Multiobjective Constrained MPC with Simultaneous Closed Loop Identification for MIMO Processes

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Abstract
In most Model Predictive Control (MPC) configurations, the objective function is single and incorporates the different items that need to be minimized, by multiplying them with appropriate weights. In this paper an alternative multiobjective MPC approach is proposed. The method is based on the prioritization of the different control objectives, which reduces considerably the number of tuning parameters and guarantees that the solution meets at least the most desirable targets. The proposed methodology can be used for simultaneous closed loop process control and model identification, by considering the persistent excitation of the manipulated variables as an additional top priority objective. The method is illustrated through its application to a nonlinear, non-isothermal, continuous tank reactor. Simulation results illustrate the superiority of the method over traditional single-objective MPC approaches.

Keywords: Model Predictive Control, Multiobjective Optimization, Adaptive Control, MIMO systems, Closed Loop Identification

1. Introduction

Model Predictive Control (MPC) has become a popular control method with many successful industrial applications over the recent years since it can be used to control Multi Input Multi Output (MIMO) systems and has the ability to handle modeling errors and process constraints. The philosophy behind the various different formulations of MPC is the direct use of a process model to calculate the control moves by solving online an optimization problem that minimizes both the deviation from the desired set points and the control energy over some future horizons. A number of review papers (Morari and Lee, 1999; Qin and Badgwell, 2003) as well as recent books (Camacho and Bordons, 1999) present the various MPC configurations and their applications.

In most conventional MPC methodologies, the control goals are multiplied by appropriate weights and formulate a single objective function that is minimized. The weight selection is in essence the tuning strategy for the controller, since it affects to a great extent the closed loop performance. An alternative approach that can be used for meeting simultaneously many control targets is to describe mathematically each target as a different objective and formulate a multiobjective optimization control problem. Such a multiobjective MPC configuration is proposed in this paper. The formulation of

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the problem is based on the prioritization of the targets, so that the most important objectives are satisfied first. In the case of time varying systems, the idea is further extended to include the simultaneous closed-loop adaptation of the model into the MPC framework. This is achieved by considering the persistent excitation of the manipulated variables (Genceli and Nikolaou, 1996; Åström and Wittenmark, 1995) as an additional top priority control objective.

The proposed methodology is illustrated through the application to a nonlinear, time varying non-isothermal Continuous Stirred Tank Reactor (CSTR), where the process model is linear. The results show that using the proposed multiobjective configuration, we can obtain more successful closed-loop responses with considerably less tuning effort, compared to the standard MPC formulation with a single objective function. The methodology can be easily extended to nonlinear systems, where the basic difference is the formulation of a nonlinear multiobjective optimization problem that needs more advanced optimization techniques and more computational time to get solved.

2. Theoretical Aspects

2.1 Prioritized multiobjective optimization

A multiobjective optimization problem contains a number of objective functions and has no unique solution. In fact, there exists a set of solutions, called the Pareto Optimal set. More precisely, if a vector of \( m \geq 2 \) objective functions \( f_i : \mathbb{R}^n \rightarrow \mathbb{R} \) to be minimized is:

\[
\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T, \quad \mathbf{x} \in S \subseteq \mathbb{R}^n
\]

then the vector \( \mathbf{x}^* \) is a Pareto optimal if and only if an objective \( f_i \) can be reduced only at the expense of increasing at least one different objective (Miettinen, 1999).

There are several ways to obtain Pareto optimal solutions, such as the weighting method and the lexicographic ordering. The weighting method suggests that all objectives are combined with appropriate weights in one single objective and the produced result is proved to be Pareto optimal (Miettinen, 1999). A totally different approach is followed by the lexicographic ordering procedure, where the objectives are ordered to formulate a hierarchy. The first objective in this hierarchy is the most important one, while the last function is the one with the least priority. Then, following the hierarchy, each objective is minimized separately with the addition of proper constraints, which guarantee that the previous in rank objectives preserve their optimal values. It has been proved that if this solution exists, it is Pareto optimal and unique (Miettinen, 1999). Most MPC configurations formulate a single objective function, by adopting the aforementioned weighting method. Surprisingly, only few publications consider the lexicographic ordering procedure (Kerrigan and Maciejowski, 2002).

2.2 Model predictive control and identification

An important issue in MPC is its performance when applied to a time varying process, since the available dynamic model of the system gets weaker over time. In such cases, continuous adaptation is necessary in order to preserve the accuracy of the model. A technique that has been proposed for closed-loop identification is the persistent excitation of the manipulated variables, so that at each time step sufficient information
is collected for adapting the model parameters. In a typical MPC configuration where
the weighting method is used for handling the different objectives, the persistent
excitation condition is added as a constraint. The mathematical formulation of such an
optimization problem that includes the persistent excitation constraint is described by
the following set of equations:

\[
\min_{\{u(k), \ldots, u(k+\cdots-N-1)\}} \sum_{i=1}^{N} \left[ \|W(y(k+i|k) - y^m)\|^2 + \|R\Delta u(k+i-1|k)\|^2 \right]
\]  

(1)

\[u_{\text{min}} \leq u(k+i-1|k) \leq u_{\text{max}}, \quad i = 1, \ldots, c + N\]  

(2)

\[-\Delta u_{\text{max}} \leq \Delta u(k+i-1|k) \leq \Delta u_{\text{max}}, \quad i = 1, \ldots, c + N\]  

(3)

\[u(k+c+i-1|k) = u(k+i-1|k), \quad i = 1, \ldots, N\]  

(4)

\[
\sum_{m=0}^{i-1} \lambda^m \varphi(k+i-1-m|k) \varphi(k+i-1-m|k)^T \geq \rho \cdot \mathbf{I} > 0, \text{ for } i = 1, \ldots, c + N \text{ and } \rho > 0
\]  

(5)

\[\varphi(k+i-m)^T = \left[ u(k+i-m|k), u(k+i-m-1|k), \ldots, u(k+i-m-N+1|k) \right]\]  

(6)

In the above equations, \(c\) is the control horizon, \(u\) is the vector of manipulated variables, 
\(y\) is the vector of control variables, \(W\) and \(R\) are weight matrices, \(\varphi\) is a regression vector whose entries are process inputs, \(\lambda\) is the forgetting factor, \(\rho\) is the desirable value in the persistent excitation constraint, \(N\) is the number of pulse response coefficients in the linear process model and \(l\) is the number of past examples that are used for adapting the model parameters. In the pulse response model:

\[\hat{y}(k+i|k) = \sum_{j=1}^{N} G_j(k)u(k+i-j|k) + \mathbf{d}(k|k)\]  

(7)

\(G_j(k)\) are the time varying unit pulse response model coefficients and \(\mathbf{d}(k|k)\) is the estimated disturbance at time point \(k\). The optimization problem, solved on line, minimizes the differences between the predicted outputs and the desired set points and the control effort over the prediction horizon. Eq. (4) shows that the future inputs follow a periodic sequence in contrast to conventional MPC, where the control moves are forced to remain zero after the end of the control horizon. In the above formulation, the hard persistent excitation constraint (5) can be transformed into a soft constraint by including it in the objective function, but this introduces an extra tuning effort. The adaptation of the model parameters \([G_1, G_2, \ldots, G_N]\) is performed at each time step by applying the least squares with exponentially forgetting method on the past \(l\) input-output examples.

3. The Proposed Multiobjective MPC Configuration

The theoretical aspects given above are combined in order to develop a multiobjective
MPC configuration, which will be able to simultaneously identify the changes in the
process dynamics and achieve a good control performance. As identification is considered as the most important objective, it is placed in the top position in the hierarchy and then each output of the process is optimized separately. Thus, the persistent excitation constraint (5) is transformed to an optimization problem, which is formulated as follows:

$$\min_{u(k|\ldots,u(k+c+N-1|k))} \mu$$ \hspace{1cm} (8)

subject to:

$$\sum_{n=0}^{k} \lambda^n \varphi(k+i-1-m|k) \varphi(k+i-1-m|k)^T \geq (\rho - \mu) \cdot \mathbf{I} > 0, \text{ for } i=1,\ldots,c+N \text{ and } \rho > 0$$ \hspace{1cm} (9)

and the constraints (2), (3). The parameter $\mu$ in Eq. (8) must be nonnegative. This configuration is always feasible, since when $\mu = \rho$, Eq. (9) is guaranteed to get satisfied. This is a great advantage compared to the MPC configuration of section 2, where due to the persistent excitation hard constraint (5), infeasibility problems may occur.

The optimized value $\mu^*$ of the parameter $\mu$ is used as a constraint in the subsequent optimization problems that are formulated. Assuming that the system is square (same number $nv$ of manipulated and controlled variables) we can use our knowledge of the process and/or Relative Gain Array (RGA) to assign a different manipulated variable to each controlled variable, so that pairs $(u_j,y_j)$, $j=1,\ldots,nv$ are constructed. Then the solution of the multiobjective optimization problem proceeds in the following way:

FOR $j=1$ to $nv$

Solve the following optimization problem:

$$\min_{u(k|\ldots,u(k+c+N-1|k))} \sum_{i=1}^{c+N} \left[ w_j \cdot (\hat{y}_j(k+i|k) - y_j^*)^2 + \| R_j \Delta u(k+i-1|k) \|^2 \right]$$ \hspace{1cm} (10)

subject to Eqs. (2) and (3), the constraints on the output responses which are consecutively added to the constraint set and the persistent excitation equation:

$$\sum_{n=0}^{k} \lambda^n \varphi(k+i-1-m|k) \varphi(k+i-1-m|k)^T \geq (\rho - \mu^*) \cdot \mathbf{I} > 0, \text{ for } i=1,\ldots,c+N \hspace{1cm} (11)$$

Add the following constraint to the set of constraints:

$$\sum_{i=1}^{c+N} (\hat{y}_j(k+i|k) - y_j^*)^2 \leq \sum_{i=1}^{c+N} (\hat{y}_j^*(k+i|k) - y_j^*)^2$$ \hspace{1cm} (12)

where $\hat{y}_j^*(k+i|k)$ are the optimal response values.

END
Remark 1: Constraint (12) assures that once an optimal response profile has been calculated for a controlled variable, only improvements are allowed in subsequent iterations.

Remark 2: The above procedure can be easily extended to non-square systems by assigning more than one manipulated variables to some of the controlled variables.

Remark 3: In case of large systems, the variables can be partitioned in groups containing more than one inputs and outputs in order to reduce the number of objective functions that need to be minimized.

Remark 4: One important advantage of the proposed method is the considerable reduction of the tuning effort, since this approach eliminates the trial and error procedure for selecting an appropriate weight matrix $W$.

Remark 5: In the move suppression matrix $R$, small values are assigned for the inputs $1,2,\ldots,j$ and large values for the inputs $j+1,j+2,\ldots,nv$, so that energy is preserved to control the next outputs in the hierarchy.

4. Results

The methodology described above is applied to a control problem concerning a non isothermal continuous tank reactor with two inputs and two outputs. The differential equations that describe the process are (Kazantzis and Kravaris, 2000):

\[
\frac{dC_A}{dt} = \frac{F}{V}(C_{A,in} - C_A) - 2k_o \exp\left(-\frac{E}{RT}\right) \cdot C_A^2
\]

\[
\frac{dT}{dt} = \frac{F}{V}(T_{in} - T) + 2 \frac{(-\Delta H)}{\rho \cdot c_p} \cdot k_o \exp\left(-\frac{E}{RT}\right) \cdot C_A^2 - \frac{U \cdot A}{V \cdot \rho \cdot c_p} (T - T_j)
\]

where $V$ the volume of the reactor, $U$ is the heat transfer coefficient, $A$ is the surface of the exchanger, $k_o (\cdot \Delta H) \rho \cdot c_p$, $E/R$ are constants of the reaction and the reactants. The inlet temperature $T_{in}$, the temperature of the coolant $T_j$, the inlet concentration $C_{A,in}$ and the flow rate $F$ are the inputs of the system, while the concentration of the reactant $A$ inside the reactor $C_A$ and the temperature inside the reactor $T$ are the outputs. In our simulations we used $F$ and $T_j$ as the manipulated variables and the two remaining inputs were considered as disturbances. The steady state around which the simulations were performed and the values of the model parameters can be found in Kazantzis and Kravaris (2000). The process is assumed to be time varying due a dynamic modification of the heat transfer coefficient: $U = 20000 \cdot \exp(-0.0001 \cdot t)$. The multiobjective MPC configuration described previously is applied based on a pulse response model consisting of 20 past values of each manipulated variables: First the persistent excitation optimization problem is solved and gives an optimal value $\mu^*$ for the parameter $\mu$. Then the concentration of the reactant at the outlet of the reactor is controlled by using the flow rate as the main manipulated variable (a large move suppression coefficient is assigned to the coolant temperature). Finally the temperature of the reaction mixture is optimized, using both the manipulated variables, provided that both the persistent excitation optimal value $\mu^*$ and the optimal concentration profile are preserved.
A case study is presented in this paper involving two set point changes in both control variables. More precisely, the new set points for the concentration and the temperature are 0.07 mol/l and 370K respectively, while after some time the process is forced to return to the original set points (0.07545 mol/l and 376.3K). The responses produced by the proposed methodology are given in Fig. 1. In the same figure the results produced by the conventional Dynamic Matrix Control (DMC) approach are depicted, where in the formulation of the $W$ matrix, larger values have been assigned to the parameters that multiply the deviations of the predicted $\hat{C}_i$ values from the desired set point. Comparison results are clearly in favor of the proposed methodology.

5. Conclusions

In this work an alternative MPC approach for controlling a MIMO process is proposed, by formulating and solving on-line a multiobjective optimization problem. The approach is particularly useful when the control variables are of different importance. In this way, the most important output is controlled first, thus simplifying considerably the tuning effort. Furthermore, it can be used for simultaneous closed loop identification by considering an additional persistent excitation top priority objective. The idea can be easily extended to nonlinear MPC, which can further improve the performance of the controller.

References

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