Reconfigurable UKF for In-Flight Magnetometer Calibration and Attitude Parameter Estimation

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Abstract: In this study a reconfigurable unscented Kalman filter (UKF) based algorithm for the estimation of magnetometer biases and scale factors is proposed as a part of the attitude estimation scheme of a pico satellite. Algorithm is composed of two stages; in first stage UKF estimates magnetometer biases and scale factors as well as six attitude parameters of the satellite. Differently from the existing algorithms, scale factors are not treated together with the other parameters as a part of the state vector; three scale factors are estimated via a newly proposed extension for the UKF. After a convergence rule for the biases second stage starts and the UKF reconfigures itself for the estimation of only attitude parameters. At this stage filter regards the biases and scale factors estimated at the initial stage. Proposed algorithm is simulated for attitude estimation of a pico satellite which has three magnetometers and three rate gyros as measurement sensors.

Keywords: Attitude algorithms, Calibration, Kalman filters, Estimation parameters, Satellite applications

1. INTRODUCTION

Because of advantages such as providing continuously available two-axis attitude measurements; relative low cost and almost insignificant power demand, operating magnetometers as primary sensor in small satellite missions is a common method for achieving attitude information. However, these sensors are not error free because of the biases, scaling errors and misalignments (nonorthogonality). These terms inhibit the filter efficiency and so attitude data accuracy and even they may bring about the filter divergence in long terms. The attitude accuracy requirements demand compensation for the magnetometer errors such as misalignments and biases (Wertz, 1988). Estimating magnetometer biases and scale factors as well as the attitude of the satellite is a proposed technique to solve such problems and increase on-board accuracy.

In literature there are several methods for estimating the magnetometer bias when the attitude knowledge is not available (e.g. at the injection phase where the spacecraft is spinning rapidly). As a relatively new method, twostep algorithm, which was proposed in order to overrun quartic cost function problem met during attitude free bias estimation process, may be given as an example (Alonso and Shuster, 2002a). In (Alonso and Shuster, 2003) it is also shown that twostep algorithm can be also implemented in case of incomplete observability of the magnetometer bias vector. Moreover (Alonso and Shuster, 2002b) uses algorithm to estimate scale factors and nonorthogonality corrections as well as magnetometer biases. On the other hand, Kim and Bang integrates twostep algorithm with the genetic algorithm, from which the initial estimates of the magnetometer bias estimation are provided (Kim and Bang, 2007). Twostep algorithm like methods can be used for guessing an initial estimate for Kalman filter type stochastic attitude determination methods. Those a priori estimates for the estimator may be then corrected on an expanded state vector including attitude parameters and biases.

As another example for attitude-free bias estimation procedure, in (Crassidis et al., 2005; Huang and Jing, 2008) it is proved that a full magnetometer calibration can be performed on-orbit during typical spacecraft mission-mode operations by the use of real-time algorithms based on both the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Although these studies can estimate the magnetometer characteristics such as biases, scale factors and nonorthogonality corrections, they all disregard the attitude dynamics of the satellite and if this information is possible at any instant, their accuracies can be exceeded. Hence they may be considered as a part of spacecraft modes where attitude data is absent.

In (Steyn, 1995) both the magnetometer biases and scale factors are estimated by recursive least squares method. The vector of unknown parameters is formed of magnetometer biases and scale factors. Method is not attitude free since the attitude knowledge is necessary to form the attitude matrix. However as a drawback, used attitude parameters are not estimated by the method itself and the spacecraft dynamics are not taken into consideration in this method. On the other hand, a few studies like (DaForno et al., 2004; Ma, 2005), which handle magnetometer calibration process together with the estimation of attitude parameters, also do exist. In (DaForno et al., 2004) EKF is used as a part of the estimation scheme. Via an UKF based estimation algorithm, which has advantages over EKF such as absence of Jacobian calculations etc., its accuracy may be surmounted. Ma and Jiang solve magnetometer calibration and attitude estimation problem via two UKFs working synchronously. As a
disadvantage, this approach requires a high computational effort because of two distinct UKFs and it may be not suitable for pico satellites where the processing capacity of the attitude computer is limited.

In this study a reconfigurable unscented Kalman filter (UKF) based algorithm is proposed for magnetometer calibration and attitude parameter estimation. Algorithm is composed of two stages; in first stage UKF estimates magnetometer biases and scale factors as well as six attitude parameters of the satellite. Differently from the existing algorithms, scale factors are not treated together with the other parameters as a part of the state vector; three scale factors are estimated via a newly proposed extension for the UKF. At the second stage which starts after a convergence rule for the bias estimations UKF reconfigures itself and regarding the estimated biases and scale factors, it runs for estimation of only attitude parameters. Proposed algorithm is simulated for attitude estimation of a pico satellite which has three magnetometers and three rate gyros as measurement sensors.

2. MATHEMATICAL MODEL OF THE PICO SATELLITE

If the kinematics of the pico satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 9 dimensional system vector which is made of attitude Euler angles (\( \varphi \) is the roll angle about \( x \) axis; \( \theta \) is the pitch angle about \( y \) axis; \( \psi \) is the yaw angle about \( z \) axis), the body angular rate vector with respect to the inertial axis frame, and the vector formed of bias terms of magnetometers. Hence,

- For the first stage where bias terms of three magnetometer are estimated;
  \[
  \dot{x} = \begin{bmatrix} \varphi & \theta & \omega_x & \omega_y & \omega_z & b_{m1} & b_{m2} & b_{m3} \end{bmatrix}^T, \tag{1}
  \]

- For the second stage only attitude parameters are estimated;
  \[
  \dot{x} = \begin{bmatrix} \varphi & \theta & \omega_x & \omega_y & \omega_z \end{bmatrix}^T, \tag{2}
  \]

Here subscript \( m \) for bias terms represents magnetometer. Also for consistency with the further explanations, the body angular rate vector with respect to the inertial axis frame should be stated separately as;
\[
\omega_{bi} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T,
\]
where \( \omega_{bi} \) is the angular velocity vector of the body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;
\[
J \frac{d\omega_{bi}}{dt} = \mathbb{N}_g - \omega_{bi} \times (J \omega_{bi}), \tag{3}
\]
where \( J \) is the inertia matrix consists of principal moments of inertia as \( J = \text{diag}(J_x,J_y,J_z) \) and \( \mathbb{N}_g \) is the vector of gravity gradient torque affecting the satellite as (Sekhavat et al., 2007);
\[
\begin{align*}
N_{gs} &= \begin{bmatrix} (J_y - J_z)A_{33} - A_{31} \\ (J_z - J_x)A_{13} - A_{32} \\ (J_x - J_y)A_{23} - A_{31} \end{bmatrix} \\
N_{gs} &= -3 \frac{\mu}{r_0^3} \begin{bmatrix} (J_y - J_z)A_{33} - A_{31} \\ (J_z - J_x)A_{13} - A_{32} \\ (J_x - J_y)A_{23} - A_{31} \end{bmatrix}. \tag{4}
\end{align*}
\]
Here \( \mu \) is the gravitational constant, \( r_0 \) is the distance between the centre of mass of the satellite and the Earth and \( A_{ij} \) represents the corresponding element of the direction cosine matrix of;
\[
A = \begin{bmatrix}
c(\varphi)c(\psi) & c(\varphi)s(\psi) & -s(\varphi) \\
c(\theta)c(\psi) - s(\varphi)s(\theta)c(\psi) & c(\varphi)c(\psi) - s(\varphi)s(\theta)s(\psi) & s(\theta)c(\psi) + s(\varphi)s(\theta)c(\psi) \\
s(\theta)c(\psi) - s(\varphi)s(\theta)c(\psi) & s(\varphi)c(\psi) - c(\varphi)s(\theta)s(\psi) & -s(\theta)c(\psi) - s(\varphi)s(\theta)c(\psi) \end{bmatrix}. \tag{5}
\]

In matrix \( A \), \( c(\cdot) \) and \( s(\cdot) \) are the cosines and sinus functions, respectively. Kinematic equations of motion of the pico satellite with the Euler angles can be given as
\[
\begin{bmatrix}
\dot{\varphi} \\
\dot{\theta} \\
\dot{\psi} \\
\end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)c(\psi) & -c(\varphi) \\
0 & c(\varphi)c(\psi) & s(\varphi) \\
0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix} \begin{bmatrix} p \\
q \\
r \end{bmatrix}, \tag{6}
\]
Here \( t(\cdot) \) stands for tangent function and \( p \), \( q \) and \( r \) are the components of \( \vec{\omega}_{bi} \) vector which indicates the angular velocity of the body frame with respect to the reference frame. \( \vec{\omega}_{bi} \) and \( \vec{\omega}_{bi} \) can be related via,
\[
\vec{\omega}_{bi} = \vec{\omega}_{bi} + A \begin{bmatrix} 0 & -\omega_y & 0 \end{bmatrix}^T, \tag{7}
\]
where \( \omega_y \) denotes the angular velocity of the orbit with respect to the inertial frame, found as \( \omega_y = \left( \mu / r_0^3 \right)^{1/2} \).

3. MEASUREMENT SENSOR MODEL

3.1 The Magnetometer Model

As the satellite navigates along its orbit, magnetic field vector differs in a relevant way with the orbital parameters. If those parameters are known, then, magnetic field tensor vector that affects satellite can be shown as a function of time analytically (Sekhavat et al., 2007). Note that, these terms are obtained in the orbit reference frame.
\[
H_i(t) = \frac{M_s}{r_0^3} \left[ \begin{bmatrix} \cos(\omega_y) \cos(i) - \sin(\varepsilon) \cos(i) \cos(\omega_y) \\ -\sin(\omega_y) \sin(\varepsilon) \sin(\omega_y) \end{bmatrix} \right], \tag{8}
\]
\[
H_s(t) = -\frac{M_s}{r_0^3} \left[ \begin{bmatrix} \cos(\omega_y) \cos(i) + \sin(\varepsilon) \sin(i) \cos(\omega_y) \\ -\sin(\omega_y) \sin(\varepsilon) \sin(\omega_y) \end{bmatrix} \right], \tag{9}
\]
\[
H_i(t) = \frac{2M_s}{r_0^3} \left[ \begin{bmatrix} \sin(\omega_y) \cos(i) - \sin(\varepsilon) \cos(i) \cos(\omega_y) \\ -2\sin(\omega_y) \sin(\varepsilon) \sin(\omega_y) \end{bmatrix} \right]. \tag{10}
\]
Here \( M_s \) is the magnetic dipole moment of the Earth as \( M_s = 7.943 \times 10^{27} \text{ Wb.m} \), \( \mu \) is the Earth Gravitational constant as \( \mu = 3.98601 \times 10^4 \text{ m}^3 \text{s}^{-2} \), \( i \) is the orbit inclination as \( i = 97^\circ \), \( \omega_y \) is the spin rate of the Earth as \( \omega_y = 7.29 \times 10^{-5} \text{ rad/s} \), \( e \) is the magnetic dipole tilt as \( e = 11.7^\circ \) and \( r_0 \) is the distance between the centre of mass of the satellite and the Earth as \( r_0 = 6,928,140 \text{ m} \).

Three onboard magnetometers of pico satellite measures the components of the magnetic field vector in the body frame. Therefore for the measurement model, which characterizes the measurements in the body frame, gained magnetic field
terms must be transformed by the use of direction cosine matrix, $A$ . Overall measurement model may be given as;

$$S = \begin{bmatrix}
H_x(ϕ, θ, ψ, t)

\end{bmatrix} = \begin{bmatrix}
A

\end{bmatrix} \begin{bmatrix}
H_i(t)

\end{bmatrix} \begin{bmatrix}
H_x(ϕ, θ, ψ, t)

\end{bmatrix} \begin{bmatrix}
H_i(t)

\end{bmatrix} + \begin{bmatrix}
\vec{b}_m

\end{bmatrix} + \begin{bmatrix}
\vec{η}_i

\end{bmatrix} ,$$

(11)

where, $H_i(t)$ and $H_x(t)$ represent the Earth magnetic field vector components in the orbit frame as a function of time, and $H_x(ϕ, θ, ψ, t)$ show the measured Earth magnetic field vector components in body frame as a function of time and varying Euler angles. Furthermore, $S$ is the diagonal scale matrix as,

$$S = \text{diag}(s_{1}, s_{12}, s_{13}),$$

(12)

$\vec{b}_m$ is the magnetometer bias vector as $\vec{b}_m = [b_{m_1}, b_{m_2}, b_{m_3}]^T$ and $\vec{η}_i$ is the zero mean Gaussian white noise with the characteristic of

$$E[\vec{η}_i^T \vec{η}_i] = I_{3 \times 3} \sigma_m^2 \delta_i .$$

(13)

$I_{3 \times 3}$ is the identity matrix with the dimension of $3 \times 3$, $\sigma_m$ is the standard deviation of each magnetometer error and $\delta_i$ is the Kronecker symbol.

3.2 The Rate Gyro Model

Three rate gyros are aligned through three axes, orthogonally to each other and they supply directly the angular rates of the body frame with respect to the inertial frame. Hence the model for rate gyros can be given as;

$$\vec{ω}_{m, \text{meas}} = \vec{ω}_{m} + \vec{η}_2 .$$

(14)

where, $\vec{ω}_{m, \text{meas}}$ is the measured angular rates of the satellite, and $\vec{η}_2$ is the zero mean Gaussian white noise with the characteristic of

$$E[\vec{η}_2^T \vec{η}_2] = I_{3 \times 3} \sigma_ω^2 \delta_i .$$

(15)

Here, $\sigma_ω$ is the standard deviation of each rate gyro random error.

4. UKF BASED BIAS AND SCALE FACTOR ESTIMATION ALGORITHM

4.1 Unscented Kalman Filter

In order to utilize Kalman filter for nonlinear systems without any linearization step, the unscented transform and so Unscented Kalman Filter is one of the techniques. UKF uses the unscented transform, a deterministic sampling technique, to determine a minimal set of sample points (or sigma points) from the $a \text{ priori}$ mean and covariance of the state. Then, these sigma points go through nonlinear transformation. The posterior mean and the covariance are obtained from these transformed sigma points (Julier et al., 1995).

As it is stated, UKF procedure begins with the determination of $2n+1$ sigma points with a mean of $\hat{x}(k|k)$ and a covariance of $P(k|k)$. For an $n$ dimensional state vector, these sigma points are obtained by

$$x_i(k|k) = \hat{x}(k|k) \pm \sqrt{\kappa(n+\kappa)} P_{x_i}(k|k)^{1/2}$$

(16)

where, $x_i(k|k)$, $x_i(k|k)$ and $x_i(k|k)$ are sigma points, $P(k|k)$ is the process noise covariance matrix, $n$ is the state number and $\kappa$ is the scaling parameter which is used for fine tuning and the heuristic is to chose that parameter as $n+\kappa = 3$ (Julier et al., 1995). Also, $i$ is given as $i = 1…n$.

Next step of the UKF process is transforming each sigma point by the use of system dynamics,

$$x_i(k+1|k) = f(x_i(k|k), k).$$

(19)

Then these transformed values are utilized for gaining the predicted mean and the covariance (Crassidis and Markley, 2003; Soken and Hajiyev 2009).

$$\hat{x}(k+1|k) = \frac{1}{n+\kappa} \left[ x_{i_1}(k+1|k) + 2 \sum_{i=2}^{n} x_{i}(k+1|k) \right],$$

(20)

$$P(k+1|k) = \frac{1}{n+\kappa} \left[ x_{i_1}(k+1|k) - \hat{x}(k+1|k) \right] \left[ x_{i_1}(k+1|k) - \hat{x}(k+1|k) \right]^T + \frac{1}{\kappa} \sum_{i=2}^{n} \left[ x_{i}(k+1|k) - \hat{x}(k+1|k) \right] \left[ x_{i}(k+1|k) - \hat{x}(k+1|k) \right]^T .$$

(21)

Here, $\hat{x}(k+1|k)$ is the predicted mean and $P(k+1|k)$ is the predicted covariance. Nonetheless, predicted observation vector is,

$$\hat{y}(k+1|k) = \frac{1}{n+\kappa} \left[ y_{i_1}(k+1|k) + 2 \sum_{i=2}^{n} y_{i}(k+1|k) \right],$$

(22)

where,

$$y_{i_1}(k+1|k) = h(x_i(k+1|k), v(k), k).$$

(23)

After that, observation covariance matrix is determined as,

$$P_{y_i}(k+1|k) = \frac{1}{n+\kappa} \left[ y_{i_1}(k+1|k) - \hat{y}(k+1|k) \right] \left[ y_{i_1}(k+1|k) - \hat{y}(k+1|k) \right]^T + \frac{1}{\kappa} \sum_{i=2}^{n} \left[ y_{i}(k+1|k) - \hat{y}(k+1|k) \right] \left[ y_{i}(k+1|k) - \hat{y}(k+1|k) \right]^T .$$

(24)

where innovation covariance is

$$P_{\lambda_1}(k+1|k) = P_{y_i}(k+1|k) + R(k+1)$$

(25)

Here $v(k)$ is the white Gaussian measurement noise and it is non-correlated with the process noise. Besides, $R(k+1)$ is the measurement noise covariance matrix. The cross correlation matrix can be obtained as,

$$P_{\lambda_1}(k+1|k) = \frac{1}{n+\kappa} \left[ y_{i_1}(k+1|k) - \hat{y}(k+1|k) \right] \left[ y_{i_1}(k+1|k) - \hat{y}(k+1|k) \right]^T + \frac{1}{\kappa} \sum_{i=2}^{n} \left[ y_{i}(k+1|k) - \hat{y}(k+1|k) \right] \left[ y_{i}(k+1|k) - \hat{y}(k+1|k) \right]^T .$$

(26)
Following part is the update phase of UKF algorithm. At that phase, first by using measurements, \( y(k+1) \), residual term (or innovation sequence) is found as

\[
e(k+1) = y(k+1) - \tilde{y}(k+1|k),
\]

and then Kalman gain is computed via equation of,

\[
K(k+1) = P_{\omega}(k+1|k)P_{\omega}^{-1}(k+1|k).
\]

At last, updated states and covariance matrix are determined by,

\[
\begin{align*}
\hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + K(k+1)e(k+1), \\
P(k+1|k+1) &= P(k+1|k) - K(k+1)P_{\omega}(k+1|k)K^T(k+1).
\end{align*}
\]

Here, \( \hat{x}(k+1|k+1) \) is the estimated state vector and \( P(k+1|k+1) \) is the estimated covariance matrix.

### 4.2 Scale Factor Estimation

When the condition of the measurement system does not correspond to the model used in the synthesis of the filter that means the incoming measurements are scaled and they need to be re-scaled after estimation of scale factors.

If the system operates normally, the real and the theoretical innovation covariance matrix values match as in (31).

\[
\frac{1}{\mu_k} \sum_{j=k-\mu_k+1}^k e(k+1)^T e(k+1) \leq P_{\omega}(k+1|k) + R(k+1),
\]

here, \( \mu_k \) is the width of the moving window.

However, when the real measurements are scaled as in (11) the real error will exceed the theoretical one. In this case the matrix built of scale factors, \( S(k) \), is added into the algorithm as,

\[
\frac{1}{\mu_k} \sum_{j=k-\mu_k+1}^k e(k+1)^T e(k+1) = P_{\omega}(k+1|k) + S(k)R(k+1)S^T(k).
\]

Then, since it is known that \( S(k) \) is diagonal (there in no misalignment/nonorthagonality for magnetometers) and also \( R(k+1) \) is diagonal by its nature, it is possible to rewrite (32),

\[
\frac{1}{\mu_k} \sum_{j=k-\mu_k+1}^k e(k+1)^T e(k+1) = P_{\omega}(k+1|k) + [S(k)]^2 R(k+1).
\]

Finally, scale factor matrix can be found as given below,

\[
S(k) = \left[ \frac{1}{\mu_k} \sum_{j=k-\mu_k+1}^k e(k+1)^T e(k+1) - P_{\omega}(k+1|k) \right] R^{-1}(k+1).
\]

### 4.3 Stopping Rule for Bias Estimation

The following stopping rule may be introduced for the bias estimation problem (Hajiyev, 1994; 2001):

\[
r^2_k = (\hat{b}_k - \hat{b}_{k-1})^T D_{\hat{b}}^{-1}(\hat{b}_k - \hat{b}_{k-1}) = \varepsilon,
\]

where \( D_{\hat{b}} \) is the covariance matrix of the discrepancy between two successive bias estimates \( \hat{b}_k \) and \( \hat{b}_{k-1} \), and \( \varepsilon \) is a predetermined small number.

Since Kalman filter estimates the parameters from a sequence of observations with Gaussian tolerances of the measurement errors and system noise, the filter yields an estimate with an expected value equal to the estimated quantity and a Gaussian distribution function. The discrepancy \( \hat{b}_k - \hat{b}_{k-1} \) then has a normal distribution as well, since it is a linear combination of two Gaussian random variables. With these considerations in mind it is known that the statistic \( r^2 \) has a \( \chi^2 \) distribution with \( n \) degrees of freedom (\( n \) is the number of dimensions of the bias vector \( b \)), and the threshold values of \( r^2 \) can be found by determining the tabulated values of the \( \chi^2 \) distribution for a given level of significance.

It is evident from relation (35) that the smaller the value of \( r^2 \), the greater will be the consistency of the estimates. Usually in the testing of consistency in such cases the lower limit of the confidence interval must be equal to zero, and the upper limit is determined by the level of significance \( \alpha \).

To test the consistency of the estimates, the level of significance \( \alpha \) is adopted, which corresponds to the confidence coefficient \( \beta = 1-\alpha \). The threshold \( \chi^2 \) is specified in terms of this probability, using the distribution of the investigated statistic \( r^2 \):

\[
P(\chi^2 < \chi^2_\beta) = \beta, 0 < \beta < 1.
\]

The estimation process is stopped when \( r^2_k < \chi^2_\beta \), since further observations yield insignificant improvement of the identified model. If the quadratic form \( r^2 \) is larger than or equal to the specified threshold \( \chi^2_\beta \), estimation should be continued.

Nonetheless computation of the covariance matrix of the discrepancy between two successive bias estimates \( \hat{b}_k \) and \( \hat{b}_{k-1} \) is investigated in (Hajiyev, 2001). The following expression for the covariance matrix \( D_{\hat{b}} \) is obtained in this work:

\[
D_{\hat{b}} = P_{\hat{b}} - P_{\hat{b}}.
\]

Described stopping rule can be used to make a timely decision to stop the bias estimation process and it does not require large computational expenditures.

### 5. SIMULATIONS

Simulations are realized for 500 seconds with a sampling time of \( \Delta t = 0.1 \) sec. As an experimental platform a cubesat model is used and the diagonal terms of the inertia matrix is taken as \( J_x = 2.1 \times 10^{-3} \), \( J_y = 2.0 \times 10^{-3} \), \( J_z = 1.9 \times 10^{-3} \), where all non-diagonal terms are zero since the rotation is about the
principal axis of the satellite. Nonetheless the orbit of the satellite is a circular orbit with an altitude of $r_a = 550km$. Other orbit parameters are same as it is presented in the section for the Magnetometer Model (Section 3.1).

Simulations can be categorized in two: Estimation of scale factors, biases and attitude parameters without stopping rule for the filter (without UKF reconfiguration) and estimation realized by an UKF with stopping rule (with UKF reconfiguration). Results gained via these two distinct simulation scenarios are compared in order to clarify the effectiveness of the proposed reconfigurable UKF based method.

Firstly, via a single UKF algorithm it is possible to calibrate the magnetometers. In this algorithm, state vector is formed of 9 states; Euler angles, angular rates and magnetometer biases. As well scale factors are estimated via (34). Bias estimation example is given in Fig.1. As it is apparent, bias terms are estimated accurately.

Fig. 1. Estimation of the bias for magnetometer aligned through “z” axis.

Besides, in simulations, scale factors are implemented to the magnetometer measurements in order to simulate estimation process ($S$). In return, scale factors given at the right-hand side ($\hat{\Sigma}$) are estimated via the proposed method (given results are the mean of scale factor estimation samples gained till the bias estimation stopping instant):

$$
S = \begin{bmatrix}
1.2 & 0 & 0 \\
0 & 1.3 & 0 \\
0 & 0 & 1.5 \\
\end{bmatrix} \Rightarrow \hat{\Sigma} = \begin{bmatrix}
1.2042 & 0 & 0 \\
0 & 1.3056 & 0 \\
0 & 0 & 1.5050 \\
\end{bmatrix}
$$

As seen, estimation error for the scale factors is not high. Difference between the actual value and the Kalman estimation is certainly caused by the random process white noise of the measurements. On the other hand, it is possible to estimate also all other parameters accurately via this algorithm. As seen in Fig. 2 attitude parameter estimation errors are in acceptable limits for a pico satellite. Note that similar results have been obtained for all other parameters.

Fig. 2. Pitch angle estimation via UKF without reconfiguration.

It is obvious that with an UKF algorithm it is possible to estimate all 9 parameters of the state vector (1) and also the scale factors precisely. However, a 9 dimensional state vector and the additional part to UKF for scale factor estimation demand a high computational effort. In general, Kalman filter's computational burden increases in a direct relation with the cube of the number of states. Hence, constituting a state vector with 9 states means more computational burden than a state vector with 6 states. That is an important redundancy for considerably limited attitude computer of a pico satellite. As a result it is better to reconfigure filter regarding the scale factors and biases of magnetometers and to proceed the estimation procedure by only estimating Euler angles and angular rates. As given in section 4.3, a stopping rule is proposed in order to reconfigure UKF after convergence of the bias estimations to the actual values. In that second stage, previously estimated magnetometer bias terms and scale factors are taken into account by UKF algorithm and number of states to be estimated reduces to 6 as shown with (2). Pitch angle estimation result is given in Fig.3.

Fig. 3. Pitch angle estimation via UKF with reconfiguration.
Here in Fig. 3 reconfiguration occurs at about 80th second and it is marked on figure with dashed line. As it is apparent from the estimation variance, reconfiguration brings about an enhancement to filter accuracy. So as to understand the effect of reconfiguration more clearly, absolute values of error are tabulated for two filters; UKF with and without reconfiguration (Table 1).

Table 1. Comparison of Absolute Values of Error for UKFs with and without reconfiguration.

<table>
<thead>
<tr>
<th>Par.</th>
<th>Abs. Values of Err. for UKF with Reconfiguration</th>
<th>Abs. Values of Err. for UKF without Reconfiguration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200 s.</td>
<td>400s.</td>
</tr>
<tr>
<td>$\phi$(deg)</td>
<td>0.0408</td>
<td>0.0673</td>
</tr>
<tr>
<td>$\theta$(deg)</td>
<td>0.0052</td>
<td>0.1223</td>
</tr>
<tr>
<td>$\psi$(deg)</td>
<td>0.0522</td>
<td>0.1085</td>
</tr>
<tr>
<td>$\omega_x$(deg/s)</td>
<td>0.0004</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\omega_y$(deg/s)</td>
<td>0.0003</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\omega_z$(deg/s)</td>
<td>0.0009</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

As seen from table reducing the size of the state vector without any change at measurement vector brings about an increment in the accuracy of estimation of the states. Nonetheless, a smaller state vector means lower computational burden and risk for the divergence of the filter.

6. CONCLUSIONS

In this study a reconfigurable unscented Kalman filter (UKF) based algorithm is proposed for magnetometer calibration and attitude parameter estimation. Simulation results show that, it is possible to estimate both, magnetometer biases and scale factors as well as the attitude parameters via the proposed algorithm. Nevertheless, proposed reconfigurable UKF algorithm is advantageous because simulation results proves that reducing the size of the state vector without any change at measurement vector brings about an increment in the accuracy of estimation of the states. Also a smaller state vector means lower computational burden and risk for the divergence of the filter. Since magnetometer utilization has significance especially for pico satellite missions, proposed algorithm may affect the mission performance and reliability in a considerable degree.

ACKNOWLEDGMENT

This work was supported in part by TUBITAK (The Scientific and Technological Research Council of Turkey) under Grant 108M523.

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