Decentralized random control of refrigerator appliances

D. Angeli ∗ P.A. Kountouriotis ∗∗

∗ EEE Department, Imperial College, London, SW7 2AZ, UK (e-mail: d.angeli@imperial.ac.uk).
∗∗ EEE Department, Imperial College, London, SW7 2AZ, UK (e-mail: pk201@imperial.ac.uk)

Abstract: This paper considers the problem of dynamic demand management of domestic refrigerators. Dynamic demand management is a promising research direction, which aims to improve power system resilience, by dynamically controlling the energy consumption of devices that exhibit thermal energy storage. In this approach, the operating temperature of appliances, and thus their energy consumption, is modified dynamically, within a safe range, in response to mains frequency fluctuations. Such an operating scheme aims at reducing the strain that the demand side introduces in a power network, when the corresponding supply is temporarily disrupted. Previous research has highlighted the potential of this idea for responding to sudden power plant outages. However, deterministic control schemes have proved inadequate as individual appliances tend to “synchronize” with each other, leading to unacceptable levels of overshoot in energy demand, when they “recover” their steady-state operating cycles. In this paper we describe random controllers that are able to respond to sudden plant outages and which avoid the instability phenomena associated with other feedback strategies. Stochasticity is used to achieve desynchronization of individual refrigerators while keeping overall power consumption tightly regulated.

Keywords: Power system control, stochastic jump processes, Markov models, large-scale systems.

1. INTRODUCTION

Dynamic demand management is a very promising research direction for improving power system resilience. In an AC power network, the system frequency (mains frequency) can be interpreted as a measure of the balance between demand (load) and supply (generator), with perfect balance corresponding to the nominal value of 50Hz. In cases where demand exceeds the available supply, the frequency drops below 50Hz, while excess supply leads to frequency rising above 50Hz. As a result, system frequency continuously fluctuates around the nominal level, and the system operator ensures that the balance between demand and supply is continuously maintained, stabilizing the frequency within narrow bands around 50Hz, by regulating the available supply.

In order for such (supply) regulation to be possible, however, it is required that ‘frequency response services’, as well as sufficient reserves, are included in the system. This is essential not only for instantaneous frequency balancing, but, more importantly, for the ability to respond to sudden power plant failures, which would otherwise lead to severe blackouts.

These ‘support’ services, however, significantly add to the cost of power generation, and any method which manages to reduce the magnitude of these services, without sacrificing system stability, is of significant importance (Short et al. (2007)). “Dynamic demand control” is a recent research direction, which focuses on the possibility of using frequency responsive loads, so as to reduce the amount of frequency response and reserve services that are required.

In this paper, we consider the problem of managing power demand by means of “smart” thermostatic control of domestic refrigerators. In this approach, the operating temperature of these appliances, and thus their energy consumption, is modified dynamically, within a safe range, in response to mains frequency fluctuations. Previous research (Short et al. (2007), Infield et al. (2007), Aunedi et al. (2008)) has shown that this is an effective way to respond to sudden power plant outages, reducing the cost of reserve power required to deal with such events.

Short et al. (2007) and Infield et al. (2007) have investigated the potential of dynamic demand control of domestic refrigerators, when the thermostat’s temperature thresholds (and, thus, the duty cycle of appliances) are varied as linear functions of mains frequency deviation from its nominal value, while they also perform an assessment of the control method in scenarios with significant supply variability, due to power generated by wind turbines. In both cases, their results demonstrate that the amount of standing reserve required by the power system can be...
safely reduced. A similar approach is followed in Aunedi et al. (2008), where the economic impacts of such control strategies are also quantified depending on the types of generation units in the system (nuclear plants, coal plants, combined cycle gas turbine plants, etc.).

Simple feedback schemes, however, such as those employed in Short et al. (2007), in which the operating temperature is varied linearly with respect to mains frequency deviations, can prove inadequate in achieving desired performance, as individual appliances tend to “synchronize”, leading to unacceptable levels of overshoot in energy demand, when they “recover” their steady-state operating temperatures. The appearance of such phenomena can be slow, but they do ultimately lead to unstable oscillations in the frequency of the overall system.

The problem of dynamic demand management is also addressed in Stadler et al. (2009), in the context of (centralized) model predictive control (MPC). In this case, the appliances are assumed to be connected to a communications network, and are able to receive and execute commands that are generated by a central processing node. As expected, the closed loop behaviour is far superior to that corresponding to the simpler schemes of Short et al. (2007) and Infield et al. (2007), but the prospects of immediate utilization of such ideas are not enhanced, not least by the extra costs that would be required for widespread implementation.

In contrast to Stadler et al. (2009), we adopt a framework in which there is no communication between the controlled devices, and so each device has to act in an autonomous setting. We note that the quantity of interest is the temperature distribution of the whole population of appliances at each particular time point and pose the problem in a probabilistic framework, in which we try to find control schemes that steer the probability densities involved towards desired distributions. The advantage of this approach is that it greatly reduces the dimensionality of the original problem, while it allows for simple, yet successful implementation.

A viable control scheme in this setting, is the replacement of classical hysteresis-based controllers with controls that randomly jump between the “on” and “off” states of the appliances. Careful selection of the jump propensities allows for the decentralized control of individual appliances’ duty cycles (and, therefore, power consumption), while, during “recovery”, the “population” of refrigerators is sufficiently diversified (mixed) with respect to temperature, thereby avoiding undesirable overshoot phenomena.

The performance of the proposed controllers is assessed via simulations, when coupled with a simple model of the power grid. Initial results verify the theoretical underpinnings of our approach, and clearly illustrate the robustness of the method when compared to earlier approaches.

The rest of the paper is organized as follows. Section 2 elaborates the mathematical results enabling the proposed solution. A model of refrigerators is outlined in Section 2.1, with the corresponding analysis shown in Section 2.2. The interconnected system and the random controller are described in Section 3, while Section 4 shows results of simulation studies. Conclusions can be found in Section 5.

2. A STOCHASTIC APPROACH TO REFRIGERATOR CONTROL

2.1 Refrigerator modeling

In order to derive a random control algorithm, we model refrigerators as Markov jump linear systems (do Valle Costa et al. (2005); Mariton (1990)), or to be more precise jump affine systems. Roughly speaking these are switched affine systems whose driving signal is the stochastic process associated to a finite Markov chain.

The model we consider consists of two states, an OFF and an ON state, and transition probability rates between them which are denoted by \( \lambda_1 \) and \( \lambda_2 \) respectively. A graphical illustration is shown in Fig. 1. Letting \( T(t) \) denote the temperature of a single appliance at time \( t \), its evolution in each of the two states is described by an affine, first-order ordinary differential equation, as

\[
\dot{T}(t) = -\alpha(T - T_{ON}) \quad \text{when ON}
\]

\[
\dot{T}(t) = -\alpha(T - T_{OFF}) \quad \text{when OFF. (1)}
\]

In (1), \( T_{OFF} \) and \( T_{ON} \) denote respectively the ambient temperature and the steady-state temperature reached by a refrigerator which is always ON. The positive coefficient \( \alpha \) is a thermal dispersion coefficient.

![Markov chain illustration](image)

Fig. 1. Markov chain illustration

We also use \( \pi_{ON}(t) \) and \( \pi_{OFF}(t) \) to denote the probability of a single refrigerator being in the ON and OFF state respectively. Obviously, \( \pi_{ON}(t) + \pi_{OFF}(t) = 1 \) for all times \( t \). The equations governing the evolution in time of such probabilities are therefore given as:

\[
\dot{\pi}_{ON}(t) = -\lambda_1 \pi_{ON}(t) + \lambda_2 \pi_{OFF}(t)
\]

\[
\dot{\pi}_{OFF}(t) = -\lambda_2 \pi_{OFF}(t) + \lambda_1 \pi_{ON}(t). \quad (2)
\]

Due to ergodicity of the underlying Markov Chain, the vector \( (\pi_{ON}(t), \pi_{OFF}(t))^T \) for each given pair \( (\lambda_1, \lambda_2) \in (0, +\infty)^2 \), converges to a unique stationary distribution \( (\pi_{ON}, \pi_{OFF})^T \). In particular, \( \pi_{ON} \) also represents the average duty cycle of a single appliance. It is straightforward to see that:

\[
\pi_{ON} = \frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \pi_{OFF} = \frac{\lambda_1}{\lambda_1 + \lambda_2}. \quad (3)
\]

2.2 Open loop behaviour: analytical results

As our goal is to regulate the overall behaviour of a population of refrigerators, it is convenient to derive formulae that describe the time-evolution of the probability distribution of temperatures of a single appliance, and expressions for the associated first two moments. To this end, \( \rho_+(t, T) \) and \( \rho_-(t, T) \) denote the unnormalized pdf
of the temperature of a device in the ON and OFF state respectively, at time $t$. In particular note that
\[ \pi_{ON}(t) = \int_{-\infty}^{+\infty} \rho_{+}(t,T) dT, \quad \pi_{OFF}(t) = \int_{-\infty}^{+\infty} \rho_{-}(t,T) dT. \]
These (unnormalized) temperature distributions satisfy a form of Kolmogorov’s forward equation, such that
\[ \frac{\partial \rho_{+}}{\partial t} = \alpha(T - T_{ON}) \frac{\partial \rho_{+}}{\partial T} + (\alpha - \lambda_{1}) \rho_{+} + \lambda_{2} \rho_{-}, \]
\[ \frac{\partial \rho_{-}}{\partial t} = \alpha(T - T_{OFF}) \frac{\partial \rho_{-}}{\partial T} + (\alpha - \lambda_{2}) \rho_{-} + \lambda_{1} \rho_{+}. \] (4)

Even though equations (4) admit no closed-form solution, it is possible to obtain ODEs that describe the evolution of the first two moments associated with these distributions, as well as asymptotic (steady-state) values for these moments. To this end, we define $T_{+}(t)$ and $T_{-}(t)$ as
\[ T_{+}(t) = \int_{-\infty}^{+\infty} T \rho_{+}(t,T) dT, \]
\[ T_{-}(t) = \int_{-\infty}^{+\infty} T \rho_{-}(t,T) dT, \] (5) (6)
so that $E[T(t)] = T_{+}(t) + T_{-}(t)$. Differentiating the previous formulae with respect to time, and using (4), we obtain a differential equation for the time evolution of the expected temperature $E[T(t)]$, as
\[ E[T(t)] = -\alpha E[T(t)] - \pi_{ON}T_{ON} - \pi_{OFF}T_{OFF}. \] (7)
Taking into account the steady-state values of $\pi_{ON}$ and $\pi_{OFF}$ given in (3), the expected value of $T$ converges asymptotically to
\[ E[T(\infty)] = \frac{\lambda_{1}}{\lambda_{1} + \lambda_{2}} T_{OFF} + \frac{\lambda_{2}}{\lambda_{1} + \lambda_{2}} T_{ON}. \] (8)

Now consider the variance $v(t) = \text{var}[T(t)]$ of $T$. This is given as
\[ v(t) = \int_{-\infty}^{+\infty} (T - E[T])^{2} \rho_{+}(t,T) + \rho_{-}(t,T) dT \]
\[ = \int_{-\infty}^{+\infty} T^{2}(\rho_{+}(t,T) + \rho_{-}(t,T)) dT - (E[T])^{2}. \]
Differentiating the above with respect to $t$, and using (4), yields
\[ \dot{v}(t) = -2 \left[ \alpha v(t) + (E[T])^{2} - T_{ON}T_{+} - T_{OFF}T_{-} \right] + E[T] \dot{E}[T], \] (9)
leading to an asymptotic value for the variance equal to
\[ \bar{v} = (T_{ON} - T_{OFF})^{2} \frac{\alpha \lambda_{1} \lambda_{2}}{(\lambda_{1} + \lambda_{2})^{2}(\alpha + \lambda_{1} + \lambda_{2})}. \] (10)

Overall, the first and second population’s moments are governed by the following block-triangular set of ODEs:
\[ \dot{\pi}_{ON} = -\lambda_{1} \pi_{ON} + \lambda_{2} \pi_{OFF}, \]
\[ \dot{\pi}_{OFF} = -\lambda_{2} \pi_{OFF} + \lambda_{1} \pi_{ON}. \]
\[ T_{+} = -(\alpha + \lambda_{1}) T_{+} + \lambda_{2} T_{-} + \alpha \pi_{ON} T_{ON}, \]
\[ T_{-} = -(\alpha + \lambda_{2}) T_{-} + \lambda_{1} T_{+} + \alpha \pi_{OFF} T_{OFF}, \]
\[ v = -2 \left[ \alpha v(T_{+} + T_{-})^{2} - \pi_{ON} T_{ON}^{2} - \pi_{OFF} T_{OFF}^{2} \right] \]
\[ - \left[ (T_{+} + T_{-} - \pi_{ON} T_{ON} - \pi_{OFF} T_{OFF}) \right] T_{+} + T_{-} \right]. \] (11)

Due to the cascaded structure of this system and linearity of its diagonal terms, it is easy to see that the system is globally asymptotically convergent.

The formulae derived so far will be useful also for the derivation of a control strategy which is developed in the next Section.

3. THE INTERCONNECTED SYSTEM

The power grid is modeled by a 3rd order linear system (see, e.g., Jones (2005)), corresponding to the block diagram shown in Figure 2. (Values for the parameters in Figure 2 are given in Section 4. Also note that the model shown is expressed in a ‘per-unit’ basis (Aunedi et al. (2008)), so that $\Delta f = 50 \times \Delta \omega$.)

![Model of power grid](image)

Fig. 2. Model of power grid

The appliances load the grid at the summing junction via the variable $\Delta P_L$, which represents the deviation in overall power (consumed by all the appliances that are connected to the network) from their nominal consumption level. The additional variable $\Delta P_L$ is used so as to simulate a sudden loss of power in the system.

Standard refrigerator controllers operate on a hysteretic basis, in which two temperature levels $T_{max}$ and $T_{min}$ trigger the motor ON and OFF respectively. Initial attempts at dynamic demand refrigerator control (Short et al. (2007), Aunedi et al. (2008)) focused on dynamically adjusting these threshold levels, by imposing a linear dependence on mains frequency deviations, as
\[ \dot{T}_{max}(t) = T_{max} + K \Delta f(t) \]
\[ \dot{T}_{min}(t) = T_{min} + K \Delta f(t), \] (12)
where $K$ is a constant of proportionality.

Even though such strategies are effective in the short-term (i.e. when for the first time a plant failure occurs), simulation results indicate that they eventually lead to unstable overall behaviour of the closed-loop system. This phenomenon takes two different forms:

- long-term phase synchronization of refrigerators: indeed even non-identical population of refrigerators with duty cycles of comparable duration will tend to asymptotically synchronize their oscillations, giving rise to the so-called phase-locking phenomenon, see for instance Pikovsky et al. (2003).
- uncontrolled modifications of the population’s temperature distribution: the uniform in phase distribution that one expects of a population of randomly switched on utilities gets unpredictably modified by

\[ \Delta f = 50 \times \Delta \omega. \]

\[ \text{Note that a sudden power loss is equivalent to a sudden increase in demand, of the same magnitude.} \]
the occurrence of load disturbances and leads to significant oscillations in power demand even in the medium term.

In the rest of this Section, we describe an alternative control strategy that avoids such instabilities.

3.1 Random control strategy

In what follows, it is constructive to define the control variables $u_1$ and $u_2$ as

$$u_1 = \frac{\lambda_1}{\alpha} \quad \text{and} \quad u_2 = \frac{\lambda_2}{\alpha}.$$  

(13)

Now consider equations (8) and (10). In terms of the new variables, these can be rewritten as

$$\bar{v} = (T_{ON} - T_{OFF})^2 \frac{u_1 u_2}{(u_1 + u_2)^2(1 + u_1 + u_2)}.$$  

(15)

Note also that equation (3), which determines the duty cycle of each appliance as a function of the transition rates, can be rewritten as

$$\pi_{ON} = \frac{u_2}{u_1 + u_2}.$$  

(16)

Control of the appliances can be achieved via the selection of the transition rates $\lambda_1$ and $\lambda_2$ as functions of the grid frequency deviation $(f_{nom} - f(t))$. By fixing a desired value for the variance in operating temperatures, $v_{des}$ in (15), the transition rates $\lambda_1$ and $\lambda_2$ can be determined by postulating a desired average temperature, $T_{des}$, or a desired average duty cycle, $\pi_{des}$. In particular, if the latter is adopted, the following expressions are obtained by inverting the previous formulas:

$$u_1 = (\pi_{des}^2 - \pi_{des} + v_{des} / (T_{ON} - T_{OFF}))^2 / v_{des},$$

(17)

Our decentralized control strategy is therefore to vary either $T_{des}$ or $\pi_{des}$ as linear functions of the frequency deviation:

$$T_{des}(t) = T_{nom} + K_T (f_{nom} - f(t)),$$

(18)

$$\pi_{des}(t) = \pi_{nom} + K_s (f_{nom} - f(t)),$$

(19)

where $K_T$ and $K_s$ are proportionality constants, and $T_{nom}$ and $\pi_{nom}$ are the nominal values of the average temperature and the (corresponding) duty cycle when there is no frequency deviation in the grid ($\Delta f = 0$).

In the simulations shown in Section 4, $\pi_{des}$ was chosen as the reference variable, as it led to faster responses.

The control scheme described above results in a time-inhomogeneous Markov chain, with rate functions $\lambda_1(t)$ and $\lambda_2(t)$ which can be computed as functions of the instantaneous mains frequency $f(t)$, just by composing equations (17) and (19).

Individual appliances, then, will run the following algorithm ($RND$ denotes a random number, uniformly distributed in the interval $[0, 1]$):

- When device switches to ON mode:
  1. Set $t_0 = t$ and $r = RND$.
  2. Start evaluating the integral $I(t) = \int_{t_0}^{t} \lambda_1(\tau)d\tau$. (3) Switch to OFF at time $t'$, for which $I(t') \geq -\ln(r)$.
- When device switches to OFF mode:
  1. Set $t_0 = t$ and $r = RND$.
  2. Start evaluating the integral $I(t) = \int_{t_0}^{t} \lambda_2(\tau)d\tau$. (3) Switch to OFF at time $t'$, for which $I(t') \geq -\ln(r)$.

Note that the scheme is computationally simple, in that it only involves a random number generator and a standard quadrature routine.

Under this scheme, the closed-loop system can be shown to be locally asymptotically stable (assuming the model of Fig. 2), regardless of parameters values and control gains.

3.2 Algorithm variations

Based on the random control strategy of Section 3.1, several ‘hybrid’ algorithms can be constructed.

For example, it might be desirable in practical applications to introduce safety thresholds $T_{max}$ and $T_{min}$, which would serve to prohibit temperature excursions beyond ‘safe’ levels. This leads to a random controller with temperature constraints, according to which, if any of the $T_{max}$ or $T_{min}$ thresholds is exceeded, the appliance forcibly switches ON (or, respectively, OFF), overriding the random control action.

In addition, initial simulations indicated that, even though the proposed random controller performs overall better than simpler linear feedback schemes, the latter respond faster at the onset of a failure, where the initial frequency drop is very sharp. To cater for the slower response, the $T_{min}$ ‘safety’ threshold can be made frequency dependent, resulting in a random controller with variable constraints:

$$T_{min} = T_{min} - K_s \Delta f.$$  

4. SIMULATION RESULTS

In this section, we present preliminary results on the performance of the random controller and its variations, outlined in Sections 3.1 and 3.2, and compare with the deterministic controller of Short et al. (2007), when these algorithms are employed to control a population of 40 million refrigerators (90% of which is assumed to be of the ‘dynamic demand’ type), connected to a power supply network as shown in Section 3.

The parameters of the power grid model were set to the values used in Aunedi et al. (2008), and are given in Table 1. The scenario considered involved small values for the total power available in the system, so as to highlight the effects of the refrigerators in the closed loop system.

The nominal parameters in the refrigerator model (1), namely $\alpha$, $T_{ON}$, and $T_{OFF}$, were set to $2.4 \times 10^{-4}$, $-38.3$

$^3$ The stability analysis relies on the passivity of the power system model, and applies to a wide class of models of the power network. The simplified model of Fig. 2 is used here for illustration purposes.
Fig. 3. Power Loss

Table 1. Grid parameter values

<table>
<thead>
<tr>
<th>$T_g$</th>
<th>$T_t$</th>
<th>$R_{eq}$</th>
<th>$M$</th>
<th>$D$</th>
<th>$P_{tot}$ (GW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>50</td>
<td>0.177</td>
<td>6.7</td>
<td>1</td>
<td>25</td>
</tr>
</tbody>
</table>

A trade-off is identified between the absence of overshoot of the consumed power and the time required for the average temperature to recover its steady-state value, which is (significantly) longer for the random controller. The proposed scheme does not allow for the control of the recovery time, as the time constant in the expression for $E[T]$ (7) is equal to $1/\alpha$, which is a device constant.

The ‘variable constrained’ version of the random controller responds as fast as the deterministic algorithm, while the introduction of ‘safety’ temperature thresholds does not adversely affect the closed loop performance.

5. CONCLUSIONS

A new algorithm for dynamic-demand control of refrigerator appliances has been presented. The proposed algorithm adopts a probabilistic description of the problem, resulting in a relatively simple control scheme. Application of this control strategy ensures sufficient diversification (mixing) of the temperature across the controlled appliances, and does not lead to overshoot or instability phenomena associated with simpler deterministic schemes.

Initial simulation results verify the theoretical underpinning of the proposed approach. The random controller is capable of maintaining the power system’s frequency for a longer period of time, when compared to the deterministic scheme, and results in faster recovery at the end. Contrary to deterministic feedback, which breaks down in cases where the total average power consumed by refrigerators is large relative to the overall system demand, the random controller was shown to perform robustly.

REFERENCES


### Table 2. Controller parameter values

<table>
<thead>
<tr>
<th>Type</th>
<th>( T_{\text{min}} )</th>
<th>( T_{\text{max}} )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteretic</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Deterministic</td>
<td>4</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Random</td>
<td>5, ( T_{\text{des}} ) = 5, ( v_{\text{des}} ) = 1</td>
<td>5, ( T_{\text{min}} ) = 1, ( T_{\text{max}} ) = 9</td>
<td></td>
</tr>
<tr>
<td>Constrained Random</td>
<td>5, ( T_{\text{des}} ) = 5, ( v_{\text{des}} ) = 1</td>
<td>1, ( T_{\text{min}} ) = 1, ( T_{\text{max}} ) = 9, ( K_{s} ) = 10</td>
<td></td>
</tr>
<tr>
<td>Variable Constrained</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

![Graphs](image-url)  
**Fig. 4. Simulation results**