Case-study based performance assessment of an event-triggered MPC scheme for freeway systems
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Abstract—The paper is aimed at the definition of an efficient control framework for freeway systems. To this end, a Model Predictive Control scheme based on the cell transmission model is adopted in which the considered control action refers to ramp metering. Two major aspects characterize the paper. First, the prediction model is formulated as a mixed logical dynamical system (i.e., it is described by linear dynamic equations and linear inequalities in which both real and integer variables are involved) in order to obtain a finite-horizon optimal control problem with the structure of a mixed-integer quadratic programming problem. In such a problem the objective function quadratically penalizes the deviation of the state variables from a specific equilibrium point. Secondly, an event-triggered scheme is used to further reduce the computational load of the control scheme making it more adequate for real applications. Numerical results based on data coming from a real case study relevant to Grenoble South Ring are also presented and discussed.

I. INTRODUCTION
Traffic congestion is nowadays a very common problem in freeways and interurban roadways, with the associated bad consequences related to the increase of travelling times, consumption and environmental problems. This issue has been dealt with by many researchers and in the last decades various control approaches have been developed in order to reduce congestion phenomena in freeway systems. In particular, different traffic control measures have been proposed and implemented, such as ramp metering, variable speed limits, route guidance and vehicle-infrastructure integration systems (see, for instance, [1], [2], [3], [4]).

In this paper the control of freeway traffic systems via ramp metering is studied; ramp metering is a control action in which the traffic flow entering the freeway from on-ramps is appropriately regulated adopting traffic lights. Ramp metering has been studied for some decades and it has been successfully applied in many real cases [5], [6]. The ramp metering control schemes developed by the research community can be divided between those relying on the second-order macroscopic traffic model, as in [7], [8], and those based on the first-order cell transmission model (CTM), as [9].

In the present paper, an event-triggered Model Predictive Control (MPC) scheme is considered; such scheme has been proposed by the same authors in [10]. The main novelty of the present paper stands in the application of the proposed control framework to a real case that is a portion of the Grenoble South Ring.

In the proposed scheme, firstly a classical MPC is defined. The prediction model considered in the finite-horizon optimal control problem is the CTM, restated as a mixed logical dynamical (MLD) system, i.e. a system described by linear equations and inequalities involving both real and binary variables, according to the framework proposed in [11]. Besides, in the finite-horizon optimal control problem the objective function quadratically penalizes the deviation of the decision variables from a specific equilibrium point. In this way, a mixed-integer quadratic programming problem must be solved at each time step. For such problem, it is possible to prove some convergence results. Specifically, in case all the problem data are known or exactly estimated, it is proven that the predictive control law makes the equilibrium state stable. On the contrary, if it is not possible to estimate all the problem data, the system can be considered as affected by bounded disturbances; in this case, the control law derived by adopting the proposed scheme preserves the input-to-state practical stability of the system.

In order to reduce the computational load of the classical MPC scheme for real applications, an event-triggered control approach is proposed. The idea of event-triggered strategies is almost new in the literature and is basically motivated by the difficulty of applying classical MPC schemes because of communication and computation difficulties. In event-driven control systems the sampling is event-triggered instead of being time-triggered. The research works [12], [13], [14] in the literature dealing with event-triggering control are focused to prove some theoretical results about system stability and convergence, mainly in the continuous-time case [15], [16]. The extension of these techniques to discrete-time systems has been proposed in [17], where event-driven strategies for MPC approaches are also investigated.

The proposed event-triggered MPC scheme has been extended to the distributed case in [18] by the same authors. In that paper, clusters of freeway stretches, i.e. sets of subsequent freeway cells, are separately modeled and controlled. Freeway stretches can be of different type (for instance, with or without ramps), can be characterized by different parameters and can also be subject to different kinds of control strategies (such as variable speed limit (VSL) based control, or ramp metering), then, the overall freeway network
can be considered and studied as a System of Systems.

The paper is organized as follows. In Section II the MPC scheme is described and the relative convergence results are reported. In Section III the event-triggered approach is presented and described. The application of the proposed control approach to the real case of the Grenoble South Ring is discussed in Section IV, where numerical results are reported and analysed in detail. Finally, some conclusive remarks are drawn in Section V.

II. THE MPC SCHEME BASED ON THE CTM MODEL IN MLD FORM

As already proposed, the control scheme considered in this paper and applied to the real case of Grenoble South Ring has been previously introduced by the same authors in [10]. Then, the proposed MPC scheme and its event-triggered version are summarized in the present paper, where the convergence results are also reported (only the statement of the theorems, without their proofs) since all these issues represent the theoretical basis for the present paper (the interested reader can find all the details of this approach in [10]).

In the proposed MPC scheme the finite-horizon optimal control problem adopts the cell transmission model in MLD form for the prediction. In the cell transmission model the freeway is supposed to be divided into cells with homogeneous traffic features and the time horizon is discretized as well (see [19] and [20] for a detailed model description). Let \( N \) be the number of cells (index \( i = 1, \ldots, N \) refers to the generic cell) and let \( K \) be the number of time steps (index \( k = 1, \ldots, K \) refers to the generic time step); let \( T \) denote the sample time and \( L_i \) denote the length of cell \( i \). Let us define the following quantities (see Fig. 1): \( \rho_i(k) \) is the traffic density of cell \( i \) [veh/km], \( \Phi_i^+(k) \) is the total flow entering cell \( i \) [veh/h], \( \Phi_i^-(k) \) is the total flow exiting cell \( i \) [veh/h], \( \phi_i(k) \) is the mainstream flow entering cell \( i \) from cell \( i - 1 \) [veh/h], \( l_i(k) \) is the queue length in the on-ramp of cell \( i \) [veh], \( r_i(k) \) is the flow exiting cell \( i \) from the on-ramp [veh/h], \( d_i(k) \) is the on-ramp demand referred to cell \( i \) [veh/h], \( s_i(k) \) is the flow exiting cell \( i \) through the off-ramp [veh/h], \( D_i(k) \) is the demand of cell \( i \) [veh/h], \( S_i(k) \) is the supply of cell \( i \) [veh/h]. Other parameters are: \( \beta_i \in [0, 1] \) is the split ratio, \( F_i \) the capacity [veh/h], \( \bar{\rho}_i \) the jam density [veh/km], \( \psi_i \) the congestion wave speed [km/h], \( \psi_f \) the free flow speed [km/h], \( p_i \in [0, 1] \) the priority of on-ramp flow with respect to mainstream flow of cell \( i \).

\[
\phi_i(k) = (1 - \beta_i) l_{i-1}(k) \rho_i(k) \\
\phi_i(k) = \phi_i(k) + r_i(k) \\
\phi_i(k) = \phi_i(k) + s_i(k) \\
s_i(k) = \frac{\beta_i}{1 - \beta_i} \phi_i(k) + 1
\]

(1)

(2)

(3)

(4)

(5)

Fig. 1. Sketch of the division of the freeway into cells.

The CTM is given by the state equations for the traffic density \( \rho_i(k) \) and the queue length \( l_i(k) \), \( i = 1, \ldots, N \), \( k = 1, \ldots, K \)

\[
\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i} \left[ \Phi_i^+(k) - \Phi_i^-(k) \right]
\]

\[
l_i(k+1) = l_i(k) + T [d_i(k) - r_i(k)]
\]

where

\[
\Phi_i^+(k) = \phi_i(k) + r_i(k)
\]

\[
\Phi_i^-(k) = \phi_i(k) + s_i(k)
\]

\[
s_i(k) = \frac{\beta_i}{1 - \beta_i} \phi_i(k) + 1
\]

(6)

(7)

(8)

Referring to cell \( i \), it is useful to define the demand of cell \( i - 1 \) and the supply of cell \( i \), as follows

\[
D_{i-1}(k) = \min \left\{ (1 - \beta_i) l_{i-1}(k) \rho_i(k), F_{i-1} \right\}
\]

\[
S_i(k) = \min \left\{ w_i (\bar{\rho}_i - \rho_i(k)), F_i \right\}
\]

(6)

(7)

According to Daganzo’s merge connection model, the mainstream and on-ramp flows are obtained as

\[
\phi_i(k) = D_{i-1}(k) + d_i(k) + \frac{l_i(k)}{T} \\
\phi_i(k) = \phi_i(k) + r_i(k) = d_i(k) + \frac{l_i(k)}{T}
\]

else

\[
\phi_i(k) = \min \left\{ D_{i-1}(k), S_i(k) - d_i(k) - \frac{l_i(k)}{T} \right\}
\]

\[
(1 - p_i) S_i(k) \}
\]

\[
r_i(k) = \min \left\{ d_i(k) + \frac{l_i(k)}{T}, S_i(k) - D_{i-1}(k), p_i S_i(k) \right\}
\]

(6)

(7)

(8)

where the function \( \text{mid} \) returns the middle value.

This model is nonlinear, due to the presence of the \( \min \) function in (6) and (7) and the logical conditions and \( \text{mid} \) function in (8). Anyway, it can be viewed as a mixed logical dynamical system, i.e. it can be described by linear equations and inequalities involving both continuous and binary variables, according to [11]. Specifically, in the CTM, each nonlinear function is replaced by equalities and inequalities which relate the original variables with some new auxiliary variables that are either binary or continuous. Referring to our MLD formulation for the CTM model, let us denote all the auxiliary binary variables as \( \delta_j(k) \), \( j = 1, \ldots, \Delta, k = 1, \ldots, K \).
The FHOCP to be solved at the generic time step \( k \) can then be summarized as follows. Given the initial conditions on the density and the queue length \( \rho_i(k) \) and \( l_i(k) \), \( i = 1, \ldots, N \), the demand of the cell before the first one \( D_0(h) \), \( h = k, \ldots, k + K_p - 1 \), the supply of the cell after the last one \( S_{N+1}(h) \), \( h = k, \ldots, k + K_p - 1 \), the on-ramp demands \( d_i(h) \), \( i = 1, \ldots, N \), \( h = k, \ldots, k + K_p - 1 \), find the optimal control variables \( r_i(h) \), \( i = 1, \ldots, N \), \( h = k, \ldots, k + K_p - 1 \), minimizing the cost function corresponding to the quadratic deviation of the system variables from their equilibrium values.

The constraints of the problem include all the equalities and inequalities of the CTM in MLD form, define lower and upper bounds for the decision variables and impose that the state variables at time \( k + K_p \) assume the equilibrium value.

The FHOCP is a mixed-integer quadratic programming problem. At each time step \( k \), the optimal control sequence \( U^o(k) \) = \{ \( y^o(h) \), \( h = k, \ldots, k + K_p - 1 \) \}, is derived, with \( \sum h^o(h) \triangleq \text{col} [r_i^o(h), i = 1, \ldots, N] \). In a classical MPC scheme, only the first element of the control sequence \( \sum h^o(k) \) is applied and the overall procedure is repeated. The derived control law is denoted as the mixed-integer predictive control (MIPC) law. In general, let us also denote with \( x(h|k) \) the value of quantity \( x \) for the \( h \)-th time interval computed by solving the \( k \)-th FHOCP.

To solve the FHOCP, it is necessary to know some data such as the initial conditions, the demand of the cell before the first one, the supply of the cell after the last one and the on-ramp demands. If we suppose that such quantities are known, it is possible to exploit the convergence result reported in [11] and, then, to guarantee that the proposed control scheme makes the considered equilibrium state stable (again by following the definition provided in [11]), as stated in the following result.

**Theorem 1:** Let \( \rho_i^k, l_i^k, i = 1, \ldots, N \), be the equilibrium states for the considered system (i.e., the CTM in the MLD form) subject to the on-ramp volumes \( r_i^k, i = 1, \ldots, N \). If, at each time step \( k \), all the problem data are known and are such that a feasible solution of the FHOCP exists, then, the MIPC law makes the equilibrium state stable.

It is worth noting that it is not very realistic to suppose that the problem data of the FHOCP are known or exactly estimated. On the contrary, it seems more proper to consider such quantities as bounded disturbances affecting the system behaviour. Moreover, in the MLD formulation such disturbances are always additive. To this end, the input-to-state practical stability (ISpS) property of the proposed MPC scheme is analysed. In particular, it is possible to verify that the optimal cost function resulting from the solution of the FHOCP in an ISpS Lyapunov function for the system, since it verifies the conditions defined in [22], leading to the following result.

**Theorem 2:** Let \( \rho_i^k, l_i^k, i = 1, \ldots, N \), be the equilibrium states for the considered system subject to the on-ramp volumes \( r_i^k, i = 1, \ldots, N \). If, at each time step \( k \), the initial conditions on the density and the queue length are known and are such that a feasible solution of the FHOCP exists, then the MIPC law makes the system ISpS.

### III. EVENT-TRIGGERED CONTROL

The MPC scheme proposed in the previous section is computationally demanding since the FHOCP problem must be solved at each time step \( k \). To overcome this drawback, an event-triggered control scheme is proposed. According to this scheme, the control law is not updated at each time step but only when a given set of conditions is no more satisfied; let us call such set of conditions as **triggering rule** and the time intervals in which the triggering rule is not met as **triggering time steps**.

Hence, the control law can be defined as follows. At \( k = 0 \), the FHOCP is solved determining the control sequence \( U^o(0) \) and applying it in the subsequent time steps. At each time step \( k, k = 1, \ldots, K - 1 \), the triggering rule is verified; if it is fulfilled, the control action already available is applied, otherwise time interval \( k \) becomes a triggering time step, the defined FHOCP is solved and a new optimal control sequence \( U^o(k) \) is derived. The values of the control actions composing such a control sequence are applied to the system until the next triggering time step.

A first consideration about the triggering rule is that it is not verified whenever the real value of at least one variable \( \delta_j(h|k) \) is different from the predicted one (corresponding to an operating condition different from the predicted one, for instance a cell that from the free-flow case becomes congested). Moreover, the triggering condition is defined by finding in each time step an upper bound on the distance between the real state trajectory derived by using new measurements and the predicted one. At this purpose, let us denote with \( \hat{\rho}_i(h|k) \) and \( \hat{l}_i(h|k) \), respectively, the predicted values for the traffic density and the queue length found by solving the FHOCP at time interval \( h \). Then, let us define the density error \( e_i^\rho(h|k) \) and the queue length error \( e_i^l(h|k) \), \( i = 1, \ldots, N, k = 0, \ldots, K - 1, h = k, \ldots, k + K_p - 1 \), as

\[
e_i^\rho(h|k) = \frac{|\rho_i(h) - \hat{\rho}_i(h|k)|}{\hat{\rho}_i(h|k)}
\]

\[
e_i^l(h|k) = \frac{|l_i(h) - \hat{l}_i(h|k)|}{\hat{l}_i(h|k)}
\]

The above errors can be bounded, respectively with threshold values \( \epsilon^\rho \) and \( \epsilon^l \). The set of conditions to be verified at a generic time interval \( h \) with respect to the last triggering time step \( h > k \) is defined as **Triggering rule**

\[
\delta_j(h) = \tilde{\delta}_j(h|k) \quad \forall j = 1, \ldots, \Delta \quad (10)
\]

\[
e_i^\rho(h|k) \leq \epsilon^\rho \quad \forall i = 1, \ldots, N \quad (11)
\]

\[
e_i^l(h|k) \leq \epsilon^l \quad \forall i = 1, \ldots, N \quad (12)
\]

Then, the FHOCP is solved only in time steps in which at least one condition in the above set is violated.

The proposed control law guarantees that the deviations between the predicted system state and the real one are lower than the predefined thresholds. This property, together with
the fact that the MIPC law guarantees the ISpS stability of the system, provides a good performance of the event-triggered control scheme whose stabilizing properties are being analyzed.

IV. SIMULATION RESULTS

A significant part of our research activity has been devoted, within the framework of the HYCON2 Network of Excellence, to assess the performance of the proposed traffic control approach. To this end, a case study based on the Grenoble South Ring, the so called “Roçade sud”, has been considered and carefully analysed. More specifically, in this section we refer to a segment including exit number 4, which corresponds to Saint-Martin-d’Hères, with a total length of about 3 km. This section is composed of twelve CTM cells, and includes a single off-ramp and a single on-ramp, located approximatively in the second part of the section. A detailed view of the ramps is shown in Fig. 2. In 2012, a process for improving the actual traffic control system has started, along with the installation of new sensors and actuators. Wireless magnetic sensors are employed for the measurement of the vehicle flows, while traffic lights are installed at the on-ramps in order to perform ramp metering. Fig. 3 shows the subdivision into cells of the considered freeway segment. The wireless sensors are positioned between cells, and on the ramps. The traffic light is on the on-ramp. Table I reports a description of the cells, along with their length.

Since the physical system is still under development, a model of the freeway, using a commercial microscopic traffic simulator (Aimsun v 7.0, developed by TSS-Transport Simulation Systems), has been implemented by the researchers of INRIA involved in HYCON2, relying on the data of the actual traffic monitoring system. The commercial simulation tool allows one to generate different scenarios upon which it is possible to test control algorithms.

In order to evaluate the performance of our control scheme, we compare the results obtained by applying the proposed control scheme to the Grenoble freeway traffic model, with those obtained on the same system by using a standard MPC approach. We use the nonlinear CTM as freeway model, while we use the CTM in the aforementioned MLD form as prediction model for the MPC algorithm. The CTM has been identified using the data generated by the microscopic traffic model and following the identification approach introduced in [23]. The sample time is 8 s.

As boundary conditions, we adopt the mainstream and on-ramp demands, along with the supply of the freeway cell following the segment we consider. In this way, we assume that congestion can be caused also by the conditions of the freeway after cell C27, the supply of which varies over time.

The performance assessment is based on the following performance indexes: the Computational Burden Index (CBI) and the Control Performance Index (CPI).

The CBI is a simplified measure of the computational burden of the MPC algorithm, defined as follows

\[
\text{CBI}(k) = \sum_{h=1}^{k} N(h)
\]

where \(N(\cdot)\) is the length of the optimization horizon. Note that \(N(\cdot)\) is defined as a piecewise constant function that is equal to zero at the time instants in which the control law is not updated, and to 120 s otherwise. To define the CBI, we are practically assuming that the computational burden of the optimization problem is mainly dependent on the length of the optimization horizon. Although more complex index functions could be used for this evaluation, (13) seems reliable enough to carry on comparisons in the considered applicative case.

The CPI is an index of the overall control performance,
TABLE II
EQUILIBRIUM STATE COMPONENTS USED AS SET POINT.

<table>
<thead>
<tr>
<th>Cell</th>
<th>( \rho^e )</th>
<th>( l^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C16</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>C17</td>
<td>59.44</td>
<td>0</td>
</tr>
<tr>
<td>C18</td>
<td>59.73</td>
<td>0</td>
</tr>
<tr>
<td>C19</td>
<td>60.44</td>
<td>0</td>
</tr>
<tr>
<td>C20</td>
<td>61.70</td>
<td>0</td>
</tr>
<tr>
<td>C21</td>
<td>64.06</td>
<td>0</td>
</tr>
<tr>
<td>C22</td>
<td>67.09</td>
<td>0</td>
</tr>
<tr>
<td>C23</td>
<td>57.92</td>
<td>0</td>
</tr>
<tr>
<td>C24</td>
<td>66.08</td>
<td>0</td>
</tr>
<tr>
<td>C25</td>
<td>71.98</td>
<td>0</td>
</tr>
<tr>
<td>C26</td>
<td>64.96</td>
<td>0</td>
</tr>
<tr>
<td>C27</td>
<td>64.77</td>
<td>0</td>
</tr>
</tbody>
</table>

defined as follows

\[
\text{CPI} = \sqrt{\frac{1}{K_s} \sum_{h=1}^{K_s} J(h)^2}
\]  \hspace{1cm} (14)

where \( K_s \) is the number of simulation steps and \( J(\cdot) \) is the objective function of the MPC algorithm. In this paper, we adopt the CPI, i.e. the normalized CPI, defined as follows

\[
\overline{\text{CPI}} = 100 \cdot \frac{\text{CPI}}{\max_{i=1,2} \{\text{CPI}_i\}}
\]  \hspace{1cm} (15)

where \( \max_{i=1,2} \{\text{CPI}_i\} \) is the maximum between the CPI of the standard MPC with receding horizon and the CPI of the proposed event-triggered MPC approach.

The components of the equilibrium point used as set point in our simulation are reported in Table II, considering a density of 60 vehicles per kilometer in the first cell.

The thresholds used for the triggering conditions (11) and (12) of the triggering rule are \( \epsilon^\rho = \epsilon^l = 0.1 \). Note that these thresholds have been determined experimentally, but future work will be devoted to determine them taking into account considerations about the stability of the controlled system.

The open-loop simulation is shown in Fig. 4. The results produced by applying a standard MPC algorithm with receding horizon to the test case are shown in Fig. 5. Such a figure shows that the control algorithm is able to regularize the traffic conditions (note that all the cells have a controlled density lower than their critical density, which is about 120 veh/km).

Fig. 6 shows the results obtained by applying the proposed approach.

Fig. 7. Comparison of the CBI obtained with the standard MPC with receding horizon and the event-triggered MPC.
event-triggered MPC algorithm which achieves satisfactory performances. The main advantage of the event-triggered MPC scheme stands in requiring a lower computational load with respect to the standard MPC algorithm, as shown in Fig. 7, while preventing an excessive degradation (only the 12%) of the CPI, as illustrated in Fig. 8.

V. CONCLUSIONS

This paper presents a control framework for freeway systems based on a MPC scheme, adopting ramp metering as control measure. The proposed finite-horizon optimal control problem in the MPC scheme is stated as a mixed-integer quadratic programming problem where the CTM in MLD form is adopted for the prediction. The resulting control law makes the system stable in the case without disturbances and input-to-state practically stable when bounded perturbations (relevant to traffic data) affect the system behaviour. In order to reduce the computational load necessary to solve the FHOCP, an event-triggered version of the proposed scheme is derived, in which the control law is updated only when the error between the real system evolution and the predicted one exceeds a pre-specified threshold. In the present paper, the performance of the proposed traffic control approach is assessed considering real data regarding the Grenoble freeway South Ring, the so-called “Roçade sud”. To this end, a set of performance indexes is introduced, in order to evaluate the computational burden and the control efficacy. Numerical results about this case study are reported in the paper and analysed.

REFERENCES