Error Dynamics-Based Lyapunov Guidance Law for Stationary Target Observation

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Abstract: Lyapunov guidance law based on error dynamics is proposed to perform observation missions. A new guidance law is designed in the consideration of the characteristics of distance error dynamics. Lyapunov stability theorem is used to analyze the guidance law. The turning direction is related to the difference between the Line-of-sight rate and the heading angle of each UAV. Linear-quadratic regulator is used to design a controller for a linear six degree-of-freedom UAV system. Numerical simulations are performed to validate the performance of the proposed guidance law, and are compared with that of pure error dynamics-based guidance law.

Keywords: Error Dynamics, Lyapunov Stability, Guidance Systems, Automated Guided Vehicles, Target Tracking

1. INTRODUCTION

A lot of research on Unmanned Aerial Vehicle (UAV) guidance and control has been performed, because the guidance and navigation system is very important to accomplish various UAV missions including aerial reconnaissance, surveillance, and attack. The flight path of UAV should be generated in the consideration of the specified mission, and the guidance law is designed to make UAV track the generated path. Recently, multiple UAVs are involved to perform a very complicated mission, and therefore various guidance laws for multiple UAVs have been developed.

Proportional Navigation Guidance (PNG) and pursuit guidance laws are widely used for the missile and UAV guidance systems. PNG constructs a command that the UAV’s head should rotate at a rate proportional to the rotational rate of the Line-of-Sight (LOS) that joins the UAV and the target (Pastrick et al, 1981). Pursuit guidance generates a command that the velocity vector of the UAV always points at the target.

Recently, nonlinear guidance laws are used for solving various constrained and complicated problems. Lyapunov stability theorem has been applied for developing nonlinear guidance laws. A quadratic Lyapunov candidate function was used to design nonlinear guidance laws with free-singularity and to analyze the stability of missile-target systems (Lechevin et al, 2004). Geometric analysis was used to consider the waypoint passing angles as the impact angle guidance law. The impact time guidance law based on PNG was also introduced (Jung and Kim, 2006). Conventional guidance law based on Lyapunov stability theorem was developed to consider finite time convergence in two dimensions as well as in three dimensions (Zhou et al, 2009).

Lyapunov vector fields were also applied for the autonomous UAV control (Lawrence et al, 2008). To perform a stationary target observation or an arbitrary waypoint following, Lyapunov vector fields were constructed by considering the contraction and circulation, and the guidance vector fields were applied for the cooperative tracking of multiple UAVs (Frew et al, 2008; Frew, 2008). While keeping vehicle formation using a rigid graph theory, multiple UAVs were engaged in a stationary target monitoring in the constructed Lyapunov guidance vector fields (Summers et al, 2009), and it has been modified to account for a moving target.

Error dynamics can be also utilized to design a guidance law. Assuming that the velocity of a UAV is constant, the guidance command can be made by making the distance error or the line-of-sight (LOS) error have desirable characteristics (Kim et al, 2010). However, selection of guidance gain was not systematic.

In this study, nonlinear guidance law based on the distance error dynamics and Lyapunov stability theorem is proposed. The proposed guidance law is much simpler than the other nonlinear guidance laws, because only the Lyapunov stability theorem is used to design a guidance law.

This paper is organized as follows. In Section 2, distance error dynamics is formulated, and the error dynamics-based guidance law is proposed to reduce the distance error. Lyapunov stability theorem is used to modify the guidance law. Linear-quadratic regulator (LQR) controller is used to design a stability augmentation system and autopilots for a linear six degree-of-freedom (DOF) UAV model in Section 3. Numerical simulations for a point mass UAV model and a linear 6DOF UAV model are presented in Section 4 to verify the performance of the proposed guidance law. Finally, conclusions are made in Section 5.
2. GUIDANCE LAW

2.1 Point Mass Model

Consider a point mass model as shown in Fig. 1 (Jung and Kim, 2006). Assuming that the velocity is constant, the angle-of-attack of vehicle is small enough to be omitted, and the acceleration vector is perpendicular to the velocity vector, the equations of motion can be represented as

\[ \dot{R} = -V_m \cos(\sigma - \theta_m) \]  
\[ \dot{\sigma} = \frac{V_m}{R} \sin(\sigma - \theta_m) \]  
\[ \dot{\theta}_m = a_m / V_m = a_c \]

where \( R \) denotes the distance between the UAV and the target, \( \theta_m \) denotes the heading angle of the UAV, \( V_m \) denotes the velocity, \( a_m \) denotes the acceleration, and \( \sigma \) represents the line-of-sight to the target. Distance error is defined as

\[ e_R = R - R_d \]  

where \( R_d \) is the constant radius when the UAV goes around the target for observation. Differentiating (4) with respect to time once and twice yields

\[ \dot{e}_R = \dot{R} \]  
\[ \ddot{e}_R = \ddot{R} \]

In order to apply to a 6 DOF UAV system, it is assumed that the autopilot and sensor dynamics of the vehicle are much faster than UAV dynamics so that they can be neglected.

Let us propose a centrifugal acceleration command \( a_c \) as

\[ a_c = \frac{1}{R\sigma} \left( k_1 \ddot{e}_R + k_2 e_R + R\dot{\sigma}^2 \right) \]

Then, substituting (9) into (8), the distance error dynamics can be represented as

\[ \ddot{e}_R + k_1 \dot{e}_R + k_2 e_R = 0 \]

Two gains, \( k_1 \) and \( k_2 \), should be carefully selected considering the characteristics of the error dynamics. In this study, Laplace transform is used to analyze the guidance gains.

\[ s^2 L(e_R) - se_R(0) - \dot{e}_R(0) + k_1 \{ sL(e_R) - e_R(0) \} + k_2 L(e_R) = 0 \]

The first derivative of the distance error is same as the first derivative of the distance between the vehicle and the target. Equation (11), therefore, can be rewritten as

\[ L(e_R) = \frac{(s + k_1)e_R(0) + \dot{R}(0)}{s^2 + k_1s + k_2} \]

In this study, we restrict the system that the above error dynamics has two negative real poles to prevent an overshoot condition. Therefore, to make the guidance law have the characteristics of exponentially decreasing, let us select the guidance gains as

\[ k_1 = p_1 + p_2 \]  
\[ k_2 = p_1 p_2 \]

In (13) and (14), \( p_1 \) and \( p_2 \) are positive real numbers. Equation (12), then, can be rewritten as the following form

\[ L(e_R) = \frac{a + e_R(0)}{s + p_1} - \frac{a}{s + p_2} \]

Using (12) and (15), the relations of two poles and a guidance gain, \( k_1 \), can be represented as

\[ p_1 = \frac{a}{2a + e_R(0)} k_1 - \frac{\dot{R}(0)}{2a + e_R(0)} \]  
\[ p_2 = \frac{a + e_R(0)}{2a + e_R(0)} k_1 + \frac{\dot{R}(0)}{2a + e_R(0)} \]

The guidance gain, \( k_1 \), should be satisfied the following conditions to guarantee the guidance stability.

\[ a > 0 \]  
\[ k_1 > \frac{\dot{R}(0)}{a} \]
Because the UAV flies to the target, let us assume that the initial heading angle of each UAV is within $\pm 90$ degrees from its LOS angle. Then, the initial value of the first derivative of the distance error of each UAV is always negative. Hence, if $k_i$ satisfies (19), equation (20) could be always satisfied.

Laplace inverse transform of (15) can be represented as

$$e_k(t) = (a + e_k(0))e^{-pt} - ae^{-pt}$$

(21)

Note that two poles are in LHF (Left Half Plane), and therefore the distance error converges to zero. Thus, it can be stated that the proposed guidance law makes the error go to zero exponentially.

Note that the first derivative term of the LOS to the target is in the denominator of the guidance command. It is important to reduce the distance error at an early stage. In this stage, however, the LOS does not change much, or its change is rather infinitesimal. Therefore, the limit of the guidance command should be included by considering the dynamics of the UAV (Kim et al., 2010).

2.3 Lyapunov Guidance Law

In this study, Lyapunov stability theorem is used to modify the error dynamics-based guidance law. The main object of the guidance law is to reduce the distance error, and therefore, let us consider the following Lyapunov candidate function.

$$V = c_1\left(\frac{V_m}{R}\right)^2 e_k^2 + c_2 e_k^2 \geq 0$$

(22)

where $c_1$ and $c_2$ are positive constants. Considering the dimension of each term, $\left(\frac{V_m}{R}\right)^2$ is multiplied to the first term.

The first derivative of the Lyapunov candidate function can be represented as

$$\dot{V} = 2c_1\frac{V_m^2 e_k R d}{R^3} + 2c_2 \dot{R}\left(\frac{V_m}{R}\right)^2 + c_3 \frac{V_m}{R} \dot{R}$$

(23)

Let us propose the centrifugal acceleration command $a_c$ as

$$a_c = \frac{1}{c_2 \dot{R}}\left(c_1 V_m^2 e_k R d + c_2 R \dot{\sigma}^2 + c_3 \frac{V_m}{R} \dot{R}\right)$$

(24)

Using (24) in (23), we have

$$\dot{V} = -2c_3 \frac{V_m}{R} \dot{R}^2$$

(25)

where $c_3$ is a positive constant design variable.

As shown (25), the first derivative of the Lyapunov candidate function is negative semi-definite, and therefore, the proposed guidance command makes the distance error zero by the Lyapunov stability theorem (Khalil, 2001).

2.4 Relation between the heading angle and turning direction

Turning direction is very important for multiple UAVs to perform a target observation, because they should not collide with each other. The direction of the turn can be chosen by setting the initial heading angle of each vehicle, because the given Lyapunov candidate function is defined as a positive definite quadratic function and its first derivative is a negative semi-definite quadratic function.

It can be conjectured that the turning direction is related to the initial heading angle. The difference between the LOS angle and the initial heading angle determines the signs of the first derivative of the distance and the LOS rate. When a UAV turns counter clockwise, the LOS rate will be positive. On the other hand, when UAV turns clockwise, the LOS rate will be negative. Hence, the turning direction of each UAV is dependent on its initial heading angle.

![Fig. 2. Relation between the angle $(\sigma - \theta_u)$ and Turning direction](image)

Although the initial heading angle of a UAV is opposite to the stationary target, the distance error would be converted to zero exponentially. To observe a stationary target, however, UAVs should fly to the target and their points of departure may be far from the desired keeping distance. Therefore, in this study, the situation for within $\pm 90$ degrees of the difference between their heading angle and LOS is only considered.

3. CONTROL LAW

The proposed guidance law is associated with the lateral command of roll angle. To apply the guidance law to the linear 6-DOF UAV model, longitudinal and lateral controllers are designed.
First, longitudinal distance error, \(d\), as shown in Fig. 3, is defined as the difference between the actual flight path and the reference path. The distance error is considered to control the flight path in the longitudinal controller. The dynamics of longitudinal distance error, \(d\), can be represented as

\[
\frac{d}{dt} = V_M \sin(\gamma - \gamma_R) \approx V_M (\gamma - \gamma_R) \tag{26}
\]

where \(\gamma_R\) is the reference flight path angle.

Fig. 3. Longitudinal distance error

In this study, the output feedback controller based on LQR is designed to track the guidance command of the flight path angle while maintaining the distance error zero (Stevens et al., 2003). When the longitudinal commands, \(r_v\) and \(r_d\), are given, the longitudinal tracking errors, \(e_v\) and \(e_d\), should be kept zero. The longitudinal tracking errors are defined as

\[
e_v = r_v - V_M \tag{27}
\]

\[
e_d = r_d - d \tag{28}
\]

Lateral controller is also designed to track the roll command, \(r_\phi\), while maintaining zero wash-out yaw rate, \(r_\phi\). When the lateral commands, \(r_\phi\) and \(r_\phi\), are given, the lateral tracking errors, \(e_\phi\) and \(e_\phi\), should be kept zero. The lateral tracking errors are defined as

\[
e_\phi = r_\phi - r_\phi \tag{29}
\]

\[
e_\phi = r_\phi - \phi \tag{30}
\]

The following lateral heading angle, \(\lambda_{\text{lat}}\), is used to make the roll angle command.

\[
a_\phi = \tan(\lambda_{\text{lat}}) \tag{31}
\]

where \(g\) is a gravitational acceleration constant.

Finally, the following roll angle command is adapted to guide the lateral direction.

\[
\dot{\phi} = K_\phi (\lambda_{\text{lat}} - \phi_{\text{lat}}) \tag{32}
\]

The longitudinal control block diagram and the lateral control block diagram are shown in Figs. 4 and 5.

4. NUMERICAL SIMULATIONS

Numerical simulations are performed to verify the performance of the proposed nonlinear guidance law. It is assumed that there exists no wind, and a stationary target is considered. The target observation by the coordinated turn with an arbitrary radius is considered as an UAV mission. Simulations using the point mass model are first performed to show the performance of the proposed guidance laws, and then the modified guidance law is applied to the linear 6-DOF UAV model.

4.1 Point mass model simulation

In this simulation, the velocity of the UAV is 22m/s, and the initial position of each UAV, target position, and the turning radius are summarized in Table 1. The initial heading angle of each UAV is set as 1 degree larger than its LOS angle.

\[
k_1 = \frac{-\dot{R}(0)}{a + e_\phi(0)} + 1 \tag{33}
\]

The limit of the guidance command is chosen as 2 \(g\). \(a\) in the pure error dynamics-based guidance law is 1, and \(k_1\) is selected using (33). In the Lyapunov guidance law, the guidance gains \(c_1\), \(c_2\), and \(c_3\) are set as 2, 1, and 3, respectively.
Table 1. The initial conditions for the point mass model

<table>
<thead>
<tr>
<th>Position and distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAV 1 (0,0)</td>
</tr>
<tr>
<td>UAV 2 (1600,0)</td>
</tr>
<tr>
<td>UAV 3 (300,1600)</td>
</tr>
<tr>
<td>Target (800,600)</td>
</tr>
<tr>
<td>Radius 300m</td>
</tr>
</tbody>
</table>

Figures 6 and 7 show the two-dimensional trajectories of both guidance laws. The turning directions of UAV 1 and UAV 3 based on the pure guidance law are clockwise, but that of UAV 2 is counter clockwise as shown in Fig. 6. In Fig. 7, however, all UAVs using the modified Lyapunov guidance law turn clockwise.

Fig. 6. 2D trajectories of pure error dynamics-based guidance law

Fig. 7. 2D trajectories of Lyapunov guidance law

Figure 8 shows the guidance command histories of both guidance laws. For the pure error dynamics-based guidance law and Lyapunov guidance law, the average computational time is summarized in Table 2. Matlab R2008a is used, and core2 quad 2.67GHz CPU and 3.25GB memory are used for simulation. As shown in Table 2, the computational time of pure error dynamics-based guidance law is shorter than that of Lyapunov guidance law, because command calculation of pure error dynamics-based guidance law is much simpler than that of Lyapunov guidance law.

Table 2. Computational time comparison

<table>
<thead>
<tr>
<th>Pure error dynamics-based guidance law</th>
<th>Lyapunov guidance law</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0013 sec</td>
<td>0.0020 sec</td>
</tr>
</tbody>
</table>

4.2 6-DOF UAV simulation

The modified guidance law is applied to a linear 6-DOF UAV model, which is the identified model using the flight test data using UAV in Fig. 9 (Kim et al., 2010). The states vectors and control inputs vectors are

$$x_{\text{long}} = [v_M \quad \alpha \quad \theta \quad q]^T$$  

$$x_{\text{lat}} = [\beta \quad \phi \quad p \quad r]^T$$  

$$u_{\text{long}} = [\delta_e \quad \delta_t]^T$$  

$$u_{\text{lat}} = [\delta_a \quad \delta_r]^T$$

The states, $v_M$, $\alpha$, $\theta$, $q$, $\beta$, $\phi$, $p$, and $r$, are the velocity, the angle of attack, pitch angle, pitch rate, sideslip angle, roll angle, roll rate, and yaw rate, respectively. Control inputs, $\delta_e$, $\delta_t$, $\delta_a$, and $\delta_r$, are the elevator displacement, thrust displacement, aileron displacement, and rudder displacement, respectively.
The velocity of each UAV is 22m/s, and the initial position of each vehicle, the position of the target, the turning radius are summarized in Table 3.

<table>
<thead>
<tr>
<th>Position and distance</th>
<th>UAV 1 (0,0)</th>
<th>UAV 2 (-2000,10000)</th>
<th>UAV 3 (-10000,1000)</th>
<th>Target (-5000,5000)</th>
<th>Radius 3000m</th>
</tr>
</thead>
</table>

The limit of the guidance command is set as 2 g, and the initial heading angle of each UAV is set as 1 degree smaller than its LOS angle. The guidance gains $c_1$, $c_2$, and $c_3$ are selected as 1000, 0.5, and 300, respectively. The limit of displacement of all control surfaces is 0.5 radian.

Figure 10 shows the two-dimensional trajectories of UAVs. All UAVs turn clockwise while keeping its specified observation radius. Using the 6-DOF simulation, the relation of the difference between heading angle and the LOS angle and the direction of turn as shown in Fig.2 is verified.

5. CONCLUSIONS

The modified nonlinear guidance law is proposed based on the distance error dynamics. The equations of motion and error dynamics are formulated to derive a guidance law. To reduce the distance error while keeping the observation radius, the centrifugal acceleration is used. The turning direction of UAV is closely related to the difference between the LOS angle and the heading angle. To avoid a collision between UAVs, the turning direction of each UAV should be same. The turning direction can be determined by setting the initial heading angle of UAV in the modified guidance law. The guidance commands of the pure error dynamics-based guidance law have much fluctuation, however those of the Lyapunov guidance law are very smooth and small. The proposed guidance law will be modified for the moving target and wind condition.

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