\partial e_x} e_x + \frac{\partial V_0}{\partial e_1} e_1 + \frac{\partial V_0}{\partial e_2} e_2 \]

\[ = \frac{\partial V_0}{\partial e_x} e_x + \frac{\partial V_0}{\partial e_1} e_1 + \]

\[ \frac{\partial V_0}{\partial e_2} \left[ -M^{-1}(h + K(q - \theta)) - \dot{q}_d \right] , \]  

and is generally of the form

\[ \dot{V}_0(e_x, e_1, e_2) = L(e_x, e_1, e_2) + \varepsilon_1 , \]

if \( \theta \) and \( \theta_d \) are identical, where \( L \) is a negative definite function of \( e_x \), \( e_1 \) and \( e_2 \), and \( \varepsilon_1 \) is either zero or a positive real constant, which can be made arbitrarily small by the control parameters. In the literature, \( \varepsilon_1 \) is encountered only in the control algorithms considering system uncertainties.

**Assumption 4:** Eq. (10) holds, if \( \theta = \theta_d \) for all \( t \in [t_0, \infty) \). Furthermore, \( \varepsilon_1 = 0 \), if the system model of (3) is perfect. \( \varepsilon_1 \geq 0 \), if the model and system equations concerning (3) are not identical, and thus, Eq. (4) and (5) correspond to a robust and/or adaptive control method.

In general, Eq. (10) does not hold, since \( \theta \) differs from \( \theta_d \). Let the vector \( e_3 \) denote the difference between them

\[ e_3 = \theta - \theta_d . \]  

Substituting (11) into (9) yields for the time derivative of \( V_0 \) along the trajectories of (3) and (4)

\[ \dot{V}_0 = L + \varepsilon_1 + \frac{\partial V_0}{\partial e_3} M^{-1} K e_3 . \]  

The last error variable is defined by

\[ e_4 = \dot{\theta} - \alpha , \]

where the vector \( \alpha \) is a function of \( x, q, \dot{q}, \theta, \) and \( t \), and denotes the desired values for the motor velocities \( \dot{\theta} \). \( \alpha \) will be given in Section 4 and 5 in two different ways, and will play a crucial role for the proposed algorithm.

4. CONTROLLER DESIGN IN THE CASE OF PERFECT SYSTEM KNOWLEDGE

The feedback control law
\[ \tau = Tu + B\dot{\theta} - K(q - \theta) \] (14)

converts Eq. (2) into
\[ \ddot{\theta} = u , \] (15)

where the vector \( u \in \mathbb{R}^n \) denotes the new system inputs. Let the before mentioned function \( \alpha \) be
\[ \alpha(x, q, \dot{q}, \dot{\theta}, t) = \dot{\theta}_d - K_3 e_3 - K M^{-1} \left( \frac{\partial V_0}{\partial \theta_2} \right)^T , \] (16)

where \( K_3 \in \mathbb{R}^{n \times n} \) is a diagonal and positive definite control matrix. The term \( \dot{\theta}_d \) in (16) is calculated with the help of the system and controller equations (3), (4) and (5) as follows
\[ \dot{\theta}_d(x, q, \dot{q}, \dot{\theta}, t) = \frac{\partial g}{\partial x} f + \frac{\partial g}{\partial q} \dot{q} + \frac{\partial g}{\partial \theta} M^{-1} [ - h - K(q - \theta) ] + \frac{\partial g}{\partial t} . \] (17)

In the following, it will be shown that the control law
\[ u(x, q, \dot{q}, \dot{\theta}, \dot{\theta}, t) = \dot{\alpha} - K x e_4 - e_3 \] (18)

globally asymptotically stabilizes the system (1)-(2) (or equivalently (3)-(15)), where \( K_4 \in \mathbb{R}^{n \times n} \) is a diagonal and positive definite control matrix, and \( \dot{\alpha} \) is calculated with the help of the system and controller equations (3) and (4) as follows
\[ \dot{\alpha}(x, q, \dot{q}, \dot{\theta}, \dot{\theta}, t) = \frac{\partial \alpha}{\partial x} f + \frac{\partial \alpha}{\partial q} \dot{q} + \frac{\partial \alpha}{\partial \theta} M^{-1} [ - h - K(q - \theta) ] + \frac{\partial \alpha}{\partial t} . \] (19)

Theorem 1. The equilibrium point of the error dynamics \( e_x = 0, e_1 = 0, e_2 = 0, e_3 = 0, e_4 = 0 \) with respect to the control laws (14) and (18) is globally asymptotically stable.

Proof. Consider the globally positive definite function (Lyapunov function candidate)
\[ V(e_x, e_1, e_2, e_3, e_4) = V_0 + \frac{1}{2} e_x^T e_x + \frac{1}{2} e_4^T e_4 . \] (20)

The identities
\[ \dot{e}_3 = \dot{\theta} - \dot{\theta}_d = e_4 + \alpha - \dot{\theta}_d \] (21)
\[ \dot{e}_4 = \dot{\theta} - \dot{\alpha} = u - \dot{\alpha} \] (22)

and Eq. (12) (with \( e_1 = 0 \)) yield for the time derivative of \( V \) along the system trajectories
\[ \dot{V} = \dot{V}_0 + e_x^T \dot{e}_x + e_4^T \dot{e}_4 = L + \frac{\partial V_0}{\partial \theta_2} M^{-1} K e_3 + e_3^T ( e_4 + \alpha - \dot{\theta}_d ) + e_4^T ( u - \dot{\alpha} ) . \] (23)

Substituting (16) and (18) into (23) gives
\[ \dot{V} = L(e_x, e_1, e_2) - e_3^T K_3 e_3 - e_4^T K_4 e_4 . \] (24)

Eq. (24) implies that \( V \) is globally negative definite. Thus, it can be stated that \( V \) is a Lyapunov function, and the origin of the error dynamics is globally asymptotically stable.

Note that, due to Assumption 2 and 3, the control variables \( \theta_d, \dot{\theta}_d, \dot{\theta}_d, \alpha \) and \( \dot{\alpha} \) remain bounded, as the error variables converge to zero.

5. CONTROLLER DESIGN UNDER SYSTEM UNCERTAINTIES

In the following, the control algorithm in Section 4 will be extended by introducing possible system uncertainties. If the system model and the system dynamics are not identical, it can not be guaranteed that the algorithm given in the previous section is stabilizing.

In this section, the model matrices and vectors are denoted by \( M(q), h(q, \dot{q}), \dot{K}, \dot{T} \) and \( B \), and they may differ from the relating system matrices and vectors \( M(q), h(q, \dot{q}), \dot{K}, \dot{T} \) and \( B \), which are supposed to be unknown, and therefore, are not allowed to be used in the control law. The differences between the system and model components may arise due to both parameter mismatching and unmodeled (or neglected) parts of the system dynamics.

The following assumptions will enable us to find appropriate upper bound functions for the uncertain terms \( \delta_1 \) and \( \delta_2 \) defined subsequently.

Assumption 5: The structures of the unmodeled terms are known, even though their parameters are unknown. \( \square \)

Assumption 6: Upper and lower bounds for the unknown system parameters can be determined. \( \square \)

The main difficulty due to system uncertainties is that one cannot calculate \( \dot{\theta}_d \) exactly. However, \( \dot{\theta}_d \) can be calculated approximately, if the unknown system matrices \( M \) and \( K \), and vector \( h \) in (17) are replaced by their model counterparts \( \hat{M}, \hat{K} \) and \( \hat{h} \). Let \( \dot{\theta}_d^\delta \) denote this approximate value (known part) of \( \dot{\theta}_d \)
\[ \dot{\theta}_d^\delta(x, q, \dot{q}, \dot{\theta}, \dot{\theta}, t) = \frac{\partial g}{\partial x} f + \frac{\partial g}{\partial q} \dot{q} + \frac{\partial g}{\partial \theta} M^{-1} [ - \hat{h} - \hat{K}(q - \theta) ] + \frac{\partial g}{\partial t} , \] (25)

and \( \Delta \dot{\theta}_d \) denote the approximation error
\[ \Delta \dot{\theta}_d = \dot{\theta}_d - \dot{\theta}_d^\delta . \]

Determining the motor torques as
\[ \tau = Tu + B\dot{\theta} - K(q - \theta) \] (26)

converts Eq. (2) into
\[ \ddot{\theta} = u + \Delta u , \] (27)

where \( \Delta u \) is defined by
\[ \Delta u = (T^{-1} T - I) u + T^{-1} ( \hat{B} - B ) \dot{\theta} - ( \hat{K} - K ) (q - \theta) . \]

The definitions of the vectors \( \delta_j, b_j, \beta_j \) (\( j = 1, 2 \)), the modified \( \alpha, \dot{\alpha}, K, \) and \( \Delta \alpha \), which will be required by the robust control law, are given as follows: \( \delta_1 \) is the unknown vector
\[ \delta_1(x, q, \dot{q}, \dot{\theta}, \dot{\theta}, t) = K M^{-1} \left( \frac{\partial V_0}{\partial \theta_2} \right)^T - \Delta \dot{\theta}_d \]
and \( b_1(x, q, \dot{q}, \dot{\theta}, \dot{\theta}, t) \) is a known upper bound vector for \( \delta_1 \) such that \( b_1 \) is bounded in \( t \), and the inequalities
\[ |\delta_1| < b_1 \quad i = \{ 1, \ldots, n \} \] (28)
hold, where the index \( i \) points to the \( i \)th element of the vectors \( \delta_1 \) and \( b_1 \). The vector \( \beta_1 \) is defined by
\[ \beta_1 = -b_1, \tan \left( \frac{c_1 b_1}{\varepsilon_2} \right) \quad i = \{ 1, \ldots, n \} , \] (29)
where \( \varepsilon_2 \in \mathbb{R}^n \) is a control vector with positive real elements. Differently to the previous section, the function \( \alpha \) is given by

\[
\alpha(x, q, \dot{q}, \theta, \dot{\theta}, t) = \dot{\theta}^k - K_3 e_3 + \beta_1 .
\]

(30)

Similar to \( \dot{\theta}_d \), the vector \( \dot{\alpha} \) cannot be calculated exactly in the presence of system uncertainties. On the other hand, \( \dot{\alpha} \) can be calculated approximately, if the unknown system matrices and vectors arising in \( \dot{\alpha} \) are replaced by their model counterparts. \( \dot{\alpha}^k \) denotes this approximate value of \( \dot{\alpha} \), and given by

\[
\dot{\alpha}^k(x, q, \dot{q}, \theta, \dot{\theta}, t) = \frac{\partial \alpha}{\partial x} f + \frac{\partial \alpha}{\partial q} \dot{q} + \frac{\partial \alpha}{\partial \theta} M^{-1} [-h - K(q - \theta)] + \frac{\partial \alpha}{\partial \theta} \dot{\theta} + \frac{\partial \alpha}{\partial t} ,
\]

(31)

and \( \Delta \dot{\alpha} \) is defined by

\[
\Delta \dot{\alpha} = \dot{\alpha} - \dot{\alpha}^k .
\]

(32)

\( \delta_2 \) is the unknown vector

\[
delta_2(x, q, \dot{q}, \theta, \dot{\theta}, t) = \Delta u - \Delta \dot{\alpha} ,
\]

and \( \delta_2(x, q, \dot{q}, \theta, \dot{\theta}, t) \) is a known upper bound vector for \( \delta_2 \) such that \( \delta_2 \) is bounded in \( t \), and the inequalities

\[
|\delta_2| < b_2 , \quad i = \{1, \ldots, n\}
\]

(33)

hold. In fact, \( \delta_2 \) also depends on the input \( u \). On the other hand, \( u \) will be determined as a function of \( x, q, \dot{q}, \theta, \dot{\theta}, \) and \( t \). Therefore, the notation of \( \delta_2 \) in (32) is relevant. Note that, due to Assumptions 5 and 6, the upper bound functions \( b_1 \) and \( b_2 \) can easily be determined by appropriate simple algebraic estimates. At last, the vector \( \beta_2 \) is defined by

\[
\beta_2 = -b_2 \tanh\left(\frac{e_4^T b_2}{\varepsilon_3}\right), \quad i = \{1, \ldots, n\},
\]

(34)

where \( \varepsilon_3 \in \mathbb{R}^n \) is a control vector with positive real elements. With the help of the above definitions, the robust control law is given by

\[
u(x, q, \dot{q}, \theta, \dot{\theta}, t) = \dot{\alpha}^k - K_3 e_4 - e_3 + \beta_2.
\]

(35)

Theorem 2. The solutions of the error dynamics with respect to the robust control laws (26) and (35) are globally uniformly ultimately bounded.

Proof. Let the Lyapunov function candidate be

\[
V = V_0 + \frac{1}{2} e_3^T e_3 + \frac{1}{2} e_4^T e_4 .
\]

(36)

The identities

\[
\dot{e}_3 = \dot{\theta} - \dot{\theta}_d = e_4 + \alpha - \dot{\theta}_d^k - \Delta \dot{\theta}_d
\]

(37)

\[
\dot{e}_4 = \dot{\alpha} - \dot{\alpha}^k = \Delta u - \alpha^k - \Delta \dot{\alpha}
\]

(38)

and Eq. (10) yield for the time derivative of \( V \) along the system trajectories

\[
\dot{V} = \dot{V}_0 + e_3^T (e_3 + e_3^T e_3) + e_4^T (e_4 + e_4^T e_4)
\]

\[
= L + \varepsilon_1 + \frac{\partial V_0}{\partial \epsilon_2} M^{-1} K e_3 + e_3^T (e_4 + \alpha - \dot{\theta}_d^k - \Delta \dot{\theta}_d) +
\]

\[
e_4^T (u + \Delta u - \alpha^k - \Delta \dot{\alpha}) ,
\]

(39)

Substituting (30) and (35) into (39) yields

\[
\dot{V} = L + \varepsilon_1 + e_3^T e_4 - K_3 e_3 + \beta_1 + K M^{-1} \left( \frac{\partial V_0}{\partial \epsilon_2} \right)^T
\]

\[
- \Delta \dot{\theta}_d] + e_3^T (-K_3 e_4 - e_4 + \beta_2 + \Delta u - \Delta \dot{\alpha})
\]

\[
\leq L + \varepsilon_1 - e_3^T K_3 e_3 - e_4^T K_4 e_4 + e_3^T \beta_1 + \sum_i |e_3, b_i| +
\]

\[
e_4^T \beta_2 + \sum_i |e_4, b_i|,
\]

\[
< L - e_3^T K_3 e_3 - e_4^T K_4 e_4 + c ,
\]

(40)

where \( c = \varepsilon_1 + 0.3 \sum \varepsilon_2, + 0.3 \sum \varepsilon_3 \). Note that the inequality (40) follows from the fact

\[
0 \leq |y| - y \tanh(\gamma/y) < 0.3 \varepsilon ,
\]

where \( y \) is a real number, and \( \varepsilon \) is a positive real number. The Lyapunov function \( V \) decreases if the error variables satisfy the inequality

\[
-L(e_3, e_1, e_2) + e_3^2 K_3 e_3 + e_4^2 K_4 e_4 = c .
\]

Let \( \mathcal{S} \) denote the smallest level surface of the Lyapunov function \( V \), which includes the ellipsoid

\[
-L(e_3, e_1, e_2) + e_3^2 K_3 e_3 + e_4^2 K_4 e_4 = c .
\]

The solutions of the error dynamics will be in \( \mathcal{S} \) within finite time, and then, stay in it. \( \square \)

6. APPLICATION TO THE METHOD OF SLOTINE AND LI

In this section, the well-known adaptive control method of Slotine and Li [1987], which has been proposed for position control of rigid joint robots with unknown parameters, is adapted for flexible joint robots. The method requires splitting the system vector \( h(q, \dot{q}) \) as follows

\[
h(q, \dot{q}) = C(q, \dot{q}) \ddot{q} + g(q) ,
\]

(41)

where the matrix multiplication \( C(q, \dot{q}) \ddot{q} \) includes the Coriolis and centrifugal forces, and the vector \( g(q) \) the gravitational forces. This notation allows us to use the fact that the matrix \( M - 2C \) is skew symmetric.

Before giving \( \theta_d \), the matrices \( Y \) and \( Z \), and the vectors \( p \) and \( r \) are introduced: The fact that the robot dynamics is linear in parameters, and Eq. (41) make possible to write the first two terms of (3) in the form

\[
K^{-1} M(q, \dot{q}) \ddot{q} + K^{-1}[C(q, \dot{q}) \ddot{q} + g(q)] = Y(q, \dot{q}, t) p .
\]

(42)

where \( p \in \mathbb{R}^l \) is the vector of the unknown system parameters, and \( Y(q, \dot{q}, t) \in \mathbb{R}^{n \times l} \) is the matrix of the known functions multiplied by the elements of \( p \). The \( n \)-dimensional vectors \( v \) and \( r \) are defined by

\[
v = \dot{q}_d - K_1 (q - q_0) ,
\]

(43)

\[
r = \dot{q} - v = e_1 - K_1 e_2 ,
\]

(44)

where \( K_1 \in \mathbb{R}^{n \times n} \) is a diagonal and positive definite control matrix. The matrix \( Z(q, \dot{q}, t) \in \mathbb{R}^{n \times l} \) is obtained by replacing \( \ddot{q} \) and \( \dot{q} \) with \( v \) and \( \dot{v} \) in the following manner

\[
K^{-1} M(q) \ddot{v} + K^{-1}[C(q, \dot{q}) \ddot{v} + g(q)] = Z(q, \dot{q}, t) p .
\]

(45)

With the help of the above definitions, the controller states \( x \) and the desired motor positions \( \theta_d \) are given by

\[
\dot{x} = -\Gamma^{-1} [\Gamma^T r + \Psi (x - p^2)]
\]

(46)

\[
\dot{\theta}_d = q + Z x - K_2 r ,
\]

(47)

where \( \Gamma \) and \( \Psi \) are \( l \times l \), diagonal, and positive definite control matrices, and \( p^2 \) is an initial estimate of the
unknown parameters $p$. Eq. (46) is actually a parameter adaptation law. Therefore, the state $x$ corresponds to a time-varying estimate of $p$. Thus, $e_x = x - p = x - x_d$ refers to the parameter estimation error. Subtracting Eq. (45) from Eq. (42) yields

$$K^{-1}M\dot{x} + K^{-1}C\dot{r} = Yp - Zp.$$  (48)

Due to (3) and (42), the equation

$$Yp = \theta_d + e_3 - q$$  (49)

holds for $Yp$. Eq. (47) and the definition of $e_x$ yield

$$Zp = \theta_d - q + K_2 x - Ze_x.$$  (50)

Substituting (49) and (50) into (48) gives the first part of the error dynamics as follows

$$K^{-1}M\dot{x} + K^{-1}C\dot{r} = -K_2 x + Ze_x + e_3.$$  (51)

Consider the globally positive definite function

$$V_0(e_1, e_2, e_3) = \frac{1}{2}e^T K^{-1} M r + \frac{1}{2}e^T \dot{r} e_x.$$  (52)

which will form the first part of the Lyapunov function $V$. The time derivative of $V_0$ along the trajectories of (46) and (51) is

$$\dot{V}_0 = r^T K^{-1} M \dot{r} + \frac{1}{2}r^T K^{-1} \dot{r} e_x + e^T \dot{e}_x = r^T (\dot{K}^{-1} \dot{C} r - K_2 \dot{r} + Z e_x + e_3) + \frac{1}{2}r^T K^{-1} \dot{M} r = -r^T K_2 \dot{r} + \frac{1}{2}r^T e_x - e^T \dot{e}_x.$$  (53)

The last term of (53) satisfies the inequality

$$-e^T \dot{e}_x \leq -\frac{1}{2}e^T \dot{e}_x \quad \text{due to the relationship } 2(a - b)(a - c) = (a - b)^2 + (a - c)^2 - (b - c)^2, \text{ where } a, b \text{ and } c \text{ are real numbers, and}$$

$$e_1 = \frac{1}{2}(p - p^o)^T \dot{y}(p - p^o).$$

Thus, the inequality

$$\dot{V}_0 \leq -r^T K_2 \dot{r} - \frac{1}{2}e^T e_x + e_1 + r^T e_3.$$  (54)

is obtained, as required by the proposed method in the previous section. Eq. (54) is of the form (12). Therefore, the system trajectories are globally uniformly ultimately bounded with respect to the control laws (26) and (35).

7. SIMULATIONS

The adaptation scheme described in the previous section has been simulated on an one link robot with a flexible joint, whose equations of motion are

$$ml^2 \ddot{q} + mgl \sin(q) + K(q - \theta) = 0$$

$$T\dot{\theta} + B\dot{\theta} - K(q - \theta) = \tau,$$

with the system parameters $m = 0.8, l = 0.8, g = 9.80, T = 0.8, B = 0.8, K = 8$. On the other hand, the model and control parameters are chosen as $\dot{m} = 1.0, l = 1.0, \dot{g} = 9.81, T = 1.0, B = 1.0, K = 10, K_1 = 5, K_2 = 8, K_3 = 15, K_4 = 10, \Gamma = 0.1I_4, \Psi = I_4, e_2 = 25, e_3 = 25$. The unknown functions $\delta_1$ and $\delta_2$, for which the upper bound functions $b_1$ and $b_2$ have been calculated by simple algebraic estimates, are obtained as follows:

$$\delta_1 = -(K_1 x_1 - K_2)[\frac{q}{l} - \dot{q}_{\theta} \sin(q)] + \frac{K}{ml^2} - \frac{\dot{K}}{ml^2}(q - \theta)$$

$$\delta_2 = (T^{-1} \dot{r} - 1)(\dot{\theta} - K_1 e_3 - e_3 + 1) + \frac{\partial \alpha}{\partial q} \left[ \frac{q}{l} \dot{\theta} \sin(q) + \frac{K}{ml^2} - \frac{\dot{K}}{ml^2} (q - \theta) \right].$$

The desired link position $q_0$ and the arising $q, \theta_d, \theta, \alpha, \dot{\theta}$, and $\tau$ trajectories are depicted in Fig. 1. As seen in Fig. 1, the control objective is achieved by a continuous trajectory of $\tau$.

8. CONCLUSIONS AND FUTURE WORK

A method to adapt position control algorithms known from rigid robots to flexible ones is proposed. It is shown that an adaptation is possible, provided that the original control law is of class $C^2$, and a Lyapunov function is known. The approach can even be applied in the presence of system uncertainties. The modification of a control law of Slotine and Li [1987] is explained in detail. Future work will extend the method by including a robust observer which estimates the motor positions and velocities. Such an observer makes the measurements of the motor variables unnecessary.

REFERENCES


Fig. 1. Trajectories of the link position $q$, the desired link position $q_d$, the motor position $\theta$, the desired motor position $\theta_d$, the motor velocity $\dot{\theta}$, the desired motor velocity $\alpha$, and the motor torque $\tau$ for the one link flexible robot.


