NONLINEAR OBSERVER FOR NONLINEAR ADAPTIVE GUIDANCE LAW CONSIDERING TARGET UNCERTAINTIES AND CONTROL LOOP DYNAMICS

DongKyoung Chwa*, A.G. Sreenatha*, Ki Hong Im†, Jin Young Choi†, and Jin H. Seo †

*School of Aerospace, Civil, and Mechanical Engineering, UNSW@ADFA (The University of New South Wales at Australian Defence Force Academy), Australia
e-mail: dkchwa@neuro.snu.ac.kr
Fax: +82-2-883-3251
†School of Electrical Engineering, Seoul National University, Korea

Keywords: nonlinear observer, nonlinear adaptive guidance, target maneuver, control loop dynamics, integrated guidance and control model.

Abstract

This paper proposes a nonlinear observer design method for nonlinear adaptive guidance. Several states of the previously proposed nonlinear adaptive guidance law are estimated by a nonlinear observer, which is designed based on the integrated guidance and control model. Using the estimated states and uncertainties, desired engagement performance of the nonlinear adaptive guidance law can be obtained against target maneuver and the limited performance of control loop. The performance and stability analyses of the proposed observer and simulations are included to demonstrate the practical application of our scheme.

1 Introduction

There has been much research on guidance area [13, 6] including proportional navigation (PN), true proportional navigation (TPN), augmented proportional navigation (APN), optimal guidance law (OGL), nonlinear guidance laws using Lyapunov method [14], nonlinear geometric method [2,9,10], nonlinear $H_\infty$ method [15], and sliding mode guidance (SM) [3,1,16].

All the above guidance laws, however, do not consider the actual dynamics of missile control systems and have limitation in their performance of the overall guidance and control loop. Accordingly, the integrated guidance and control approach is suggested in [12,8,11], where the optimal control technique together with gain scheduling approach is used. In [4], another approach to integrated guidance and control is suggested including the actual missile control loop in [7]. That is, an integrated guidance and control loop, which is valid for all flight conditions and also includes the uncertainties in both control loop dynamics and target acceleration, is formulated and then a nonlinear adaptive guidance law is designed. This approach is shown to achieve better interception performance than PN guidance. This, however, assumes that all states in the guidance law are available.

In this paper, a nonlinear observer is proposed for the nonlinear adaptive guidance law based on integrated guidance and control model in [4]. First, the integrated guidance and control model is re-formulated as a normal form with respect to available states by considering unavailable information as parametric and non-parametric uncertainties. Then, a nonlinear observer is designed and the estimated states and uncertainties are used in the nonlinear adaptive guidance law. The performance and stability of the proposed adaptive observer are analyzed and simulation results are also performed to demonstrate the proposed approach.

2 Integrated guidance and control model

In this section, an integrated model for guidance and control loop proposed in [4] is reviewed, and then it is further reformulated for the design of nonlinear observer.

First, using the controller in [7], [4] shows that the control loop consisting of the nonlinear controller and missile dynamics has output response given by

$$\ddot{a}_m + 2\xi\omega_n a_m + \omega_n^2 a_m = \omega_m^2 a_{mc} + \Delta_e$$  (1)

where $a_m$ is acceleration command; $a_m$ is acceleration output; $\xi$ and $\omega_n$ are design parameters of the control loop; and $\Delta_e$ is a bounded uncertainty. Equation (1) can be expressed as

$$\dot{X}_e = \begin{bmatrix} 0 & 1 \\ -a_{c1} & -a_{c2} \end{bmatrix} X_e + \begin{bmatrix} 0 \\ b_c \end{bmatrix} u_c + \begin{bmatrix} 0 \\ \Delta_e \end{bmatrix}$$  (2)

where $X_e = (x_{e1}, x_{e2})^T = (a_m, \dot{a}_m)^T$, $a_{c1} = \omega_n^2$, $a_{c2} = 2\xi\omega_n$, $b_c = \omega_m^2$, and $u_c = a_{mc}$. 
Secondly, [4] shows that the state equation of the guidance loop can be expressed by

\[ \dot{\mathbf{x}}_g = \left( \begin{array}{c} 0 \\ a_{g1}(t) \\ a_{g2}(t) \end{array} \right) \mathbf{x}_g + \left( \begin{array}{c} 0 \\ -b_g(t) \end{array} \right) \mathbf{a}_T \\
= A_g \mathbf{x}_g + B_g \mathbf{u}_g + D_g \]

(3)

where \( \mathbf{x}_g = (x_{g1} \ x_{g2} \ y_{g3})^T = (\sigma \ \sigma)^T ; \sigma \) is a line-of-sight (LOS) angle; \( a_{g1}(t) = \dot{R}(t)/R(t) \), \( a_{g2}(t) = 2\dot{R}(t)/R(t) \), \( b_g(t) = l/R(t) \); \( A_g \) is target acceleration; \( u_g = \mathbf{a}_m \); and \( R \) is the relative distance and velocity between the target and the missile. Since from Equation (2) \( u_g = x_{g1} = C_g X_c \) holds for \( C_g = [1 \ 0] \), the control loop in Equation (2) and the guidance loop in Equation (3) can be combined as

\[ \dot{\mathbf{x}}_{igc} = \left( \begin{array}{c} 0 \\ a_{g1} \\ a_{g2} \end{array} \right) \mathbf{x}_{igc} + \left( \begin{array}{c} 0 \\ -b_g \end{array} \right) \mathbf{a}_T \\
\]

\[ = A_{igc} \mathbf{x}_{igc} + B_{igc} \mathbf{u}_c + D_{igc} \]

(4a)

\[ Y_{igc} = x_{g2} \]

(4b)

where

\[ \mathbf{x}_{igc} = \begin{pmatrix} X_g \\ X_r \end{pmatrix}, \quad A_{igc} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_{g1} & -a_{g2} & -b_g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_{g1} & -a_{g2} \end{bmatrix}, \]

\[ B_{igc} = \begin{pmatrix} 0 & 0 & b_g \end{pmatrix}^T, \quad C_{igc} = (0 \ 1 \ 0 \ 0), \]

\[ D_{igc} = (0 \ b_g \mathbf{a}_T \ 0 \ \mathbf{A}_c)^T. \]

In [4], the guidance law is designed by deriving an integrated guidance and control model from Equation (4). However, all parameters in Equation (4) are not available. While \( a_{g1}, a_{g2} \) in \( A_{igc} \) of Equation (4) are available since they are design parameters of the control loop, \( a_{g1}, a_{g2}, b_g \) are decomposed into known parts \( \hat{a}_{g1}, \hat{a}_{g2}, \hat{b}_g \) and unknown parts \( \tilde{a}_{g1}, \tilde{a}_{g2}, \tilde{b}_g \) such as

\[ a_{g1} = \hat{a}_{g1} + \tilde{a}_{g1} \]

(5a)

\[ a_{g2} = \hat{a}_{g2} + \tilde{a}_{g2} \]

(5b)

\[ b_g = \hat{b}_g + \tilde{b}_g \]

(5c)

For easier application, a normal form of integrated guidance and control model with uncertainties is formulated in the following.

In proportional navigation, acceleration commands are generated to make the rate of rotation of the LOS (line-of-sight) be zero and this guarantees the interception performance. So, the output is chosen by

\[ y = Y_{igc} = x_1 \]

(6)

as in [16], which will be made to be zero by the guidance law. Differentiating the output, we have

\[ \dot{x}_1 = -a_{g1} x_{g1} - a_{g2} x_{g2} - b_g x_{g3} + b_g a_T \]

(7a)

where

\[ \dot{x}_2 = \dot{\mathbf{x}}_g = \hat{a}_{g1} x_{g1} - \hat{a}_{g2} x_{g2} - \hat{b}_g x_{g3} \]

(7b)

\[ \dot{x}_3 = \dot{\mathbf{a}}_T = \mathbf{a}_m \]

(7c)

\[ \mathbf{a}_m = \begin{pmatrix} \sigma \ 
\mathbf{a}_m \end{pmatrix} \]

(7d)

\[ \theta^T = \begin{pmatrix} \theta_{11} \ 
\theta_{12} \ 
\theta_{13} \ 
\theta_{14} \end{pmatrix} \]

(7e)

\[ \phi^T = \begin{pmatrix} \phi_{11} \ 
\phi_{12} \ 
\phi_{13} \ 
\phi_{14} \end{pmatrix} \]

(7f)

\[ \Delta = b_g a_T. \]

In the same way, we can have

\[ x_1 = \tilde{a}_{g1} x_{g1} - \tilde{a}_{g2} x_{g2} - \tilde{b}_g x_{g3} \]

(8a)

\[ \theta^T = \begin{pmatrix} \theta_{21} \ 
\theta_{22} \ 
\theta_{23} \ 
\theta_{24} \end{pmatrix} \]

(8b)

\[ \phi^T = \begin{pmatrix} \phi_{21} \ 
\phi_{22} \ 
\phi_{23} \ 
\phi_{24} \end{pmatrix} \]

(8c)

\[ \Delta = \tilde{a}_{g2} \]

(8d)

\[ \dot{x}_1 = -\hat{a}_{g1} x_{g1} - \hat{a}_{g2} x_{g2} - \hat{b}_g x_{g3} \]

(9a)

\[ \dot{x}_2 = \dot{\mathbf{x}}_g = \hat{a}_{g1} x_{g1} - \hat{a}_{g2} x_{g2} - \hat{b}_g x_{g3} \]

(9b)

\[ \dot{x}_3 = \dot{\mathbf{a}}_T = \mathbf{a}_m \]

(9c)
In this section, a nonlinear observer is designed for the integrated guidance and control model under the following assumptions:

\begin{align*}
\phi^T &= [\theta_{3i} \theta_{32} \theta_{33} \theta_{34} \theta_{35}]^T \\
\dot{\phi}^T &= [x_{\phi} \hat{x}_1 x_2 x_3 x_4], \\
\Delta_3 &= -\hat{a}_{i3}\Delta_1 - \hat{a}_{i3}\Delta_2 - \hat{b}_i\Delta_3.
\end{align*}

Thus, an integrated guidance and control model with uncertainties can be described by

\begin{align}
\dot{X} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u_x \\
&+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \cdots \\ 0 & 0 & 0 \\ \cdots \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ m_{x_1} \\ m_{x_2} \end{bmatrix} + \begin{bmatrix} 0 \\ -b_p x_3 \\ \cdots \end{bmatrix} u_x \\
&+ \begin{bmatrix} \hat{a}_{i3} x_3 - \hat{a}_{i3} x_3 + \hat{b}_i a_1 x_3 + \hat{b}_i a_2 x_2 \end{bmatrix} + \begin{bmatrix} \theta_1^T \phi_1 \\ \theta_2^T \phi_2 \\ \cdots \end{bmatrix}, \quad (10a)
\end{align}

where $X = [x_1 \ x_2 \ x_3]^T$.

3 Nonlinear observer based on integrated guidance and control model

In this section, a nonlinear observer is designed for the integrated guidance and control model under the following assumptions.

**Assumption 3.1:** $\theta_i$ and $\Delta_i$ are bounded as $|\theta_i| = |\theta_{ji}|$, $\dots$, $|\theta_{ji}| \leq \mu_i = [\mu_{i1}, \dots, \mu_{ij}]$, and $|\Delta_i| \leq \Delta_i$ where $1 \leq i \leq 3$ and $3 \leq j \leq 5$.

The proposed observer is given by

\begin{align}
\dot{\hat{x}} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \hat{X} + \begin{bmatrix} k_1 \\ k_2 \\ m_{x_1} \end{bmatrix} e + \begin{bmatrix} 0 \\ 0 \\ m_{x_2} \end{bmatrix} u_x \\
&+ \begin{bmatrix} \mu_i \phi_1 + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \\ \mu_i \phi_2 + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \\ \mu_i \phi_j + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \end{bmatrix}, \quad (11)
\end{align}

and the adaptation law by

\begin{align}
\dot{\hat{\mu}} &= \gamma \hat{X}, \quad \dot{\hat{D}} = \gamma \hat{X}, \quad (12)
\end{align}

where $\hat{X} = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3]$; $e$ is obtained from $\dot{e} = -a_1 e + a_2 \hat{x}_1$ with a positive constant $a_0$; $\hat{\mu}_i = [\hat{\mu}_{i1}, \dots, \hat{\mu}_{ij}]$ and $\hat{D}_i$ are estimates of $\mu_i$ and $D_i$.

Proof: From Equations (10a) and (11), we have

\begin{align}
\frac{d}{dt} E &= AE + \begin{bmatrix} \theta_1^T \phi_1 \\ \theta_2^T \phi_2 \\ \theta_j^T \phi_j \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_j \end{bmatrix} - \begin{bmatrix} \mu_1 \phi_1 + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \\ \mu_2 \phi_2 + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \\ \mu_j \phi_j + \hat{D}_i \operatorname{sgn}(\hat{x}_i) \end{bmatrix}.
\end{align}

We choose a Lyapunov function
for \( P = \text{diag}(P_1, P_2, P_3, P_4) > 0 \) and take its time derivative to have

\[
\dot{V} = \frac{1}{2} E^T P E + \sum_{i=1}^3 \frac{P}{2} \left( \frac{\mu_i^T \bar{\mu}_i + \bar{D}_i^2}{\gamma_{\text{in}}} \right)
\]

This section presents simulation results for the proposed observer and guidance law. The output or the rate of line-of-sight can be made to converge to zero by the following guidance law, which is based on [4].

\[
\hat{\mu}_v = \gamma_{\text{in}} [\bar{x}_i, \bar{p}_i, \hat{D}_i] \frac{\bar{\zeta}}{\bar{\zeta}}.
\]

\[
Q.E.D.
\]
depend on the conditions of the missile and the target. The miss distance and flight time are chosen as performance indices. The actual missile control system in [7] is employed in a closed-loop guidance and control simulation environment described in [5]. The magnitude and rate saturation of the guidance commands \( \dot{u} \) are included as \( \dot{u} \leq 40g \) and \( |\ddot{u}| \leq 400g/sec \). The performance of the proposed nonlinear adaptive guidance (NAG) law is compared with that of proportional navigation guidance (PNG) law. Design parameters of control loop in Equation (4) are \( \xi = 0.7 \) and \( \omega_n = 15 \). Also, those of observer in Equations (13) and (14) are \( k_1 = 50 \), \( k_2 = 30 \), \( k_3 = 10 \), \( m_1 = m_2 = m_3 = 3 \), \( a_0 = 50 \), \( \gamma_{\mu_1} = \gamma_{\mu_2} = \gamma_{\mu_3} = 0.01 \), \( \gamma_{D_1} = \gamma_{D_2} = \gamma_{D_3} = 1 \), \( d_{\dot{u}} = d_{\ddot{u}} = 0.01 \), \( d_{\dot{u}} = 0.1 \), and those of guidance law in Equation (15) are \( a_s = 250 \), \( b_s = 1 \), \( k_s = 5 \), \( k_f = 50 \), \( a_{\mu_1} = 0.01 \), \( a_{\mu_1} = 200 \), \( a_{\mu_2} = 1 \).

Here, we selected several scenarios shown in Table I, where the target initially travels at constant velocity with 200 m/sec and make step-changes in acceleration. Each vector component represents the value along the y and z axis, respectively. The control start time of the missile is 0.5 sec. Parameters in Equation (5) are chosen as \( \bar{a}_{s1} = 0 \), \( \bar{a}_{s1} = \dot{R}/R \), \( \bar{a}_{s2} = 2\dot{R}/R \), \( \bar{a}_{s2} = 0 \), \( \bar{b}_{s} = 1/R \), and \( \bar{b}_{s} = 0 \) by assuming that only \( R \) and \( \dot{R} \) are available. Table II compares the miss distances and flight time of PNG and NAG under each scenario. Although the estimated states and uncertainties from nonlinear observer are used in NAG, NAG exhibits better performance than PNG. Fig. 1 shows the acceleration commands and actual accelerations, and three-dimensional missile-target trajectories for PN guidance (PNG) and proposed guidance (NAG) for Scenario I in Table I. We also performed for other scenarios and we could see that in overall cases the proposed scheme have better performance over PNG.

5 Conclusion

We proposed a nonlinear observer for a nonlinear adaptive guidance law. The simulation results show that the nonlinear adaptive guidance law using a proposed nonlinear observer can perform as much as the one where all of states are assumed to be available. More rigorous design and analysis of the guidance law combined with the nonlinear observer needs to be done and can be pursued as a further study.

Acknowledgements

This work was supported by the visiting fellow program of UNSW@ADFA (The University of New South Wales at Australian Defence Force Academy), Post-doctoral Fellowship Program of Korea Science & Engineering Foundation (KOSEF), the Brain Korea 21 Project, and the Automatic Control Research Center (ACRC) in Seoul National University.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>First evasive time (sec.)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First evasive acceleration (m/ sec^2)</td>
<td>[4 –4]</td>
<td>[0 8]</td>
<td>[0 –10]</td>
</tr>
<tr>
<td>Second evasive time (sec.)</td>
<td>2</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>Second evasive acceleration (m/ sec^2)</td>
<td>[8 –8]</td>
<td>[–8 0]</td>
<td>[15 0]</td>
</tr>
</tbody>
</table>

(a) Target conditions

<table>
<thead>
<tr>
<th>Scenario</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off-boresight angle (deg)</td>
<td>-30</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>Aspect angle (deg)</td>
<td>-90</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>Elevation angle (deg)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Azimuth angle (deg)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Initial relative distance (m)</td>
<td>2000</td>
<td>3000</td>
<td>1500</td>
</tr>
<tr>
<td>Initial relative altitude (m)</td>
<td>1300</td>
<td>2500</td>
<td>1000</td>
</tr>
</tbody>
</table>

(b) Target-Missile geometry

Table I. Scenarios for missile-target interception

<table>
<thead>
<tr>
<th>Scenario</th>
<th>PNG</th>
<th>NAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD</td>
<td>FT</td>
<td>MD</td>
</tr>
<tr>
<td>I</td>
<td>7.7048m</td>
<td>4.9505sec.</td>
</tr>
<tr>
<td>II</td>
<td>4.3244m</td>
<td>5.8085sec.</td>
</tr>
<tr>
<td>III</td>
<td>1.8637m</td>
<td>3.9255sec.</td>
</tr>
</tbody>
</table>

Table II. Performance of PNG and NAG (MD: Miss Distance, FT: Flight Time)

References


Fig. 1 Performance of Scenario I
(in (a),(c), Solid: actual acceleration, Dotted: acceleration command; in (b),(d), M: missile, T: target)