Stability of Economically-Oriented NMPC with Periodic Constraint

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Abstract: In smart grid environment, it is economically attractive to manage the load with respect to prices and demands that could change periodically. Thus it is desired to operate some plants and system applications in a cyclic fashion. Model predictive control (MPC) and nonlinear MPC (NMPC) are widely accepted advanced control tools in process industry, due to their advantages of easily handling constraints and multi-input-multi-output systems. Nevertheless, the objective of the majority of NMPC applications is to track a predefined set point. This work proposes an economically-oriented NMPC formulation with periodic constraints that directly deal with systems that exhibit cyclic steady state behavior. The periodic constraint ensures the system converges to a cyclic steady state. In addition, nominal stability of the proposed NMPC formulation is analyzed, where we introduce a transformed system with the origin as the steady state. Hence, a Lyapunov function can be established at the steady state for the transformed system. As a result, asymptotic stability of the original system at the cyclic steady state can be inferred from the stability of the transformed system.

1. INTRODUCTION

The recent push for smart grid systems requires more efficient operation on both supply and demand sides. On the supply side, research that attempts to dispatch intermittent resources, such as wind and solar energy as active generation supplies in the power grid, has attracted much attention, e.g. Xie et al. [2010]. On the demand side, price-responsive load management, which adjusts the demand with respect to the power price to minimize the system operational cost, is of great interest as well. For example, Ierapetritou et al. [2002] proposed a price-responsive scheduling method for energy intensive applications.

Model predictive control (MPC) and nonlinear MPC (NMPC) have the advantages of handling constraint and multi-input-multi-output systems, and provide the possibility to consider resources and demand simultaneously in the smart grid environment. Applications range from short-term regularization and long-term scheduling problems. On the supply side, Xie and Ilic [2009] proposed an MPC-based economic dispatch system. On the demand side, Baumrucker and Biegler [2010] applied open-loop dynamic optimization to pipeline operations with respect to the varying power price and achieved up to 15% more profit.

Currently, most industrial NMPC applications are posed as tracking problems. The objective function of the tracking-NMPC is to regulate the plant at a predetermined steady state. In addition, after more than two decades, stability theory of the tracking-NMPC is well developed for both nominal and robust cases, e.g. Rawlings and Mayne [2009], Mayne et al. [2000] and Limon et al. [2009]. More recently, the interest on economically-oriented NMPC has increased and good practical performance has been reported by many NMPC practitioners (Zavala and Biegler [2009], Rawlings and Amrit [2009], Engell [2007], Aske et al. [2008]). In contrast to the mature body of theoretical basis for tracking-NMPC, there is little stability theory supporting economically-oriented NMPC. Recently, Diehl et al. [2010] proved the stability of a class of nonlinear systems with a single optimal steady state by establishing a Lyapunov function at the steady state. In addition, Angeli et al. [2009] considered receding horizon NMPC with periodic behavior.

In a smart grid environment, especially on the demand side, the periodically varying power price suggests that it is difficult to achieve optimal performance by running the plant at a steady state. On the contrary, a periodic operation which takes advantage of the varying electricity price during different period is preferred. In addition, periodic varying systems are also abundant in the process industry. For example, pressure swing absorption (PSA) and simulated moving bed (SMB) chromatography exhibit cyclic steady states due to their periodic operational nature. To deal with these periodic systems, Lee et al. [2001] proposed a tracking-MPC method by using the concept of repetitive control. However, stability issues for this controllers with periodic systems have yet to be addressed.

This work studies the nominal stability of economically-oriented NMPC for periodical operations. A periodic constraint is introduced to enforce the system to converge to a cyclic steady state solution. Moreover, in order to define the stability of the periodic system, a transformed system is introduced by subtracting the cyclic steady state from the original system. We show that the original system is asymptotically stable at the cyclic steady state if the transformed system is asymptotically stable at the origin. Furthermore, a Lyapunov function is established for the transformed system. As a result, asymptotic stability of the original system is proved. Finally, the concept is demonstrated by simulation study of a double-tank system.
2. ECONOMICALLY-ORIENTED NMPC WITH PERIODIC CONSTRANT

In this work, we consider the following general nonlinear dynamic system as the plant model,

$$x_{t+1} = f(x_t, u_t)$$  \hspace{1cm} (1)

where $x$ is the state variable, $u$ is the control variable. Assume that this system exhibits cyclic steady state $\{x_{0 t}, u_{0 t}\}, \{x_{1 t}, u_{1 t}\}, \ldots, \{x_{K t}, u_{K t}\}$ with a period $K$. It could come from the system’s cyclic operational nature such as SMB and PSA, or the result of optimization from the periodically fluctuating cost or demand. Therefore, it is valid to assume that the period $K$ is known. In addition, it is worth emphasizing that a unique steady state is a special case of the cyclic steady state. Hence, the NMPC formulation and the stability analysis developed in this work applies to problems with single steady states as well.

Fig. 1. Illustration of economically-oriented NMPC with periodic constraint.

To ensure the system converges to the cyclic steady state, we propose an NMPC formulation with a periodic constraint:

$$\min \sum_{i=0}^{N+K} l(x_i, u_i)$$

s.t. $x_{i+1} = f(x_i, u_i), \quad i = 0, \ldots, N+K$

$$x_0 = x_x, x_{N+K} = x_N$$

$$x_i \in X, \quad u_i \in U$$  \hspace{1cm} (2)

where $N + K$ is the horizon length, $l(\cdot, \cdot)$ is a general economic function, not necessarily a quadratic tracking term. Also, $x_{N+K} = x_0$ is a periodic constraint as shown in Fig. 1. If NMPC (2) is feasible for the plant (1) and the cyclic steady state $\{x_{0 t}, u_{0 t}\}, \{x_{1 t}, u_{1 t}\}, \ldots, \{x_{K t}, u_{K t}\}$ is locally unique, then as shown in Fig. 1, the plant is driven from its initial condition $x_0$ to $x_N$ in $N$ time steps and the periodic constraint enforces the plant to converge to the cyclic steady state from $N$ to $N + K$ time steps, e.g.

$$\{x_0, x_N, x_{N+1}, \ldots, x_{N+K}\} = \{x_{0 t}, u_{0 t}, \ldots, x_{K t}, u_{K t}\},$$

Note that the cyclic steady state also evolves according to the dynamic system $x_{t+1} = f(x_t, u_t)$ and it has a period of length $K$, that is $x_{(t+m)K} = x_{t+K}$, where $m$ is any positive integer. Thus the plant evolves according to the cyclic steady state after $N + K$ time steps towards infinite time as shown in Fig. 1. As a result, the periodic constraint in economically-oriented NMPC for cyclic steady state can be viewed as the terminal equality constraint for tracking-NMPC to mimic the plant dynamic over infinite time with a finite time horizon as described by Keerthi and Gilbert [1988].

For simplicity, we make the following assumption throughout.

Assumption 1. The optimization problem (2) satisfies linear independent constraint qualification (LICQ) (Nocedal and Wright [2006]), sufficient second order condition (SSOC) (Nocedal and Wright [2006]) and strict complementarity (SC) (Nocedal and Wright [2006]) at the solution.

Assumption 1 indicates that the NMPC (2) a well-posed optimization problem. Thus its solution is locally unique, and therefore the cyclic steady state $\{x_{N}, x_{N+1}, \ldots, x_{N+K}\}$ is also locally unique. Hence, the plant (1) controlled by NMPC (2) converges to the cyclic steady state if the horizon $N$ is long enough.

3. STABILITY ANALYSIS

In this section, we pursue nominal stability analysis for the closed-loop system. Lyapunov theory is a powerful tool for stability analysis. Mayne et al. [2000], Rawlings and Mayne [2009] present excellent reviews of Lyapunov stability results for tracking-NMPC. In a recent work, Diehl et al. [2010] established a Lyapunov function for NMPC with a general economic objective function. However, most works deal with stability at a single steady state and establish Lyapunov function around the steady state. As a single steady state is not present, it is difficult to directly define Lyapunov stability and a Lyapunov function for system (1) with cyclic steady state behavior. To overcome this problem, we introduce a transformed system by subtracting the cyclic steady state from the original system and establish the Lyapunov stability for the transformed system. Moreover, we show that the stability of the original system can be inferred from the stability of the transformed system.

The introduced transformed system is described as the following.

$$\bar{x}_i = x_i - x^{*}_i$$

$$\bar{u}_i = u_i - u^{*}_i.$$  \hspace{1cm} (3)

As a result, the transformed system evolves according to

$$\bar{x}_{i+1} = f(\bar{x}_i + x^{*}_i, \bar{u}_i + u^{*}_i) - x^{*}_{i+1}$$

$$= f(\bar{x}_i + x^{*}_i, \bar{u}_i + u^{*}_i) - x^{*}_{i+1} - mK.$$  \hspace{1cm} (4)

The second equality comes from the fact that the cyclic steady state $x^{*}$ has a period of $K$. It is worth emphasizing that the cyclic steady state $\{x^{*}_0, u^{*}_0\}$ are known parameters to the transformed system $\bar{x}_{i+1} = f(\bar{x}_i, \bar{u}_i)$.

Fig. 2. Illustration of the transformed system.

2 demonstrates the dynamic behavior of the transformed system corresponding to original system shown in Fig. 1. The transformed system starts from its initial condition $\bar{x}_0$ and converges to 0 in $N$ time steps and remains at 0 from $N$ to $N + K$ time steps. On the contrary, the original system starts from $x_0$ and converges to the cyclic steady state after $N$ time steps. From equation (4), we see when $\bar{x} = (\bar{x}_i, \bar{u}_i) \rightarrow (0,0)$,

$$x^{*}_{i+1} = f(x^{*}_i, u^{*}_i) = x^{*}_{i+1} - mK$$

or $x_{i+1} = f(x_i, u_i) = x_{i+1} - mK$, and $x_i = x^{*}_i$.  \hspace{1cm} (5)

This means that the original system converges to the cyclic steady state, i.e. $(x_i, u_i) \rightarrow (x^{*}_i, u^{*}_i)$, when $(\bar{x}_i, \bar{u}_i) \rightarrow (0,0)$ as shown in Fig. 2. As a result, the following lemma can be stated.
Lemma 1. The stability of the transformed system
\[ x_{i+1} = f(\bar{x}_i, \bar{u}_i) := f(\bar{x}_i + x^*_{ni}, \bar{u}_i + u^*_{ni}) - x^*_{ni+1} \]
(6)
at \((0, 0, 0)\) is equivalent to the stability of the original closed-loop system \(x_{i+1} = f(x_i, u_i)\) at the cyclic steady state \((x^*_{ni}, u^*_{ni})\).

The proof of the Lemma follows the arguments that both \(\bar{x}, x\) and \(\bar{u}, u\) are bounded and converge to their targets simultaneously. Therefore, the asymptotic stability of the original system is the same as that of the transformed system. As a result, we prove the stability of the original system by showing that the transformed system is asymptotically stable at the origin.

For the purpose of stability analysis, we modify the objective function in the NMPC formulation (2) by subtracting the cyclic steady state cost \(l(x^*, u^*)\), that is
\[
\sum_{i=0}^{N+K} [l(x_i + x^*_{ni}, \bar{u}_i + u^*_{ni}) - l(x^*_i, u^*_i)].
\]
(7)

Note that since Assumption 1 implies the cyclic steady state \([x^*_0, u^*_0], (x^*_1, u^*_1), \ldots, (x^*_N, u^*_N)\) is a unique solution, \(\sum_{i=0}^{N+K} l(x^*_i, u^*_i)\) is a constant. Thus, economically-oriented NMPC formulation (2) yields the same control action if the objective function is modified as (7). Consequently, stability result of the NMPC (2) remains the same even though the equation (7) is used as the objective function.

For the transformed system (3), we further modify the value function (7) as follows,
\[
V(\bar{x}) = \sum_{i=0}^{N+K} \tilde{l}(\bar{x}_i, \bar{u}_i) := \sum_{i=0}^{N+K} [l(\bar{x}_i + x^*_{ni}, \bar{u}_i + u^*_{ni}) - l(x^*_i, u^*_i)],
\]
(8)
and it is easy to infer
\[ \tilde{l}(0, 0) = 0. \]
(9)

Therefore in the following, we pursue to show the value function (8) is a Lyapunov function and the transformed system is asymptotically stable at the origin.

An input sequence \(u = (u_0, u_1, \ldots, u_{N+K})\) is termed as feasible for the initial state \(x\) if \(u \in U\), and the corresponding state sequence evolved according to the plant (1) with initial condition \(x_0 = x\) satisfies \(x_i \in \mathbb{X}, i = 0, \ldots, N + K, x_{N+K} = x_N\). Moreover, the admissible set \(\mathbb{X}_N\) can be defined as the set of \((x, u)\)
\[ \mathbb{X}_N = \{ (x, u) | x_i \in \mathbb{X} \text{ and } u_i \in U, i = 0, \ldots, N + K, x_{N+K} = x_N \}. \]

Then the projection of \(\mathbb{X}_N\) onto the space \(\mathbb{R}^n\) can be further defined as the set of admissible states \(\tilde{\mathbb{X}}_N\), i.e.
\[ \tilde{\mathbb{X}}_N = \{ x | \exists u \text{ such that } (x, u) \in \mathbb{X}_N \}. \]
(10)

In addition, we make the following commonly used assumption.

Assumption 2. \(f(\cdot, \cdot)\) and \(l(\cdot, \cdot)\) are Lipschitz continuous on the admissible set, that is there exist Lipschitz constants \(L_f\) and \(L_l\) such that for all \((z_1, v_1), (z_2, v_2) \in \mathbb{X}_N\),
\[
|f(z_1, v_1) - f(z_2, v_2)| \leq L_f \| (z_1, v_1) - (z_2, v_2) \|
\]
\[
l(z_1, v_1) - l(z_2, v_2) \leq L_l \| (z_1, v_1) - (z_2, v_2) \|.
\]
(11)

With this assumption, it is equivalent to state that \(\tilde{f}(\cdot, \cdot)\) and \(\tilde{l}(\cdot, \cdot)\) are Lipschitz continuous too, that is there exist Lipschitz constants \(L_{\tilde{f}}\) and \(L_{\tilde{l}}\) such that
\[
|\tilde{f}(z_1, v_1) - \tilde{f}(z_2, v_2)| \leq L_{\tilde{f}} \| (z_1, v_1) - (z_2, v_2) \|
\]
\[
|\tilde{l}(z_1, v_1) - \tilde{l}(z_2, v_2) | \leq L_{\tilde{l}} \| (z_1, v_1) - (z_2, v_2) \| .
\]
(12)

To pursue the stability analysis, the system is required to have some degree of controllability. Here we extend the weak controllability defined at a single steady state by Diehl et al. [2010] to the weak controllability at the cyclic steady state \([(x^*_0, u^*_0), (x^*_1, u^*_1), \ldots, (x^*_N, u^*_N)]\). Without losing generality, let \(N = cK\), where \(c\) is a positive integer. Therefore \(\sum_{k=0}^{N+K} (\cdot) = \sum_{p=0}^{c} \sum_{j=0}^{K} (\cdot)\).

Assumption 3. (Weak controllability at the cyclic steady state): There exists a \(\mathcal{X}_\infty\) function \(\gamma(\cdot)\) such that for every initial condition \(x \in \mathbb{X}_N\), there exists \(u\) such that \((x, u) \in \mathbb{Z}_N\) and
\[
\sum_{p=0}^{c} \sum_{j=0}^{K} |u_{pK+j} - u^*_j| \leq \gamma(|x_j - x^*_j|).
\]
(13)

This assumption means that starting from the admissible state set, the system can be steered to the cyclic steady state in \(N\) steps without requiring large cost of input sequence, while satisfying the constraints.

For the nominal case addressed in this paper, these assumptions can be verified in advance. For the robust case these assumptions have been discussed in a number of studies, including Limon et al. [2009], DeHaan and Guay [2010].

Equivalently, for the transformed system (3), the weak controllability assumption indicates that there exists a \(\mathcal{X}_\infty\) function \(\gamma(\cdot)\)
\[
\sum_{i=0}^{N+K} |\bar{u}_i - 0| \leq \gamma(|\bar{x} - 0|).
\]
(14)

This means that requiring the original system to be steered to the cyclic steady state is the same as requiring the transformed system to be steerable to the origin without requiring large cost input sequences.

Finally, we make a similar assumption for the transformed system with the stability analysis of the tracking NMPC.

Assumption 4. There exists a \(\mathcal{X}_\infty\) function \(\beta(\cdot)\) such that the stage cost \(\tilde{l}(\cdot, \cdot)\) satisfies
\[
\tilde{l}(\bar{x}, \bar{u}) \geq \beta(|\bar{x} - 0|).
\]
(15)

In practice, a general economic objective function may not satisfy this assumption, regularization terms can be added to the original stage cost, i.e.,
\[
l(z, v) + |z - \tilde{z}'|^2_Q + |v - \tilde{v}'|^2_R
\]
where \(\tilde{z}', \tilde{v}'\) are suitably-defined reference values, and weighting matrices \(Q\) and \(R\) can be chosen so that Assumption 4 holds.

Finally, the main theorem of this work is stated as:

Theorem 2. Let the above Assumptions 1, 2, 3 and 4 hold, then \(V(\bar{x})\) defined in equation (8) is a Lyapunov function and the transformed system \((\bar{x}, \bar{u})\) is asymptotically stable at \((0, 0, 0)\). Consequently, the original closed-loop system controlled by periodic constraint NMPC (2) is asymptotically stable at the cyclic steady state solution \((x^*_{ni}, u^*_{ni})\).

Proof: First Assumption 4 implies that
\[
\tilde{l}(\bar{x}, \bar{u}) \geq 0,
\]
(17)
then
\[
V(\bar{x}) = \sum_{i=0}^{N+K} \tilde{l}(\bar{x}_i, \bar{u}_i) \geq \beta(|\bar{x} - 0|).
\]
(18)
Moreover, note that for the transformed system \( \bar{x}_i = 0, \forall i \geq N \) as shown in Fig. 2, therefore

\[
V(\bar{f}(\bar{x}, \bar{u})) - V(\bar{x}) = -\bar{I}(\bar{x}, \bar{u}) \leq -\beta(\|\bar{x} - 0\|),
\]

the inequality also comes from Assumption 4.

Finally from equation (9), Assumption 2 and equation (12), we can see

\[
V(\bar{x}) = \sum_{i=0}^{N+K} I(\bar{x}_i, \bar{u}_i)
= \sum_{i=0}^{N+K} (I(\bar{x}_i, \bar{u}_i) - I(0,0))
\leq L_f(\sum_{i=0}^{N+K} |\bar{x}_i - 0| + \sum_{i=0}^{N+K} |\bar{u}_i - 0|).
\]

Moreover, from Lipschitz continuity of \( \bar{f}(\cdot, \cdot) \), we have

\[
|\bar{x}_i - 0| \leq L_f^1|\bar{x} - 0| + L_f^2|\bar{u}_0 - 0| + \ldots + L_f^N|\bar{u}_N - 0|.
\]

Summing this inequality gives

\[
\sum_{i=0}^{N+K} |\bar{x}_i - 0| \leq L_f|\bar{x} - 0| + \sum_{i=0}^{N+K} |\bar{u}_i - 0|.
\]

Fig. 3. Double-tank system

where \( L_f \geq 1 + L_f + \ldots + L_f^{N+K} \).

In addition, from Assumption 3 and equation (14), we have

\[
\sum_{i=0}^{N+K} |\bar{u}_i - 0| \leq \gamma(|\bar{x} - 0|)
\]

As a result, equation (20) turns out to be

\[
V(\bar{x}) \leq L_f(\sum_{i=0}^{N+K} |\bar{x}_i - 0| + \sum_{i=0}^{N+K} |\bar{u}_i - 0|)
\leq L_f L_f|\bar{x} - 0| + L_f (L_f + 1) \gamma(|\bar{x} - 0|)
\]

Hence

\[
V(\bar{x}) \leq \alpha(|\bar{x}|)
\]

where \( \alpha(\cdot) = L_f L_f(\cdot) + L_f (L_f + 1) \gamma(\cdot) \) and is a \( \mathcal{K}_\infty \) function. Therefore, equations (18), (19) and (25) indicate that \( V(\bar{x}) \) is a Lyapunov function and the transformed system is asymptotically stable. Then in view of the Lemma 1, the original closed-loop system is asymptotically stable at the cyclic steady state.

4. SIMULATION EXAMPLES

In this section, we consider a simulation study of a double-tank system as shown in Fig. 3. The two tanks are interconnected. The liquid inlet flow into the first tank is \( F_{in} \). The liquid outlet flow from the first tank is the liquid inlet flow into the second tank. It is determined by the liquid height in the first tank \( x_1 \), i.e. \( 0.4x_1^\frac{1}{3} \). The liquid outlet flow from the second tank is termed as \( F_{out} \). Similarly, \( F_{out} \) is a function of the liquid height in the second tank \( x_2 \), i.e. \( 0.4x_2^\frac{1}{3} \). It is required that \( F_{out} \) is maintained above a certain level all the time, i.e., \( F_{out} \geq 0.16 \) using \( F_{in} \) as the control variable. In addition, the operational cost is considered as \( F_{in} \) multiplied by a sinusoidally varying power price with 10 time steps as the period. This is chosen to simulate the periodic varying conditions of the double-tank system for illustration purposes. The control objective is to minimize the operational cost while satisfying all the system constraints.

\[
\begin{align*}
\frac{dx_1}{dt} & = 0.16F_{in} - 0.4x_1^\frac{1}{3} \\
\frac{dx_2}{dt} & = 0.4x_2^\frac{1}{3} - F_{out} \\
F_{out} & = 0.4x_2^\frac{1}{3}
\end{align*}
\]

where \( x_1 \) and \( x_2 \) are the liquid heights in the first and second tank, respectively; \( F_{in} \) and \( F_{out} \) are the flow rates in and out of the double-tank system. The price of the power consumed by the inlet liquid \( F_{in} \) is a sinusoidally varying function with a period of 10 sampling times, as shown at the bottom of Fig. 5.

The economically-oriented NMPC is formulated as follows:

\[
\begin{align*}
\min \quad & \text{price} \times F_{in} + \text{reg} \\
\text{s.t} \quad & \text{system (26)}, \\
& \text{periodic constraint}, \\
& F_{out} \geq 0.16, \ 0 \leq F_{in} \leq 5, \\
& 0 \leq x_1 \leq 3, \ 0 \leq x_2 \leq 3
\end{align*}
\]

where the reg term is \( 0.1 + (x_1 - 0.16)^2 + 0.1 \times (x_2 - 0.16)^2 \).

To apply the discrete-time economically-oriented NMPC (2) to a general continuous-time system \( \frac{dz}{dt} = h(z,v) \), we discretize the system using the following general scheme.

\[
z_{k+1} = z_k + \int_{t_k}^{t_{k+1}} h(z(t),v_k)dt, \quad t \in [t_k,t_{k+1}]
\]

where \( z_k \) and \( v_k \) are the discretized state and control variables, respectively. \( h(\cdot,\cdot) \) represents the general continuous-time plant, and it is integrated inside each time period. If \( h(\cdot,\cdot) \) remains bounded in the time period, and a stable integrator, such as a stiff integrator, is used, then the discretization error is asymptotically stable. As a result, in the following, we directly consider the discretized version of the double tank system (26) controlled by the economically-oriented NMPC (27).
In the NMPC formulation (27), the prediction and the control horizons are chosen to be 50 time steps and the periodic constraint in the formulation (27) is $x_1(40) = x_1(50)$ and $x_2(40) = x_2(50)$, since the period of the sinusoidally varying power price profile is 10 sampling times. The closed-loop responses of this system are shown in Figs. 4, 5 and 6. From Fig. 4, we see that both the states in the double-tank system exhibit cyclic steady state behavior with a period of 10 sampling time, which is the same as the sinusoidally varying power price profile. Moreover, after a few sampling times, the system asymptotically converges to the cyclic steady state and is stabilized. As a result, the outlet flow $F_{out}$ also changes sinusoidally with the varying power price as shown in Fig. 5. In addition, it is interesting to note from Fig. 6 that the $F_{in}$ is at the highest value when the power price is the lowest. This means that the majority liquid inlet flow is pumped into the double-tank system when it is the cheapest to do so.

![Fig. 4. State variables (levels) profiles in the tank](image)

![Fig. 5. Outlet flow profile from the tank, $F_{out}$ and power price profile](image)

![Fig. 6. Control variable profile ($F_{in}$ to first tank)](image)

5. CONCLUSION

In this work, we address operation challenges for a class of nonlinear systems that exhibit cyclic steady state behavior. This cyclic behavior may be result of periodic varying cost or demand. For example on the demand side of smart grid environment, one may want to take advantage of the periodic varying electricity price to operate energy-intensive applications when it is cheaper. On the supply side, it is desired to vary electricity production in order to meet the periodically changing demand, since electricity can not be readily stored and has to be used or wasted after production. In addition, some applications in process industry also exhibit periodic behavior due to their cyclic operational nature, such as pressure swing absorption (PSA) and simulated moving bed (SMB) chromatography system.

To deal with these systems, we propose an NMPC formulation with periodic constraint if the period of the cyclic steady state is known. Providing the prediction horizon is long enough and the cyclic steady state is locally unique, this periodic constraint ensures the system converges to the cyclic steady state. In addition, we analyze the nominal stability of this system. However it is difficult to apply Lyapunov theory at cyclic steady state since Lyapunov function is normally defined at a single steady state. Therefore we introduce a transformed system by subtracting the cyclic steady state from the original system. As a result, the transformed system has a single steady state at the origin and a Lyapunov function can be established for the transformed system at the origin. Thus, the asymptotic stability of the transformed system is proved. Moreover, we
show that the asymptotical stability of the transformed system at the origin is equivalent to the asymptotical stability of the original system at the cyclic steady state. Simulation results of a double-tank system with cyclic varying operational cost is demonstrated. It shows that the system controlled by the proposed NMPC formulation is asymptotically stable at the cyclic steady state.

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