Modelling and Optimisation of Milk Pasteurisation Processes

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Abstract

This work aims at understanding the optimal operation of milk pasteurization processes. The operating procedure consists of milk heating, followed by heat-exchanger cleaning. The duration of heating and cleaning stages are the decision variables. A dimensionless dynamic model of a continuous process is presented. The objective function incorporates the value added by pasteurization and the cost of cleaning. The optimal operating policy consists of long heating and short cleaning steps. The heating time, however, cannot exceed a limiting value. Otherwise, the required milk outlet temperature cannot be achieved, or the pressure drop becomes excessive. Additionally, a minimum cleaning time is required to avoid continuous increase of deposit thickness.

Keywords: milk pasteurization, fouling, modelling, optimization

1. Introduction

Milk pasteurization (Fig. 1) aims to destroy micro-organisms by heat, so that the milk is safe for consumption. In a typical setting, milk is pumped from a feed tank to a shell-and-tube or plate heat exchanger, where it reaches the required temperature. Steam or hot oil is used as heating medium. The hot milk enters an insulated holding tube, where it is kept for a certain time. The product is then cooled, prior to storage and packaging. During pasteurization, heat-exchanger fouling occurs. Fouling lowers the efficiency of heat transfer, causes an increase of pressure drop, limits the operational time of the plant and endangers process hygiene. Therefore, cleaning is required on a daily basis. Because a significant fraction of time is spent on cleaning, optimal operation of the pasteurization equipment is of considerable importance to the milk industry.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Typical milk pasteurization unit}
\end{figure}
Despite being the subject of many papers (de Jong, 1992, Belmar-Beiny et al., 1993, Paterson & Fryer, 1988, Toyoda & Fryer, 1997), understanding of heat-exchanger fouling during pasteurization is still incomplete. The model of Georgiadis et al. (1998a) attempts to quantify the effect of various process parameters. The model considers two zones on the milk-side, the bulk and the thermal layer near tube’s wall, where momentum, heat and mass transfer take place. The model fits well the experimental data of Belmar-Beiny et al. (1993). However, in our opinion, not all details included contribute to the predictive power of the model. Moreover, the physical significance of some parameters, for example bulk-to-layer mass- and heat-transfer coefficients, is questionable. In this work we present a new model for heat-exchanger fouling during milk pasteurization. Table 1 summarizes the differences with respect to Georgiadis et al. (1998a). Our model is simpler and easier to solve, and still able to describe correctly the experimental data.

Table 1. Comparison of models

<table>
<thead>
<tr>
<th>Property</th>
<th>Georgiadis et al. (1998a)</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bulk</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>Radial velocity profile</td>
<td>Plug flow</td>
</tr>
<tr>
<td>Mass</td>
<td>Axial and radial diffusion</td>
<td>No diffusion</td>
</tr>
<tr>
<td><strong>Thermal Layer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>Flow in the axial direction.</td>
<td>Flow is neglected</td>
</tr>
<tr>
<td>Heat</td>
<td>Uniform temperature.</td>
<td>Temperature gradient</td>
</tr>
<tr>
<td>Mass</td>
<td>Uniform concentration. Mass transfer between bulk and thermal layer</td>
<td>Radial diffusion.</td>
</tr>
</tbody>
</table>

Simultaneous optimization of the design and operation of heat exchangers under milk fouling was considered by Georgiadis et al. (1998b), based on the mathematical model cited above. The study showed that large heating times are favoured, because the cost of production interruption is dominant, while the increase of energy consumption due to fouling is not very important. The present study confirms these findings and provides additional insights. If the heating time exceeds a value \( t_{\text{h, max}} \), the fouling is so severe that the desired milk outlet temperature cannot be achieved with the available steam. In addition, for a fixed heating time, the cleaning time must exceed a critical value, \( t_{c} > t_{c}^{*}(t_{h}) \). If this condition is fulfilled, the process, viewed as a discrete time dynamical system, is stable and reaches a stationary state. Otherwise, the deposit thickness increases continuously from one heating-cleaning cycle to another. The optimal operating policy is located at the intersection of the \( t_{h, \text{max}} \) and \( t_{c}^{*} \) constraints.

2. The objective function

The objective function used in this study represents the profit per time unit achieved during one heating-cleaning cycle.

\[
Z = \frac{(\text{MPF} \cdot t_{h}) - (\text{CC} \cdot t_{c}) - (\text{CC}_{\text{aux}} \cdot t_{\text{aux}})}{t_{h} + t_{c} + t_{\text{aux}}} \tag{1}
\]
In Eq. 1, $MPF$ is the value added by pasteurization, after subtracting the cost of heating. $CC$ and $CC_{aux}$ are the costs of cleaning and auxiliary operations, including materials and their disposal. $t_h$, $t_c$ and $t_{max}$ are the extents of heating, cleaning and auxiliary steps. It is evident that the objective function reaches the maximum value $Z_{max} = MPF$ for $t_h \to \infty$ and $t_c \to 0$. This would imply that the optimal way to operate the pasteurization process is to continuously heat the milk, without any interruption for cleaning. This is not possible, because cleaning is definitely required. Therefore, the optimization problem should take into account the operational constraints. In the following, we will use a dynamic model of the process to derive the values of the maximum allowed heating time, $t_{h,max}$ and the minimum required cleaning time, $t_{c}^\ast$.

3. The process model

3.1 Heating
The model (Fig. 2) assumes two regions of fluid where fouling takes place: bulk and thermal layer; fouling occurs due to protein reactions (Patterson & Fryer, 1988): native proteins unfold, $N \to U$ and aggregate $U \to A$, with rates provided by De Jong (1992); the rate at which aggregate proteins adhere to the wall is first-order, but independent of temperature; all heats of reaction are neglected; the velocity, temperature and concentrations are uniform in a cross-section of the bulk region; there is no axial flow within the thermal layer due to negligible thickness, compared to the momentum layer; partial heat transfer coefficients are constant; in the thermal layer, gradients of concentration and temperature exist.

![Fig.2 Fouling model](image)

The variables of the model are the time $t$, bulk and layer coordinates $0 \leq \xi \leq 1$ and $0 \leq \xi^\ast \leq 1$, concentrations of the native, unfolded and aggregate proteins $C_k(\xi, t)$ and $C_k^\ast(\xi, \xi^\ast, t)$, $k = N, U, A$, fluid temperatures $\theta(\xi, t)$ and $\theta^\ast(\xi, \xi^\ast, t)$, wall temperature $\theta_w(\xi, t)$ and deposit thickness $\delta(\xi, t)$. The model parameters characterize the residence time $Da$, steam-side temperature $\theta_h$, initial heat transfer coefficient $\beta_0$, resistances to heat transfer of the milk-side thermal layer $\sigma$ and of the fouling deposit $\phi \delta_h$, thermal layer thickness $\mu$, diffusion of the proteins $\alpha_k$, and adhesion rate $\omega_d$. $r_k$, $k = N, U, A$ represent the rate of disappearance of protein form $k$ due to chemical reactions. The model is written in dimensionless form. It contains mass and heat balance for the bulk and thermal layer,
and deposit, completed with appropriate initial and boundary conditions. The results will be plotted using dimensional variables.

\[ \frac{dC_k}{dt} = -aC_k + r_{k}\left(\theta, C_N, C_U, C_A\right) + \alpha_k \frac{dC'}{d\xi}, \quad k = N, U, A \]  
(2)

\[ \frac{dC_k}{dt} = \alpha_k \frac{d^2C_k}{d\xi^2} - r_k\left(\theta', C_N', C_U', C_A'\right) \]  
(3)

\[ \frac{d\theta}{dt} = \frac{d\theta}{d\xi} + \frac{\beta_0 (\theta - \theta)}{1 + \phi\delta} \]  
(4)

\[ \theta' = \xi^2 (\theta - \theta) + \theta \]  
(5)

\[ \frac{\theta_0 - \theta}{\sigma} = \frac{\theta_0 - \theta}{1 + \phi\delta} = \beta_0 \]  
(6)

\[ \frac{d\delta}{dt} = Da \cdot \omega_\delta \left[ C_{\alpha} d\xi^2 \right] = r(\xi, t) \]  
(7)

\[ C_N(0, t) = 1; \quad C_U(0, t) = C_A(0, t) = 0; \quad \theta(0, t) = \theta_0 \]  
(8)

\[ C_N'\left(\xi, 0, t\right) = C_A'\left(\xi, 0, t\right); \quad \frac{dC_k'}{d\xi}(\xi, 1, t) = 0.0; \quad k = N, U, A \]  
(9)

\[ C_N(\xi, 0) = 1.0; \quad C_U(\xi, 0) = C_A(\xi, 0) = 0.0; \quad \theta(\xi, 0) = \theta_0; \quad \delta(\xi, 0) = \delta_0(\xi) \]  
(10)

\[ C_k'(\xi, 0, 0) = C_k'(\xi, 0, 0), \quad k = N, U, A \]  
(11)

The model can be easily solved by discretizing the spatial co-ordinates by finite differences and integrating the set of resulting ODEs. Most of the model parameters can be found from known operating conditions or literature sources. However, the model parameters related to protein diffusion \(\alpha_N, \alpha_U, \alpha_A\), and deposition rate \(\omega_d\) are difficult to evaluate. They were estimated by fitting the model to experimental data. Fig. 3 shows that our model reproduces well the experimental data provided by Belmar-Beiny et al. (1993). The average error is 0.25 g/m\(^2\), about the same as in Georgiadis et al. (1998a).

During pasteurization, milk outlet temperature is controlled (Fig. 1). This implies that the thermal duty \(q\) (Eq. 6), bulk and layer temperatures and protein concentrations are also constant in time. Therefore, it is possible to define a deposition rate \(r(\xi)\), which depends on the thermal duty \(q\) and protein content of the milk, but is constant in time,
given by the right-hand side of Eq. 7. Then, the deposit thickness at the end of heating stage is:

\[ \delta_h(\xi) = \delta_h(\xi) + r(\xi) \cdot t_h \]  

(12)

3.3. Cleaning

The model of the cleaning phase (Bird & Fryer, 1992) assumes two steps: swelling of the deposit with the rate \( \omega_y \), and removal of the swelled deposit with the rate \( \omega_x \). Integration of the Bird & Fryer model gives the dependence of the deposit thickness after cleaning \( \delta_c \) versus the initial (end of heating) thickness \( \delta_h \) and the cleaning time \( t_c \).

Note that the initial condition for a new heating-cleaning cycle becomes \( \delta_0 = \delta_c \).

\[ \delta_c(\xi) = \frac{\omega_h}{\omega_s} \cdot \exp(-\omega_x t_c) \cdot \left( \exp \left( \frac{\omega_o}{\omega_s} \cdot \delta_c(\xi) \right) - 1 \right) \]  

(13)

4. The constraints

During the heating phase, the resistance to heat transfer increases due to fouling. This is overcome by the control system, which increases the steam flow rate to raise the temperature of the condensate on the steam side, \( \theta_h \). Obviously, the steam temperature cannot be exceeded, \( \theta_h < \theta_s \). Therefore, there is a maximum deposit thickness \( \delta_{\text{max}} \) that can be handled using steam of a given temperature. For a fixed duty \( q \), this thickness can be calculated setting \( \theta_h = \theta_s \) in Eq. 6. Then, the maximum allowed heating time results from simultaneous solution of Eqs 12 and 13, with \( \delta_h = \delta_{\text{max}} \) and \( \xi = 1 \). Typical results are presented in Fig. 4a. From one heating-cleaning cycle to another, the deposit thickness remains constant (extensive cleaning, \( t_c \) large, profile A in Fig 4b) or increases continuously (\( t_c \) small, profile C in Fig 4b). The critical cleaning time \( t_{c*} \) separating the two types of behaviour (profile B) is found as follows. The pasteurization process is regarded as a discrete-time dynamic system, with the deposit thickness after cleaning as state variable (Eq. 14). The critical condition B represents a bifurcation point of the dynamic system. It is obtained by solving the steady state Eqs 14 and 15 together with the bifurcation condition Eq. 16, and leads to the correlation given in Eq 17.

\[ \delta_{c+1} = g(\delta_c, t_h, t_c, r) \]  

(14)
\[ \delta_{c}^{k+1} = \delta_{c}^{k} \]  
(15)

\[ \frac{dg(\delta_{c}, t_{h}, t_{c}, r)}{d\delta_{c}} = 1 \]  
(16)

\[ t_{c} = \frac{\omega_{c}}{r} \left( t_{c}^{*} + \frac{\exp(-\omega_{c} t_{c}^{*})}{\omega_{c}} \frac{1}{\omega_{c}} \right) \]  
(17)

5. The optimal operation

Fig 5 plots the process constraints in the space of decision variables. Lines of constant-value objective function are also displayed. It can be seen that the optimal operating policy is located at the intersection of the two constraints (\( t_{h,\text{max}} \) and \( t_{c}^{*} \)).

6. Conclusions

This work presents a new model for heat-exchanger fouling during milk pasteurization. The model is simple, easy to solve, and reproduces correctly the experimental data. The optimal operation of a milk pasteurization unit consists of long heating stages followed by short, infrequent cleaning. This strategy is constrained by the following conditions: a) The heating time should not exceed the limiting value \( t_{h,\text{max}} \). Otherwise, the required milk outlet temperature cannot be achieved (shell-and-tube heat-exchangers) or the pressure drop becomes excessive (plate heat-exchangers). b) The cleaning time \( t_{c} \) must exceed the critical value \( t_{c}^{*} \). Otherwise, the deposit thickness increases continuously from one heating - cleaning cycle to another.

References


