An Efficient Global Event-Based Continuous-Time Formulation for the Short-Term Scheduling of Multipurpose Batch Plants

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Abstract

In the last decade, the PSE community has achieved remarkable results in relation to the problem of short-term scheduling of batch processes. Numerous optimization models, able to solve different problem variants, have been reported. Currently, the great majority of the formulations adopt a continuous-time approach given its advantages over other alternative representations. This contribution presents a simple continuous-time formulation based on the global event concept that relies on general sequencing and precedence constraints. Computational results show that the proposed formulation outperforms other continuous-time approaches without affecting the solution quality.

Keywords: short-term scheduling; multipurpose batch plants; continuous-time formulation; global events; general sequence and precedence constraints.

1. Introduction

Many models able to tackle the problem of short-term scheduling of multipurpose batch plants have been developed in the past few years. Recently, Méndez et al. [1] presented an exhaustive review of the state-of-the-art in this challenging area. Likewise, Shaik et al. [2] compared different ways of dealing with time in a continuous manner; by means of time slots, unit-specific and global event points. In this contribution, a global event-based continuous-time
formulation is introduced. Due to the fact that time slot and event point-based methods require solving a given problem many times (by gradually increasing the number of events until the solution does not present an improvement), it is essential to reduce both the number of iterations and the computational load required for reaching the optimal solution at each iteration. Consequently, the aim of the proposed approach is to provide a simple and efficient formulation that is easily understandable and applicable, as well as provides good quality solutions in reasonable CPU times.

2. Fundamental Concepts of the Proposed Approach

This new formulation, like [3], is based on global event points that represent the ending of tasks. It can be applied to either sequential or network processes and is able to take into account variable batch sizes and processing times, sequence-dependent changeovers and various storage policies, such as unlimited, finite (both in dedicated and shared units) and non-intermediate storage.

The key issues of the proposed model are the following: (i) a continuous-time representation is adopted; (ii) a predefined set of event points, common to all the units across the process, captures the endings of the task executions; (iii) variables that model task start and finish times are eliminated, only global event times are included; (iv) in order to consider changeover times efficiently slack variables are added to the model. These slack variables artificially force the end of processing tasks to not coincide with the model event points. This relaxation results in a decrease on the number of global events and allows an enhanced accommodation of the event grid; (v) general sequencing constraints, which apply on tasks being executed in the same unit, add flexibility to the resulting MILP models. These constraints are imposed during an extended period that may include various global event points; (vi) big-M constraints are not required; (vii) general precedence constraints ensure the fulfillment of material balances by forcing the precedence of tasks that produce a given material over those that consume it; (viii) renewable resources (utilities, manpower, etc.) can be taken into account since the model is based on global event points; (ix) a special state-task network structure is adopted. It is based on a pre-ordering of activities that results from an analysis of the STN, the existence of initial inventory, etc. This representation, which is referred as Ordered STN (OSTN), permits estimating a number of events’ lower bound. Moreover, it allows a meaningful reduction of the number of binary variables and constraints that are posed at each iteration.

3. Mathematical Formulation

3.1. Equipment Allocation Constraints

\[ \sum_{i \in I, j \in J} Y_{i,j,k} \leq 1, \quad \forall j \in J; k \in K \]  

(1)
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3.2. Relation Among Binary Variables

$$\sum_{i=1}^{K} Y_{i,j,k} \leq |J| X_{s}, \quad \forall i \in I; k \in K; k < |K|$$ (2)

3.3. Global Event Points Sequencing Constraints

$$T_{s} \geq T_{s,i}, \quad \forall k \in K; k > 1$$ (3)

3.4. Global Event Times’ Lower Bounds

$$T_{s} \geq \sum_{i=1}^{K} \left( a_{i,j} Y_{i,j,k} + b_{i,j} B_{i,j,k} \right) + S_{j,k}, \quad \forall j \in J; k \in K$$ (4)

$$S_{j,k} \leq H \left( \sum_{i=1}^{K} Y_{i,j,k} \right), \quad \forall j \in J; k \in K$$ (5)

3.5. General Sequencing Constraints

$$T_{s} \geq T_{s} + \sum_{i=1}^{K} \left[ a_{i,j} \left( Y_{i,j,k} + \sum_{i=1}^{K} Y_{i,j,k} \right) - 1 \right] + b_{i,j} \left( B_{i,j,k} - B_{i,j,k} \right) + S_{j,k} + S_{i,k}$$ (6)

$$\forall j \in J; k, k < k; k > k$$

3.6. General Sequencing Constraints for Sequence-Dependent Changeover Times

$$T_{s} \geq T_{s} + a_{i,j} \left( Y_{i,j,k} + \sum_{i=1}^{K} Y_{i,j,k} \right) - 1 + b_{i,j} \left( B_{i,j,k} - B_{i,j,k} \right) + S_{j,k} - S_{i,k} + \sum_{i=1}^{K} \sigma_{i,j,k} Y_{i,j,k} - \sigma_{i,j,k} \left( 1 - Y_{i,j,k} \right), \quad \forall k, k < k; k > k; j \in J; i \in I; i \neq I$$ (7)

3.7. Batch Size Constraints

$$B_{i,j,k} \leq B_{i,j,k} Y_{i,j,k}, \quad \forall k \in K; i \neq I; j \in J$$ (8)

$$B_{i,j,k} \geq B_{i,j,k} Y_{i,j,k}, \quad \forall k \in K; i \neq I; j \in J^{min}$$ (9)

3.8. General Precedence Constraints

$$T_{s} - T_{s} \geq a_{i,j} \left( Y_{i,j,k} + X_{i,j,k} \right) - 1 + b_{i,j} \left( B_{i,j,k} - B_{i,j,k} \right) + S_{i,k}, \quad \forall k, k < k; i \neq I; j \in J$$ (10)
3.9. Material Balance Constraints

\[ \text{Inv}_{n,k} = \text{Inv}_{n}^{a} - \sum_{i} \sum_{j} C_{s, i} B_{i, j, k} + \sum_{i} \sum_{j} P_{d, i} B_{i, j, k} - \sum_{i} \sum_{j} C_{s, i} B_{i, j, k+1}, \quad \forall m \not\in M^{*}; k \in K; k = 1 \]

\[ \text{Inv}_{n,k} = \text{Inv}_{n,k-1} + \sum_{i} \sum_{j} P_{d, i} B_{i, j, k} - \sum_{i} \sum_{j} C_{s, i} B_{i, j, k+1}, \quad \forall m \not\in M^{*}; k \in K; 1 < k < |K| \]

\[ \text{Inv}_{n,k} = \text{Inv}_{n,k-1} + \sum_{i} \sum_{j} P_{d, i} B_{i, j, k}, \quad \forall m \not\in M^{*}; k \in K; k = |K| \]

3.10. Initial Consumption Constraints

\[ \sum_{i} \sum_{j} C_{s, i} B_{i, j, k} \leq \text{Inv}_{n}^{a}, \quad \forall m \not\in M^{*}; k \in K; k = 1 \]

3.11. Storage Constraints

\[ \text{Inv}_{n,k} \leq S_{c, m}, \quad \forall m \not\in M^{ad}; k \in K \]

3.12. Storage Constraints for Shared Tanks

\[ \text{Inv}_{n,k} \leq S_{c, m} W_{n,k}, \quad \forall m \not\in M^{**}; k \in K \]

\[ \sum_{m} W_{n,k} \leq 1, \quad \forall t \in T; k \in K \]

3.13. Demand Satisfaction Constraints

\[ \text{Inv}_{n,k} \geq D_{n}, \quad \forall m \not\in M^{*}; k \in K; k = |K| \]

3.14. Total Operating Time Constraint

\[ T_{k} \leq H, \quad k \in K; k = |K| \]

\[ T_{k} \leq M_{k}, \quad k \in K; k = |K| \]

3.15. Total Profit Calculation

\[ TP = \sum_{m} MP_{m} \text{Inv}_{n,k} + \sum_{m} MP_{m} (\text{Inv}_{n,k} - \text{Inv}_{n}^{a}) - \sum_{m} MP_{m} (\text{Inv}_{n}^{a} - \text{Inv}_{n,k}), \quad k \in K; k = |K| \]
4. Results and Discussion

A benchmark case study (Example 3 of Janak et al. [4]) is dealt with to illustrate the efficiency of the new model. This example involves sequence-dependent changeover times, shared storage tanks as well as variable batch sizes and processing times. Relevant data and comparative results with other formulations can be found in [3]. The associated OSTN representations are shown in Fig. 1. The initial pre-ordering shows that tasks T12 and T22 are unable to end at the first time point since no preliminary inventory of intermediate materials is available. Similarly, tasks T13 and T23 cannot end before the finishing of tasks T12 and T22, respectively (i.e., before time point number 3). In consequence, at least three global time points are necessary. In turn, the final sequence is depicting the latest allowed task completions. For example, it is not convenient to finish tasks T11 and T12 after the \(|K|-2\) time point, since the material produced by them would not be used before the end of the scheduling horizon.

![Figure 1. OSTN representation for the chosen example](image)

Due to lack of space only one problem instance, that pursues a maximum-profit objective, is reported in this paper. The GAMS/CPLEX 10.0 solver was used to implement the proposed MILP model on a Pentium IV (3.0 GHz) PC with 2 GB of RAM. Computational results are presented in Table 1. As seen, they exhibit a very good performance.

<table>
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<th>K</th>
<th>CPU time (s)</th>
<th>Nodes</th>
<th>RMILP (10^3 $)</th>
<th>MILP (10^3 $)</th>
<th>Binary variables</th>
<th>Continuous variables</th>
<th>Constraints</th>
<th>Nonzeros</th>
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<td>8.000</td>
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<td>94</td>
<td>452</td>
<td>2110</td>
</tr>
</tbody>
</table>

5. Conclusions

A new continuous-time formulation to tackle the short-term scheduling problem of multipurpose batch plants has been presented. Despite its simplicity, it can address various problem complexities quite efficiently. The proposal was tested by means of several examples. In all cases, a small number of variables and constraints were generated and problems were solved in a low CPU time.
Nomenclature

(a) Sets/Indices

- $K/k, kk$ = global event points
- $I/i, ii$ = tasks
- $J/j = units$
- $I_i = tasks$ that produce a material required by task $i$
- $I_{mI} = tasks$ that consume material $m$
- $M/m = materials$
- $T/t = shared$ tanks
- $I_i = tasks$ that unit $j$ can perform
- $J_i = units$ that can perform task $i$
- $J_i^{min}$ = units on which the minimum batch size condition applies for task $i$
- $M/m = materials$
- $M/m^* = sold materials$
- $M/m^* = purchased$ materials
- $M/m^* = intermediate$ materials with economic value
- $M/m^* = materials$ with “as-required” availability
- $M/m^* = materials$ stored in dedicated units with maximum capacity
- $M/m^* = materials$ stored in shared units with maximum capacity
- $M_i = materials$ that can be stored in shared tank $t$

(b) Parameters

- $H = time$ horizon
- $a_{ij} = fixed$ duration of task $i$ in unit $j$
- $b_{ij} = variable$ duration of task $i$ in unit $j$
- $\sigma_{i,j,k} = sequence$ dependent changeover time
- $\sigma_{max} = maximum$ changeover time
- $B_{max} = maximum$ batch size of $i$ in unit $j$
- $B_{min} = minimum$ batch size of $i$ in unit $j$
- $D_m = fixed$ demand of material $m$
- $MP_m = price (value)$ of material $m$
- $max_{B_i} = maximum$ batch size
- $SC_m = maximum$ storage capacity for material $m$

(c) Variables

- $Y_{i,j,k} = 1$ if task $i$ finishes in unit $j$ at $T_k$
- $X_{i,k} = 1$ if a predecessor of task $i$ is finished at $T_k$
- $W_{m,k} = 1$ if material $m$ is stored at $T_k$
- $T_k = time$ corresponding to event point $k$
- $S_{j,k} = slack$ time for unit $j$ at $T_k$
- $B_{i,k} = batch$ size of task $i$ finishing at $T_k$ in unit $j$
- $Inv_{m,k} = amount$ of material $m$ at $T_k$
- $TP = total$ profit
- $M_k = makespan$

Acknowledgements. This work has been supported by CONICET, UNL and ANPCyT (PICTs 12628 & 14717).

References