Optimization of Heat Exchanger Network with Fixed Topology by Genetic Algorithms

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Optimization of heat exchanger network (HEN) of fixed structure is necessary at final stage of designing procedure, particularly in sequential and insight-based approaches. The paper addresses an efficient solution approach based on genetic algorithms (GA). The use of this evolutionary optimization technique allows accounting for standard heat exchangers, i.e. such that feature discrete values of surface area. The surface area of heat exchangers and split ratios are decision variables. The objective of optimization is minimization of HEN total annual cost. Therefore, the approach is able to solve the more general problem than the methods developed to date.

1. Introduction

The paper addresses optimization of heat exchanger network (HEN) with fixed topology. This problem has to be solved as the last stage of sequential approaches for HEN design, see for instance Briones and Kokossis (1999a,b), Asante and Zhu (1997). Also, networks calculated by simultaneous approaches often require improvements to reduce investment expenditure. The HEN structure is fixed but heat exchanger surface areas and split ratio of splitters have to be calculated to minimize a given goal function. The optimization problem is formulated in the literature as nonlinear programming (NLP) task. The lack of binaries defining existence of heat exchangers in optimization model causes that no structural changes can be accounted for rigorously. Also, split ratio of splitters is often kept fixed in existing methods. In fact, some approaches are aimed at minimizing total area of process heat exchangers as for instance the contribution by Quessada and Grossmann (1993). It is commonly assumed that surface area is unbounded continuous variable. However, in industrial scenario heat exchanger area is bounded and is often discrete variable due to application of standard heat exchangers. Most of existing approaches fails to cope with such apparatus. Even under typical assumptions the problem of optimizing HEN of fixed structure is advantageous optimization task. Sophisticated optimization techniques have to be applied to locate global optimum, see for instance, Zamorra and Grossmann (1998 a, b).

The paper addresses the method that allows removing certain limitations of existing approaches. First, it allows optimizing standard heat exchangers, i.e. such that have discrete values of surface area. Next, the method allows optimizing split ratio in splitters. It accounts for total annual cost of HEN. Hence, removing of heat exchanger reduces total cost. The optimization has the incentive to perform such changes. Heat loads of utilities are also optimization variables. It is also worth noting that the method
has potential of dealing with discontinuous functions. These effects have been achieved due to the application of genetic algorithms (GA) as optimization technique.

2. Problem formulation

The problem of optimizing HEN of given structure is formulated here as follows. Given the topology of HEN, i.e. locations of heat exchangers, splitters and mixers. Calculate heat transfer area of exchangers and split ratio of splitters such that minimize the total annual cost (TAC) defined as the sum of utility cost and investment expenses on heat exchangers. The latter involves fixed charge and area-dependent term. Notice, that heat loads on heaters and coolers are also optimized. Any exchanger in the initial structure can be eliminated if this reduces the cost. Likewise, number of branches in existing splitters can be changed and a splitter can be removed, too. The only limitation is that no new heat exchangers, splitter and mixer can be added. Also, changes of location of existing heat exchangers, splitters and mixers are forbidden.

3. Basic features of genetic algorithms optimization procedure

The optimization algorithm applied is a version of GEN-COM solver – general genetic algorithm solver for MINLP problems developed by R. Bochenek. Its novel feature is that it applies additional population, the so-called intermediate genetically modified sub-population. Offspring population is generated from the superset of parent population and the sub-population instead of parent population only what is the case in many other GA methods. This novel feature has been designed to eliminate or at least to limit the effect of premature degeneration, which means in GA optimization the convergence to a local optimum. The method applies natural representation of variables via real-valued vector instead of binary chromosomes. For HEN problem the vector consists of surface areas and split fractions. The solver provides the user with a variety of genetic operators, however, in HEN problem it appeared sufficient to use only two from the available ones: simple crossover and uniform mutation.

4. Optimization model and solution method

In order to code the structure of HEN we have applied the representation consisting of vectors and matrices that uniquely define HEN topology. The program generates automatically all model equations for given structure code. Due to space limitation this issue isn’t address here. The reader is referred to forthcoming publication by Jezowski et al. (2007).

The goal function (1) defines the total annual cost (TAC) of HEN.

\[
TAC = \text{Cost}_{HU} \sum_i^{NHU} Q_i^{HU} + \text{Cost}_{CU} \sum_j^{NCU} Q_j^{CU} + \frac{(1 + ROR)^{PL}}{PL} \sum_k^{NA} (a_k + b_k \cdot A_k^2)
\]  

(1)

It is necessary noting that if heat exchanger \( k \) is eliminated then parameter \( a_k \) must be set at zero. This requires binaries if deterministic optimization solver is to be applied. In genetic algorithms simple logical conditions are sufficient.

The optimization model consists of the following equalities and inequalities:

A. Heat balances of heat exchangers and design equations for them

B. Mass balances of splitters (isothermal process is assumed)

C. Mass and heat balances of mixers
D. Conditions on outlet temperatures from the HEN ensuring that given outlet temperatures are met

E. Thermodynamic conditions

F. Other case specific constraints as for instance must-be matches, limits on surface area and so on

Majority of contributions on HEN applies formula (2) as design equation of heat exchanger.

\[ A = \frac{Q}{U \cdot LMTD} \]  

(2)

Calculations of \( LMTD \) values in optimization procedure cause numerical difficulties that result finally in the use of some approximations. Hence, we have applied another model of heat exchanger that has the form:

\[ T_{out} = \alpha T_{in} + (1-\alpha) T_{in}^* \]  

(3)

\[ \alpha = \frac{\gamma^* - \gamma}{\gamma^* \exp\left(\gamma^*\right) - \gamma \exp(\gamma)} ; \quad \gamma = \frac{U \cdot A}{CP} ; \quad \gamma^* = \frac{U \cdot A}{CP^*} \]  

(4)

Superscript (*) denotes one from the two streams exchanging heat while parameters of the second stream have no superscript. The main advantage of the model is that it eliminates numerical problems with calculation of \( LMTD \). The deeper discussion of the model is addressed in Bochenek and Jezowski (1999).

Thermodynamic conditions (E) are inequalities (5) that ensure feasible heat exchange:

\[ t_h - t_c \geq \Delta T_{min} \]  

(5)

In our approach \( \Delta T_{min} \) can be set at small values, even close to zero. With the total annual cost as the goal function the optimization does not allow for choosing too small temperature differences in heat exchangers.

In general inequality and equality constraints cause serious problems for GA solvers. To deal with them we have developed the following mechanisms:

a) Each constraint of type (D) was changed into the pair of inequalities

b) All inequalities were included into an augmented goal function

c) All other equality constraints are solved directly in calculation procedure

The way of dealing with equalities in point (c) requires deeper explanation since is crucial point of the overall procedure. The robustness and efficacy of the optimization requires that all equalities have to be linear. This is not the case in the model. We have applied the following solution to the problem. The variables are divided into decision (independent) ones, which are generated by the algorithm and dependent ones that are calculated from the constraints. The division has to ensure that dependent variables are calculated from linear equations or sets of simultaneous linear equations in regards of them for fixed independent variables. To achieve this we have chosen surface areas and split ratios as independent variables. The solution algorithm has been organized in the way that allows sequential solution of equalities or equation sets.
5. Example of application

In order to illustrate capabilities of the approach we will show here a modified example from Ciric and Floudas (1990). The original problem consists in retrofit design. The stream data are given in Table 1. First, we have performed some structural modifications using the genetic algorithm based approach from Jezowski et al. (2007). One of the generated structures has been chosen to further optimization. Notice, that this HEN, shown in Fig. 1, is defined only at topology level. The goal function in optimization was total annual cost defined by (1) with following values for the parameters: $P_l= 5$, $ROR=0.2$, $a=4000$, $b=1200$, $c=0.6$. The calculations were performed for population of 30 members and with the stopping criterion of 400 generations. The probability for simple crossover was 0.8 and for uniform mutation 0.2. Non-standard and standard heat exchangers were optimized. For the latter we have used values of heat transfer area changing with the step of 1 m² for process heat exchangers. Surface areas of coolers and heaters were treated as continuous variables in both cases. The final solution for non-standard heat exchangers is shown in Fig. 2. The HEN for standard apparatus is similar (not shown here due to space limitation).

<table>
<thead>
<tr>
<th>Stream</th>
<th>$T_{in}$ [°C]</th>
<th>$T_{out}$ [°C]</th>
<th>$CP$ [kW/K]</th>
<th>$h$ [kW/m²K]</th>
<th>Cost [$/(kW y)$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>500</td>
<td>350</td>
<td>10.01</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>450</td>
<td>350</td>
<td>12.00</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>400</td>
<td>320</td>
<td>8.01</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>HU</td>
<td>540</td>
<td>540</td>
<td>-</td>
<td>1.6</td>
<td>80</td>
</tr>
<tr>
<td>C1</td>
<td>300</td>
<td>480</td>
<td>9.00</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>340</td>
<td>420</td>
<td>10.00</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>340</td>
<td>400</td>
<td>8.00</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>CU</td>
<td>300</td>
<td>320</td>
<td>-</td>
<td>1.6</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 1 Structure of the initial HEN for the example
It is of importance that the final solutions involve significantly smaller number of heat exchangers. This decrease effects in the reduction of investment expenses.

Table 2 gathers the values of surface areas of heat exchangers in both networks. Finally, important cost items are given in Table 3. It is of importance, that the both solutions feature utility cost very close to the global optimum.

5. Summary and conclusions

The approach has been developed for optimizing HEN of given structure. It is based on the use of genetic algorithms as optimization technique. The method allows optimizing HEN with both standard and non-standard heat exchangers. The goal function is total annual cost of HEN consisting of operational and investment expenses. The investment cost accounts for fixed charges. The approach allows optimizing both surface area of heat exchangers and split ratios. Elimination of a heat exchanger as well as a splitter is also possible. The method is more general than existing approaches applying deterministic optimization techniques.

Table 2. Optimal values of heat exchanger areas and split ratio

<table>
<thead>
<tr>
<th>Apparatus *</th>
<th>non-standard</th>
<th>standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>22.66</td>
<td>22</td>
</tr>
<tr>
<td>E-2</td>
<td>25.68</td>
<td>26</td>
</tr>
<tr>
<td>E-3</td>
<td>14.74</td>
<td>15</td>
</tr>
<tr>
<td>E-4</td>
<td>33.04</td>
<td>34</td>
</tr>
<tr>
<td>E-5</td>
<td>48.90</td>
<td>49</td>
</tr>
<tr>
<td>E-6</td>
<td>14.21</td>
<td>15</td>
</tr>
<tr>
<td>CU-2</td>
<td>9.53</td>
<td>9.53</td>
</tr>
<tr>
<td>Split</td>
<td>0.41</td>
<td>0.4</td>
</tr>
</tbody>
</table>

* surface area in m²
Table 3. Important cost parameters for solutions

<table>
<thead>
<tr>
<th></th>
<th>non-standard</th>
<th>standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total annual cost [$/year]</td>
<td>48 663</td>
<td>49 086</td>
</tr>
<tr>
<td>Energy cost [$/year]</td>
<td>8 872</td>
<td>9 112</td>
</tr>
<tr>
<td>Capital cost [$]</td>
<td>79 900</td>
<td>80 270</td>
</tr>
<tr>
<td>Capital cost of process exchangers [$]</td>
<td>74 314</td>
<td>74 680</td>
</tr>
</tbody>
</table>

Symbols

- $A$ – heat transfer area
- $a, b, c$ – cost parameters in equation (1)
- $Cost_{CU} / Cost_{HU}$ – unit cost of cooling / heating utility, respectively
- $CP$ – heat capacity flow rate
- $h$ – heat transfer coefficient
- $LMTD$ – logarithmic mean temperature differences
- $NA$ – total number of heat exchangers in HEN
- $NCU / NHU$ – number of coolers / heaters, respectively
- $Q_{j}^{CU} / Q_{i}^{HU}$ – heat load on cooler $j$ / heater $i$
- $ROR$ – rate of return
- $PL$ - plant life in years
- $t_{c} / t_{h}$ – temperature of cold / hot stream, respectively
- $T_{in}^{ex} / T_{out}^{ex}$ – inlet / outlet temperature in heat exchanger, respectively
- $T_{in} / T_{out}$ – inlet / outlet temperature in HEN, respectively
- $U$ – overall heat transfer coefficient
- $\Delta T_{min}$ – minimum temperature approach

References


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