Nonlinear Predictive Control of a pH Process

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Abstract

In this paper, a new control method based on a nonlinear predictive algorithm is developed for a pH neutralization process in order to control the plant to the desired setpoint with high-quality performances over the entire operation range. For testing the control structure, the process simulator together with the control algorithm were implemented in Matlab and simulation results are given.

Keywords: nonlinear predictive control, Wiener model, pH control

1. Introduction

The pH process is widely used in various areas such as the neutralization of industrial wastewater, biochemical and electrochemical processes, the paper and pulp industry, maintenance of the desired pH level at various chemical reactions, production of pharmaceuticals and biological processes, coagulation and precipitation processes and many other areas.

The control of pH is one of the most difficult challenges in the process industry because it shows a strong nonlinear behavior due to the nonlinear characteristics resulted from the feed components or total ion concentrations. The main dynamics of such a process are determined by predictable variations due to the effect of the nonlinearities in the control loop and are most often handled by using an adaptive control approach.

Various control techniques used in controlling the pH processes are reported in the literature in the last years. Thus, Sung and Lee in [1] proposed
an adaptive nonlinear PI controller, which uses the titration curve updated by on-line recursive least-squares method to control the pH process. Radhakrishnan and Wah in [2] presented the development of a combined static-dynamic hybrid model for the characterization of pH control processes. Lazar et al. in [3] designed a neuro-predictive control method that makes use of the neural model of the process in order to predict the systems behavior over a certain horizon and Kumar et al. developed in [4] a nonlinear PI control approach.

In this paper, a new solution that makes use of the nonlinear Wiener model of the process and a NEPSAC (Nonlinear Extended Predictive Self Adaptive Control) controller [5], is presented. In order to avoid a time consuming adaptive control approach, it is employed the nonlinear predictive controller NEPSAC which incorporates the nonlinear model of the plant making hence possible to take into account the predictable variations of the process dynamics and to obtain high-quality performances over the entire operation range. For testing the control structure, the process simulator together with the control algorithm were implemented in Matlab and simulation results are given.

2. Nonlinear predictive control approach

The nonlinear predictive control approach developed for pH processes is based on the nonlinear model of the process which is used to predict the future behavior of the pH plant over a horizon by means of NEPSAC method.

2.1. pH process model

Being a typical Wiener type process, the mass equilibrium of the pH process is described by approximately linear differential equations while the equilibrium equation (titration curve) is a strongly nonlinear static function. The control structure of the pH process is given in Fig. 1.

Fig. 1 Control structure for the pH process
Nonlinear Predictive Control of a pH Process

The weak acid (acetic acid - CH₃COOH) is treated with the strong base (sodium hydroxyl - NaOH) in a continuous stirred reactor. The mass balance and equilibrium equations given in [1,2] are the following:

\[
v \frac{dC_a(t)}{dt} = FC_{a0}(t) - (F + u(t - 5))C_a(t) \tag{1}
\]

\[
v \frac{dC_b(t)}{dt} = u(t - 5)C_{b0}(t) - (F + u(t - 5))C_b(t) \tag{2}
\]

\[
[H^+] + C_b = \frac{K_w}{[H^+]} + \frac{K_a C_a}{K_a + [H^+]} \tag{3}
\]

\[
\text{pH} = -\log([H^+]) \tag{4}
\]

where \(C_{a0}\) and \(C_a\), respectively \(C_{b0}\) and \(C_b\), are the ionic concentrations in the input and output acid, respectively base, flows. \(K_w\) and \(K_a\) denote the dissociation constants of the water and acetic acid. \(F\) and \(V\) are the input flow and the reactor volume, while \(H^+\) hydrogen ion concentration in the mixture. The dead time corresponds to the necessary duration for the transport in the mass equilibrium equations.

The chemical reactions in the reactor are described in relation with the nonlinear static function given by (3) and (4).

2.2. NEPSAC controller

The NEPSAC controller [5] is based on the future response, considered as being the cumulative results of two effects:

\[
y(t + k | t) = y_{\text{base}}(t + k | t) + y_{\text{optimize}}(t + k | t) \tag{5}
\]

in which the 2-nd term can optimally be made equal to zero in an iterative way for nonlinear systems. This results in the optimal solution, also for nonlinear systems, because the superposition principle is no longer involved.

The two contributions have the following origins:

- \(y_{\text{base}}(t + k | t)\): effects of past control \{\(u(t - 1), u(t - 2), \ldots\)\}, of basic future control scenario, called \(u_{\text{base}}(t + k | t), k \geq 0\), which is appropriately selected in an iterative way at the same sample instant and of future disturbances \(n(t + k | t)\);

- \(y_{\text{optimize}}(t + k | t)\): effect of the optimizing future control actions \{\(\delta u(t | t), \delta u(t + 1 | t), \ldots, \delta u(t + N_u - 1 | t)\)\} with:

\[
\{\delta u(t + k | t) = u(t + k | t) - u_{\text{base}}(t + k | t)\} \tag{6}
\]
and \( N_u \) being control horizon.

\[
y_{\text{optimize}}(t + k \mid t) = h_k \delta u(t \mid t) + h_{k-1} \delta u(t + 1 \mid t) + \ldots + g_{k-N_u+1} \delta u(t + N_u - 1 \mid t).
\]

Equation (7)

In equation (7), the parameters \( h \) are the coefficients of the unit impulse response and \( g \) of the unit step response of the system.

Using the matrix notation, the NEPSAC equation of the predictor is:

\[
\begin{align*}
\mathbf{Y} &= \mathbf{Y} + \mathbf{GU} \\
\mathbf{Y} &= \begin{bmatrix} y(t + N_1 \mid t), y(t + N_1 + 1 \mid t), \ldots, y(t + N_2 \mid t) \end{bmatrix}^T \\
\mathbf{Y} &= \begin{bmatrix} y_{\text{base}}(t + N_1 \mid t), y_{\text{base}}(t + N_1 + 1 \mid t), \ldots, y_{\text{base}}(t + N_2 \mid t) \end{bmatrix}^T \\
\mathbf{G} &= \begin{bmatrix} h_{N_1} & h_{N_1-1} & \ldots & h_{N_1-N_u+1} \\
           & h_{N_1+1} & h_{N_1} & \ldots & h_{N_1-N_u+2} \\
           & & h_{N_2} & h_{N_2-1} & \ldots & h_{N_2-N_u+1} \\
\end{bmatrix} \\
\mathbf{U} &= \begin{bmatrix} \delta u(t \mid t), \delta u(t + 1 \mid t), \ldots, \delta u(t + N_u - 1 \mid t) \end{bmatrix}^T
\end{align*}
\]

\( N_1 \) is the minimum prediction horizon and \( N_2 \) the prediction horizon. Using the matrix relationship between the control actions \( \Delta u \) and \( \delta u \) from [5]:

\[
\begin{bmatrix} \Delta u(t \mid t) \\ \Delta u(t + 1 \mid t) \\ \Delta u(t + N_u - 1 \mid t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \delta u(t \mid t) \\ \delta u(t + 1 \mid t) \\ \delta u(t + N_u - 1 \mid t) \end{bmatrix} + \mathbf{b}
\]

with \( \mathbf{A} \) and \( \mathbf{b} \) given by:

\[
\mathbf{A} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ -1 & 1 & \ldots & 0 \\ 0 & 0 & \ldots & -1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} u_{\text{base}}(t \mid t) - u(t-1) \\ u_{\text{base}}(t + 1 \mid t) - u_{\text{base}}(t \mid t) \\ u_{\text{base}}(t + N_u - 1 \mid t) - u_{\text{base}}(t + N_u - 2 \mid t) \end{bmatrix}
\]

the following quadratic cost function in \( \mathbf{U} \) is obtained:

\[
J = \left[ \mathbf{R} - \mathbf{Y} - \mathbf{GU} \right]^T \left[ \mathbf{R} - \mathbf{Y} - \mathbf{GU} \right] + \lambda (\mathbf{AU} + \mathbf{b})^T (\mathbf{AU} + \mathbf{b}),
\]

where \( \mathbf{R} \) is the reference vector and \( \lambda \) a weighting factor. After minimizing the cost function results the solution:
Nonlinear Predictive Control of a pH Process

\[ U^* = \left[ G^T G + \lambda A^T A \right]^{-1} \left[ G^T (R - \bar{Y}) - \lambda A^T b \right] \]  (13)

and the control action applied to the process is:

\[ u(t) = u_{\text{base}}(t \mid t) + \delta u(t \mid t) = u_{\text{base}}(t \mid t) + U^*(1) \]  (14)

The aim of NEPSAC control is to find in an iterative way a control policy \( u_{\text{base}}(t + k \mid t) \), which is as close as possible to the optimal strategy and thus bringing the optimizing control action \( \delta u(t + k \mid t) \) and the term \( y_{\text{optimize}}(t + k \mid t) \) practically to zero [5].

3. Simulation results

The process simulator, based on plant model (1)-(4), together with the control algorithm NEPSAC presented in Section 2.2 are implemented in Matlab, making use of Simulink capabilities for the real plant representation. The implementation considers also modeling errors in order to assume a close similarity with a real process control. In Fig. 2 the control system response due to stepwise changes in the reference over the entire pH range is represented.

Fig. 2 Control system response: (a) reference and controlled output; (b) control signal
In order to incorporate the dead time, the minimum prediction horizon was considered equal to this delay (in number of samples). The process model parameters from [2] and the initial conditions equal to zero for the output concentrations are used to simulate the setpoint tracking of the process output on the entire possible range of pH. For computing the optimal value of the controller output the prediction horizon over thirty samples in future is considered.

The desired reference trajectory is not known a priori, thus making the response of the process to act after the set point change. The presence of the strong nonlinearity and also of the time delay can be observed by looking at the settling time for each pH level and the control signal evolution. Despite the nonlinearity problem, the setpoint tracking is almost perfect and the absence of overshoot behavior shows the precision of the nonlinear predictive controller.

4. Conclusions

In this paper, a new control method based on NEPSAC controller is developed for a pH neutralization process in order to control the plant to the desired setpoint with high-quality performances over the entire operation range. The predictive controller uses the nonlinear model of the process in order to predict the future behavior of the pH plant over a determined horizon. The algorithm considers the minimization of the quadratic form of a cost function, based on future errors and command increment limitation, in order to obtain the optimal control action. The adaptive control principle is replaced by using a model based predictive nonlinear algorithm that is capable to capture the predictable dynamics variations of the process. The nonlinear model based predictive control method permits on-line adaptation of the controller parameters without any expert supervision. The additional benefit of keeping the performance over the entire operating range is also substantial as verified in the experimental results.

References