Optimal Scheduling of a Lube Oil and Paraffin Production Plant

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Abstract

An MILP model for the optimal scheduling of a lube oil and paraffin production plant is presented. The plant is composed of processing units that operate in continuous and discontinuous modes, and some are also duplicated. The raw materials for the process are stored in dedicated tanks, and the plant presents finite intermediate storage. The mathematical formulation was developed under a continuous time representation and compared to a discrete-time and other continuous-time MILP models. Results with significant reduction of the computational time are generated and allow the model application for realistic time horizons with a significant level of scheduling detail.

Keywords: scheduling, mixed integer optimization, refinery, lube oil, paraffin.

1. Introduction

Basic lube oils and paraffins are petroleum derivatives that require a specific process for their production. Only a few refineries are able to process these products, due to specific properties of petroleum types and specialized equipment. The lube oil types as well as the basic paraffins are raw-materials for obtaining finished products that are commercialized in the market, after mixing with additives in appropriate amounts. This work relates to the general refinery scheduling problems such as the ones addressed by Jia \textit{et al.} (2003) and Joly and Pinto (2003).

A critical aspect of mixed integer scheduling models concerns time representation. Approaches used to represent the time domain in the scheduling models can be classified into discrete and continuous time formulations. The former are based on the discretization of the time horizon into a number of intervals of equal length and all events are forced to coincide with one of the interval boundaries. In general, discrete time formulations may rely on an excessive number of time periods to achieve the required accuracy. In continuous time formulations the goal is the consideration of event points only and the length of each time interval is unknown. In this case, two classes of grids are used that are a uniform grid for all resources and a non-uniform grid in which each resource is handled independently. Among the uniform continuous time

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MILP formulations there are the works of Schilling and Pantelides (1999), Maravelias and Grossmann (2003), Giannelos and Georgiadis (2003), and Castro et al. (2001). In the latter each task must start at an event point but it may not finish at a time point. Non-uniform time grid formulations include Ierapetritou and Floudas (1998) and Giannelos and Georgiadis (2002) that in general have the advantage of requiring fewer event points at the cost of more complicated balance constraints.

This work proposes MILP formulations that rely on discrete and continuous (uniform grid) time for the scheduling of a plant that produces basic lube oils and paraffins. The plant presents continuous and discontinuous units, as well as limited intermediate storage and other operating constraints.

2. Problem Description

The plant is composed of continuous units \( U_c \), discontinuous units \( U_d \) and storage tanks as shown in Figure 1. The plant generate the following final products: light neutral lube oil (NL) from light neutral distillate (DNL), medium neutral lube oil (NM) from medium neutral distillate (DNM), bright stock oil (BS) from vacuum residue (RV), macro-crystalline paraffin (MA and MAE) from DNL and DNM, and micro-crystalline paraffin (MI and MIE) from RV. The production scheme is shown in Figure 1. In the lube oil production, each product follows the same pre-defined processing path, but with sequences that are not necessarily the same because of intermediate storage that decouples the operation of the units. In the case of paraffins, the products follow different paths and therefore sequences; moreover, deoilation can be performed in parallel units (U18 and U13).

**Figure 1: Lube oil and paraffin production plant.**
3. Mathematical Formulation

The problem results in MILP formulations (notation is provided in Table 1). The main assumptions are that the process present constant yields and that there is no significant loss of products during changeover and setup operations. Moreover, the proposed model relies on continuous time and units are divided into continuous and discontinuous.

Table 1: Nomenclature of the MILP optimization models

Indices and sets

- **e**: stages, \(E=\{dt, das, dpL, dpP, da, do, ts, htl, htP\}\); **i**: products, \(I=\{nl, nm, bs, ma, mi\}\); \(I_e\): products stored in stage \(e\); \(I_u\): products processed in unit \(u\); \(I_{ie}\): units that process \(i\) to be stored in \(e\); \(I_{ie}\): units that receive \(i\) stored in \(e\).

Parameters

- \(Camp_{i,u}^{\text{max}}\): maximum number of campaigns of \(i\) in \(u\); \(CampU_u^{\text{max}}\): maximum number of campaigns in \(u\); \(DNE_i\): demand for \(i\); \(DE_i\): demand for tabled \(i\); \(Desl_{i,u}^{\text{max}}\): maximum number of shutdowns in \(u\); \(F_{i,u}^{\text{max}}\) / \(F_{i,u}^{\text{min}}\): max and min amounts of \(i\) fed to \(u\); \(F_{i,e}^{\text{min}}\): min amount of \(i\) sent to the terminal; \(FCC_{i,e}^{\text{max}}\): max amount of \(i\) sent to FCC in \(e\); \(Feed_{i,0}\): amount of crude oil supplied to U9; \(Feed_{i,11}\): amount of RV fed to \(U_{11}\); \(H\): time horizon; \(\eta_{i,\theta}\): yield of \(i\) in \(u\); \(Ppe_{i,u}\): price of \(i\) in \(Ter\); \(Ppe_{i}\): price of tabled \(i\); \(V_{i,e}^{\text{max}}\) / \(V_i^{\text{min}}\): max / min storage capacity of \(i\) in stage \(e\); \(V_{i,e}^{\text{max}}\): max storage capacity of tank \(t\) in stage \(e\).

Binary Variables

- \(d_{u,k}\): 1 if discontinuous unit \(u\) is shut down in time \(k\); \(y_{i,u,k}\): 1 if product \(i\) is processed in unit \(u\) in time \(k\); \(y_{i,k}^{\text{in}}\): 1 if product \(i\) is stored in tank \(t\) in slot \(k\); \(z_{i,u,k}\): 1 if campaign of \(i\) in unit \(u\) starts in time \(k\).

Continuous Variables

- \(f_{i,u,k}\): amount of product \(i\) processed in unit \(u\) in slot \(k\); \(fcc_{i,e,k}\): amount of product \(i\) sent to FCC in stage \(e\) in slot \(k\); \(T_k\): time point \(k\); \(\theta_{i,u,k}\): processing time of product \(i\) in unit \(u\) in slot \(k\); \(v_{i,e,k}\): amount of product \(i\) stored in stage \(e\) at the end of slot \(k\).

\[ \text{Profit} = \max \left( \sum_{k \in K} \sum_{u \in U} \sum_{i \in I} Ppe_{i,u} \cdot f_{i,u,k} + \sum_{k \in K} \sum_{i \in I} Ppe_{i} \cdot f_{i,M1,k} \right) \]  

s.t.

\[ T_0 = T_0 = 0 \]  
\[ T_k = H \] \(\forall k \in K\) \hspace{1cm} (2.a)
\[ T_{k-1} \leq T_k \] \(\forall k \in K\) \hspace{1cm} (2.b)
\[ T_k - T_{k-1} = \sum_{i \in I_u} \theta_{i,u,k} \] \(\forall i \in I_u, u \in U_c, k \in K\) \hspace{1cm} (2.c)
\[ T_k - T_{k-1} \geq \sum_{i \in I_u} \theta_{i,u,k} \] \(\forall i \in I_u, u \in U_d, k \in K\) \hspace{1cm} (2.d)
\[
\Delta T_{\min} \cdot \gamma_{i,u,k} \leq \theta_{i,u,k} \leq H \cdot \gamma_{i,u,k} \quad \forall i \in I_u, u \in U, k \in K \\
\theta_{i,u,k} \geq \left( \frac{F_{\min}^{i,u}}{F_{\min}^{i,u}} \right) \cdot \gamma_{i,u,k} \\
\sum_{i \in I_u} y_{i,u,k} = 1 \\
\sum_{i \in I_u} y_{i,u,k} \leq 1 \\
f_{i,u,k} = F_{\max}^{i,u} \cdot \theta_{i,u,k} \\
F_{i,u}^{\min} \cdot \theta_{i,u,k} \leq f_{i,u,k} \leq F_{i,u}^{\max} \cdot \theta_{i,u,k} \\
fcc_{i,e,k} \leq FCC_{i,e} \cdot (T_k - T_{k-1}) \\
\sum_{i \in I_u} \eta_{i,u,k} - \sum_{i \in I_u} f_{i,u,k} - fcc_{i,e,k} \quad \forall i \in I_u, e \in E, k \in K \\
v_{i,e,k} = \frac{f_{i,u,k}}{f_{i,u,k} - 1} + \sum_{i \in I_u} \eta_{i,u,k} - \sum_{i \in I_u} f_{i,u,k} - fcc_{i,e,k} \\
v_{i,e,k} \leq \frac{f_{i,u,k}}{f_{i,u,k} - 1} + \sum_{i \in I_u} \eta_{i,u,k} - \sum_{i \in I_u} f_{i,u,k} - fcc_{i,e,k} \\
v_{i,e,k} \leq \sum_{i \in I_u} \gamma_{i,u,k} \cdot \gamma_{i,u,k} \\
\sum_{i \in I_u} y_{i,t,e,k} = 1 \\
z_{i,u,k} \geq y_{i,u,k+1} - y_{i,u,k} \\
z_{i,u,k} \leq 1 - y_{i,u,k} \\
z_{i,u,k} \geq y_{i,u,k+1} \\
C_{i,u,k} \sum_{k=1}^{K-1} z_{i,u,k} + y_{i,u,1} \leq Camp_{i,u}^{\max} \\
\sum_{k=1}^{K-1} z_{i,u,k} + y_{i,u,1} \leq Camp_{i,u}^{U_{i,u}^{\max}} \\
d_{i,u,1} \geq \sum_{i \in I_u} y_{i,u,k} - \sum_{i \in I_u} y_{i,u,k+1} \\
d_{i,u,1} \leq \sum_{i \in I_u} y_{i,u,k} - \sum_{i \in I_u} y_{i,u,k+1} \\
d_{i,u,1} \leq \sum_{i \in I_u} y_{i,u,k} - \sum_{i \in I_u} y_{i,u,k+1} \\
\sum_{k=1}^{K-1} d_{i,u,k} \leq Dist_{i,u}^{\max} \\
\sum_{k=1}^{K-1} d_{i,u,k} \leq Dist_{i,u}^{\max}
\]
Objective function (1) maximizes revenues. Timing constraints are presented in (2) and determine the time points and intervals, whereas (3) show the unit assignment constraints. Capacity constraints for the units (including U9 and U11 that generate feed streams) and outlet streams (to FCC) are given in (4) and (5), respectively. Material balance constraints are shown in (6) and storage of products in the stages is bounded in (7). Demands are enforced in constraints (8). The hydro-treatment stage has storage limitations that are given in (9). Finally, the number of campaigns and shut down operations are limited in (10) and (11), respectively.

The discrete model has $y_{i,u,k}$ in place of $\theta_{i,u,k}$ in constraint (4), whereas constraints (4a) for U9 and U11 present constant flow rates, and (5) is replaced by a single upper bound for the FCC flow rate. The continuous time model based on that proposed by Giannelos and Georgiadis – GG (2003) presents constraints (12) in place of (2.c-d):

$$T_k - T_{k-1} \leq \theta_{i,u,k} + H \cdot (1 - y_{i,u,k}) \quad \forall i \in I_u, u \in U - Ter, k \in K$$

(12.a)

$$T_k - T_{k-1} \geq \theta_{i,u,k} - H \cdot (1 - y_{i,u,k}) \quad \forall i \in I_u, u \in U - Ter, k \in K$$

(12.b)

$$T_k - T_{k-1} \geq \theta_{i,u,k} \quad \forall i \in I_u, u \in Ter, k \in K$$

(12.c)

Finally, the model based on Pinto et al. (2000) presents the following constraints in place of (2.c-f): $T_k - T_{k-1} \leq H$ and $T_k - T_{k-1} \geq \Delta T^{\min}$, and (13) in place of (4):

$$f_{i,u,k} \leq F_{i,u}^{\max} \cdot (T_k - T_{k-1}) \quad \forall i \in I_u, u \in U, k \in K$$

(13.a)

$$f_{i,u,k} \leq F_{i,u}^{\max} \cdot H \cdot y_{i,u,k} \quad \forall i \in I_u, u \in U - k \in K$$

(13.b)

$$f_{i,u,k} \geq F_{i,u}^{\max} \cdot (T_k - T_{k-1}) - H \cdot (1 - y_{i,u,k}) \quad \forall i \in I_u, u \in Uc, k \in K$$

(13.c)

$$f_{i,u,k} \geq F_{i,u}^{\min} \cdot (T_k - T_{k-1}) - H \cdot (1 - y_{i,u,k}) \quad \forall i \in I_u, u \in Uc - Ter, k \in K$$

(13.d)

$$f_{i,u,k} \geq F_{i,u}^{\min} \cdot y_{i,u,k} \quad \forall i \in I_u, u \in Ter, k \in K$$

(13.e)

The main difference between the proposed model and the ones presented by Giannelos and Georgiadis (2003) and Pinto et al. (2000) is based on the same approach of Castro et al. (2001) that the intervals for the discontinuous units are not explicitly calculated.

4. Model Application

The proposed mathematical formulations were applied to determine the plant production scheduling for a 30-day time horizon, whose schedule is calculated manually. Data for the case study are presented in Figure 1. The MILP models were implemented in the GAMS modeling language (Brooke et al., 1998) with the XPRESS solver (Dash, 1999). The calculations were performed on a Pentium III 500 MHz / 512 Mb RAM platform. Figure 2 shows the Gantt chart for the scheduling of the production units that also contains the timing and amounts of products to be processed from the continuous model. This chart corresponds to the best solution obtained that is given in Table 2.
Figure 2: Gantt chart for the best solution of proposed continuous time model

The model and solution statistics for all models are shown in Table 2. The results clearly show that the proposed formulation performs better than the discrete and continuous formulations. Interestingly, the proposed model presents the tightest LP relaxation (same as the discrete time model), which generates significantly lower integrality gap; consequently, less CPU time is required.

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5. Conclusions

In this work, an MILP model was developed for the scheduling of a lube oil and paraffin production plant. The proposed model, that mainly differentiates the calculation between the continuous and the discontinuous units, required a computational effort that is at least two orders of magnitude lower than the ones from the other models.

References


