Load Sharing Optimization of Parallel Compressors

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Abstract—This paper deals with the problem of optimizing the load distribution and the on/off switching sequences of parallel compressor units in natural gas pipeline compression stations. Natural gas pipelines are used to deliver gas from production sources to customers. Compression stations on these pipelines are generally composed of the interconnection of several compressors units and the aim of the load sharing optimization is to operate these units in an energy efficient way while continuously satisfying the varying demand of gas flow. As the gas demand changes start-up and shut-down of compressor units might be required and the impact of these switching events on the expected lifetime of the compressors also needs to be taken into consideration. Two types of operational constraints are involved: continuous constraints concerning the conservation equations of mass and flow and combinatorial constraints concerning the possibility of changing the number of active compressors. Overall the optimization problem can be formulated as a mixed integer nonlinear programming (MINLP) problem. In this article the generic optimization problem setup and appropriate techniques for the solution are presented. Compared to other traditional strategies such as equal load balancing or equal distance operation from surge, the MINLP approach has shown considerable improvement in terms of energy savings.

I. INTRODUCTION

In the last few years the evolution of gas boosting stations has been characterized by the replacement of fixed speed drivers such as single axis turbines and direct on-line (DOL) motors with variable speed drivers (VSD) such as double axis turbines and electrical variable frequency drives (VFD), [1]. At the time of fixed speed drivers the management of the networks was performed by acting on the number of active compressors, while the fine regulation of the throughput was achieved by control strategies such as throttling or recirculation of the compressed gas. With the introduction of VFD, the station control can be achieved by acting on the number of active compressors and on the speed of the associated drives. Since modern day boosting stations can contain both VFD on some units and double axis gas turbines on others, determining the best choice for the number of operating units and the speed of the corresponding drivers leads to a complex optimization problem. The main objective is to minimize the energy consumption for a given compression station set point. Constraints arise from the operational requirements of the compressors such as limits on the rotating speeds and safety criteria such as avoiding of surge and stonewall conditions. Boosting stations on natural gas pipelines typically consist of several compressor trains in parallel where each train is composed of a compressor unit, control valves, tanks, and coolers as illustrated in Fig. 1. Therefore, the decision variables in such systems are the set of active compressors in the parallel arrangement and their respective speed of rotation.

The optimization strategy can play an important role in the automation hierarchy of compressor network operation as depicted in Fig. 2. In this figure the dispatcher is the entity which manages the whole network by deciding on the daily production plan as well as the operating targets for the individual stations. The process control level at each station receives the production targets from the dispatcher and operates the station through various actuators such as motors, turbines and valves based on feedback control loops and heuristic rules for allocating the flow into the different compressors. The optimization systems are stepping into the compression station automation schemes as an additional layer between the dispatcher and process control in order to improve the station performance by creating asymmetries in the load distribution of parallel units.
In the literature various approaches for the optimization of compression networks can be found. Nguyen et al. [2] consider the problem of active compressor selection in natural gas pipeline operations. This selection deals with the choice of the number of operating compressors and no optimal set-points for compressor speeds are computed. Abbaspour et al. [3] derive a detailed mathematical model of compressor networks, and use the model as a basis for solving a nonlinear programming (NLP) problem. The decision variables are constituted by the compressor steady-state speeds and the objective function to be minimized is the total fuel consumption. The NLP problem is solved numerically by a sequential unconstrained minimization technique. Moritz, et al. [4] consider a mixed integer linear programming (MILP) approach for the transient optimization of compression networks, where the nonlinearities are approximated by means of piece-wise linear functions.

The present paper describes one possible optimization approach for compressor load sharing which is based on the solution of a mixed integer nonlinear programming (MINLP) problem [5]. The major difference compared to the current industry practice of having off-line schedules, is to compute the optimal load distribution together with the optimal compressor on/off states given a station set point considering all possible combinations. Moreover, the model used for the optimization is changing using online estimation of compressor map and efficiency parameters. This approach provides the most flexible framework for compressor optimization by allowing any nonlinear function to be used in defining compressor operation, discontinuous search spaces, and binary decision variables for activating and deactivating compressor units. The disadvantage is that it leads to a computationally difficult problem and finding the global optimum is generally very challenging.

A recycle line connects the compressor outlet with the compressor inlet and the flow through the recycle line is regulated by the anti-surge valve. The compressor model at steady-state is based on the operating characteristic curves of the machinery. Such maps, as the one depicted in Fig. 4, which are typically provided by the compressor manufacturer, relate the pressure ratio (i.e. the ratio between outlet and inlet pressure of the compressor) with compressor speed and flow. They also give an indication of the calculated compressor efficiency. The mass flow rate and the angular speed have been evaluated at standard inlet conditions of the compressor pressure and temperature together with the average molecular weight of the fluid also furnished by the compressor manufacturer. The mass flow rate \( \dot{m}_c \) and the angular speed \( \omega \) are intended to be substituted by the flow number and blade Mach number, which are invariant to the inlet conditions [7].

II. STEADY-STATE MODEL OF THE PLANT FOCUSED ON ENERGY CONSUMPTION

The major operating cost of a compressor station is constituted by either the energy consumption of motors in case where electrical VSD are present or by the fuel consumption if gas turbine drivers are used. Electrical motors require an external electrical power supply, while gas turbines typically consume part of the compressed gas. Accordingly, the key element for estimating the energy consumed by the station is the mathematical model of the compressor. For the present study, the model developed by Cortinovis et al. [6] for a single-compressor plant is utilized and extended to address parallel compressors in natural gas boosting stations. As depicted in Fig. 3, the core element is the compressor, surrounded by piping and valves. The main line starts with a suction side valve which is used to regulate the suction pressure followed by the compressor inlet header.

![Figure 3. Compressor system considered for steady-state modeling.](image-url)
The compressor map also provides information on operational limits. On the left side of the map a boundary called the surge line is defined; this line represents the limit on the minimum flow that can be elaborated by the compressor for a given head or pressure ratio. If this limit is exceeded then a flow instability called surge occurs, which can cause thermal and mechanical stress to compressor blades potentially leading to damages and eventually also to machine failure. On the right side of the map another limit (called choke or stonewall) is underlined; it is the line which corresponds to the maximum flow that can be reached by the compressor depending on the aerodynamic characteristics of the discharge piping. At the top and at the bottom of the map two mechanical limits are defined: the maximum operating speed (MOS) and the minimum operating speed (mos). The region within the described limits represents the subset of all feasible operating points for the compressor in examination. Starting from experimental data or estimates of the manufacturer i.e. the compressor map speed and efficiency can be defined for every feasible operating point by means of polynomial approximations. To fit the speed and efficiency data, a second degree polynomial was used:

\[ z = \alpha_0 + \alpha_1 p + \alpha_2 p^2 + \alpha_3 p_{\text{min}} + \alpha_4 p_{\text{min}}^2 + \alpha_5 p_{\text{max}} + \alpha_6 p_{\text{max}}^2 \] (1)

where \( z \) can be either the compressor speed \( \omega \) or the efficiency \( \eta \) of the working point and \( \alpha_j \) are the polynomial coefficients. Eq. 2 represents the role of the efficiency in the calculation of the shaft power as considered in [3]:

\[ \text{Pow}_{\text{shaft}} = \frac{y_p}{\eta_p} m_c \] (2)

where \( y_p \) is the polytropic head, given by:

\[ y_p = \frac{Z_m R T_m}{MW} \frac{n_c}{n_s - 1} \left( \frac{P_2}{P_1} \right)^{\frac{n_c - 1}{n_s}} - 1 \] (3)

while \( \eta_p \) and \( n_s \) are the polytropic efficiency and polytropic exponent, respectively. \( MW \) is the molecular weight of the gas mixture, \( Z_m \) the inlet compressibility factor, \( m_c \) the mass flow through the compressor, \( T_m \) the suction temperature, \( P_1 \) the suction pressure and \( P_2 \) the discharge pressure. In case of compressors driven by electrical motors, assuming an efficiency \( \eta_{\text{elect}} \), the power consumption is given by:

\[ \text{Pow}_{\text{electrical}} = \frac{\text{Pow}_{\text{shaft}}}{\eta_{\text{elect}}} \] (4)

while in case of a gas turbine driven compressors, assuming a GT efficiency \( \eta_{\text{GT}} \) and a lower heating value \( LHV \), the fuel consumption is given by the following equation as described in detail in [3]:

\[ \dot{m}_f = \frac{\text{Pow}_{\text{shaft}}}{LHV \eta_{\text{GT}}} \] (5)

The compressor station operating point is defined by using a static model of the discharge load, which has been modeled as a gas pipeline based on partial differential equations for flow and pressure taken from [4]. The pipeline model is linked to the rest of the model equations through the pressure loss as described by equation (6), where \( P_{\text{out}} \) is the compression discharge pressure and \( P_{\text{dem}} \) is the pressure at the end of the pipeline, for instance the pressure inside a gas storage tank near a city.

\[ \frac{P_{\text{out}} - P_{\text{dem}}}{L} = \frac{\lambda P T m_c Z(P_{\text{out}})}{2DA^2 T_0 h_0 \rho_i P_{\text{out}}} \] (6)

The compression discharge pressure is given by a spatial discretization of the momentum equation across the length of the pipe, as presented in [4]. The pressure loss is mainly due to friction losses which are a quadratic function of the total mass flow. Summarizing, the optimization problem has been formulated by introducing the static model of the compressor station focused on the energy consumption and the static model of the pipeline used for defining the discharge conditions. It should be noted that the partial differential equations for the pipeline are used only once to determine the system resistance for a given total flow. It is not necessary to solve them at every iteration of the optimization problem.

III. OPTIMIZATION ALGORITHM AND PARAMETER IDENTIFICATION

A. OPTIMIZATION ALGORITHM

This problem can be classified as an MINLP problem since it contains nonlinear functions in both the objective function and the constraints and also combinatorial aspects [4]. The general form of an MINLP can be stated as follows:

\[
\begin{align*}
\min & \quad f(x, y) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
& \quad x \in X \cap Z^n, y \in Y
\end{align*}
\]

where \( x \) and \( y \) are the decision variables of the optimization problem, \( X \) and \( Y \) are polyhedral subsets of \( R^n \) and \( R^p \) respectively. The functions \( f : X \times Y \rightarrow R \) and \( g : X \times Y \rightarrow R^p \) represent the cost function and constraints of the optimization problem. The optimization problem solver used in this case study is BONMIN which is an open
source code for solving general MINLP problems. More information on BONMIN and on the interface used in this study can be found in [8] and [9]. There are several algorithms that can be used with BONMIN; for the present case the approach based on branch and bound (BB) was considered. In the particular arrangement used in this case study the nonlinear solver IPOPT is used for the solution of the continuous sub problems and MUMPS for solving the linear system of equations. A reference of IPOPT can be found in [10]. The optimization formulation contains in addition to the energy costs also costs for switching on or off the compressors. For a given operating point the switching costs help to avoid the starting up of a new compressor until a certain energy saving threshold is exceeded. The switching costs included in the objective function are given by:

\[ C_{\text{START\_UP}}y_i(1-x_i) + C_{\text{SHUT\_DOWN}}x_i(1-y_i) \]  

(7)

where \( y_i \) is a decision variable that indicates if the compressor \( i \) should be switched on or off in the current operating condition and \( x_i \) is the current on or off operating state of the same compressor, while \( C_{\text{START\_UP}} \) and \( C_{\text{SHUT\_DOWN}} \) are constants that represent the start-up and shutdown costs respectively. In a similar way constraints corresponding to active or inactive compressors are handled by using the binary variable \( y_i \). As an example the surge constraint is modified as follows:

\[ s_i \hat{m}_{\text{c},i} + s_0 + (1-y_i) \hat{P}_{\text{surge}} \geq P_{\text{ratio}} \]  

(8)

where \( s_i \) and \( s_0 \) define the a surge control line depending on the compressor flow \( \hat{m}_{\text{c},i} \) and pressure ratio \( P_{\text{ratio}} \) over the compressor map, while \( \hat{P}_{\text{surge}} \) is an appropriate constant value used to satisfy the inequality also when the compressor has not been selected to operate.

Finally, a note on the performance of the solver used is in order. BONMIN is a local solver and thus it cannot guarantee global optimality of the solution. To counter this limitation, a warm starting technique is utilized along with an approach where the optimization problem is solved using various starting points randomly distributed over the search space. This method results in a better local solution even for a small set of starting points. Typical execution times with 5 to 10 initial guesses were 90.5 to 177 seconds on an Intel Core i7 processor at 2.2 GHz running on Windows 7.

B. PARAMETER ESTIMATION

As introduced in Eq. 1, the optimization uses fitted approximations for compressor maps and efficiency maps. These approximations can become inaccurate due to variations in gas properties, disturbances, equipment damage, fouling or decreased life-time of the equipment. This can be avoided by online-parameter estimation using the available measurements for each individual compressor. Without online estimation, the optimization would lead to steady-state errors which would have to be corrected by the process controller in a suboptimal way. In the following the online estimation of the map parameters is discussed. Since speed and efficiency can be approximated by polynomials, the problem of estimating the coefficients is a linear regression problem, which can be solved via least squares methods [11].

Defining the compressor characteristics as:

\[ \hat{y}(i) = \phi(i)\theta + \vartheta \]  

(9)

where \( \theta \) and \( \phi \) are vectors representing the polynomial coefficients and the actual measurements respectively, \( y \) is a measure of speed or efficiency depending on which set of polynomial coefficients are being estimated, while \( \hat{y} \) is its estimated value. \( \theta_0 \) represents the initial guess for the parameters and \( S_0 \) its relative weight. The solution of the error minimization problem is given by:

\[ \hat{\theta} = \left( S_0 + \sum_{i=1}^{N} \phi(i-1)\phi(i-1)^T \right)^{-1} \left[ \sum_{i=1}^{N} \phi(i-1)y(i) \right] \]  

(10)

IV. RESULTS

A. STEADY STATE OPTIMIZATION

In the following, the optimization results for the case of a gas boosting station composed of five compressors driven by variable speed electrical motors and discharging into a 100 km long pipe with a diameter of 1.7 m, are presented. Table I summarizes the main parameters used for the simulations. The problem size for the case study was 5 variables and 35 nonlinear constraints, 37 linear constraints and 60 bound constraints.

<table>
<thead>
<tr>
<th>Table I</th>
<th>MAIN PARAMETERS USED IN THE SIMULATION CASE STUDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suction Pressure (bar)</td>
<td>13.2</td>
</tr>
<tr>
<td>Suction Temperature (°C)</td>
<td>59.3</td>
</tr>
<tr>
<td>Nominal Weight Flow Wet (kg/sec)</td>
<td>68.2</td>
</tr>
<tr>
<td>Molecular Weight (Kg/Kmol)</td>
<td>19.62</td>
</tr>
<tr>
<td>Nominal kW Required (at compressor coupling)</td>
<td>13150</td>
</tr>
<tr>
<td>Nominal Polytropic Efficiency (%)</td>
<td>85%</td>
</tr>
<tr>
<td>Rated Speed (rpm)</td>
<td>8370</td>
</tr>
<tr>
<td>Maximum Speed (rpm)</td>
<td>8789</td>
</tr>
</tbody>
</table>

Speed and efficiency polynomials for one of the compressors are illustrated in Fig. 5. The optimization results are presented in Fig. 6 as a schedule for the full range of station mass flows that the compressor station can cover. In this figure, the individual mass flows are shown on the y axis as a function of the total station mass flow on the x axis. These plots indicate that the optimizer recognizes which compressors are more efficient for a certain interval of the total mass flow. In Fig. 7 the total power consumption of the boosting station given by the optimization strategy is compared with the one achievable with the “equal load distribution” approach. In the simulation for this traditional
The compressors have been activated in the same sequence given by the MINLP optimization, while the activation point differs between the two methods. The equal load distribution approach brings a new compressor into operation when the efficiencies of the actual operating compressors fall below a predefined threshold, while with the MINLP optimizer calculates the actual optimal point to activate a new compressor.

The comparison of the total power consumption given by the two approaches shows that the MINLP solution grants significant savings in energy consumption. As shown in Fig. 8, for the range of total mass flow considered, the percentage of saved power with the MINLP approach can reach up to 16% compared to the conventional approach. It is also intuitive that the room for power saving is limited when the station operates at low flows (most efficient compressor at low flow running alone with/without recycle) and large flows (all compressors running close to MOS without recycle).

**B. DYNAMIC SIMULATION - PARAMETER ESTIMATION AND CONTROL**

As mentioned earlier, the load sharing optimization procedure assumes that reliable models are available which describe the performance of the compressors. When this simplified assumption does not hold (i.e. due to fitting errors or due to the fact that the original performance maps are no longer valid), the optimization procedure can lead to the computation of compressor speed values that are suboptimal thus providing an inaccurate flow distribution and mismatch to the desired station flow. This mismatch is eliminated by using a process controller which corrects the errors between the measured and desired station flows. In the simulations...
such a controller is implemented along with a parameter estimation procedure that estimates the parameters describing the compressor maps of efficiency and speed. The controller then uses the deviation in flows to correct the optimized speeds to achieve the desired station flow. On the other hand, as the parameters of the compressor maps become more accurate, the corrective action of the process controller becomes negligible. Fig. 9 shows a schematic of the proposed load-sharing and map-adaptation procedure.

The load-sharing-optimization parameter-estimation/map-adaptation procedure has been tested in a dynamic simulation by introducing a mismatch in the second compressor between the real compressor characteristics and the one used by the optimization and estimation routines. A trajectory profile of the required total mass flow has been followed and the routine for the parameter identification has been applied to the acquired data. In Fig. 10 and 11 the real maps (color lines) are compared with the estimated maps (red lines) of efficiency and speed curves. The trajectory imposed on the maps represents the operating points followed by the compressor during the simulation. It is important to highlight that the estimation is most accurate in the areas of the compressor map that have been explored. This is expected since the parameter estimation is (locally) valid near the point of approximation.

**Figure 9.** Proposed load-sharing-optimization map-adaptation solution for parallel compressor boosting stations.

**Figure 10.** A comparison between the real and the estimated efficiency maps at the end of a simulation run covering an operating range indicated by the solid trajectory.

**Figure 11.** A comparison between the real and the estimated speed map at the end of a simulation run covering an operating range indicated by the solid trajectory.

## V. Conclusion

A framework for the optimization of parallel compressor operation in gas compression stations is presented. The proposed strategy is based on formulating and solving an MINLP problem, leading to optimal on/off switching sequences of the units and the corresponding optimal load distribution. Compared to conventional approaches, the proposed strategy leads to significant energy savings. Since the performance characteristics of the compressors deviate from their normal values due to wear, fouling or other factors, a procedure is presented for estimating the actual map parameters online. The updated maps are then used to change the model used in the optimization algorithm.

**References**


