Model Predictive Control of a HVAC System Based on the LoLiMoT Algorithm

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Abstract—In this paper a multi-variable control for the simultaneous control of the air temperature and the air humidity of an industrial heating ventilating and air conditioning (HVAC) system is presented. For the multi-input-multi-output (MIMO) control the model predictive control (MPC) method is applied. To model the system local linear neuro fuzzy models (LLNFM) are used and computed by the so-called Local Linear Model Tree (LOLIMOT) algorithm. The main focus of this work is the communication between the MPC and the LLNFMs and the application on a real world test plant. The proposed method is compared to a linear MPC scheme and a conventional PI control strategy.

I. INTRODUCTION

Industrial heating ventilating and air conditioning (HVAC) systems are used to adjust the air condition (temperature, humidity, pressure) in rooms and buildings to desired values. Besides the application of standard PI(D) controllers, see e.g. [1]–[4], extensive research on advanced control strategies for HVAC systems is performed [5], [6]. The actuating signals of HVAC systems are typically subject to constraints (e.g. valve position and slew rate) and the controlled variables are coupled (e.g. if the air temperature increases the relative air humidity decreases). To take into account such system properties, model based control methods, especially the model predictive control (MPC) method will be used. This strategy is established in many industries like e.g. chemicals industry, process control and building automation [7]–[10]. In addition to numerous works concerning the modeling and control of HVAC systems [11]–[14], the authors successfully applied MPC to a single-input-single-output (SISO) system for temperature control via a cooling coil [15]. However, by the use of a linear MPC the controller performance strongly depends on the model’s operating point. To counteract this problem and to improve the performance over the whole operating range an extension by means of a local linear neuro fuzzy model (LLNFM) is made [16]–[19]. Further reports dealing with the application of non-linear systems via multiple model approaches for control applications can be found in [20]–[22]. This paper deals with the application of a linear MPC in combination with LLNFMs applied to an industrial HVAC system. The paper is structured as follows: In Section II a general description of the pilot plant as well as a detailed description of the problem configuration is given. Section III describes the modeling and control design methods and Section IV shows identification and modeling results. Section V gives a comparison of the proposed control concept versus a PI controller and a linear MPC in experiment. Section VI concludes the paper and outlines the future work.

II. PLANT

A. General description

The methods discussed in this work are verified on a real world pilot plant. A photo of the plant is shown in Fig. 1. Air transportation is achieved by means of a fan powered by an asynchronous motor with a maximum speed of 2840 rpm. Two heating coils are installed to heat the air. A water-glycol mixture, with higher temperature than the air passing the heating coils is used to increase the air temperature. Two cooling coils are used to decrease the air temperature as well. In order to supply the cooling coils with cooled fluid, a water-chiller cooling 2000 liters of a water-glycol mixture stored in a buffer tank is installed. Furthermore there is a steam humidifier installed to increase the air humidity by evaporating water. The water vapor is then mixed to the air by a steam lance. Additionally, to protect the HVAC system from dust and airborne particles, two filters are installed. Finally, five air dampers are available to decide in which mode the plant is operated. There are different modes of operation indicated by white arrows in Fig. 1. Either the room in which the plant is installed (labeled ”test room” in Fig. 1) and/or the adjacent factory building can be supplied with conditioned air using recirculating air and/or fresh air from outside. The test plant is equipped with various sensors for temperature and humidity. The mass flows of the air and fluids can be measured as well.

According to the problem configuration discussed in Section II-B, the cooling coil 2 is used to cool and dehumidify the air, heating coil 2 is used for reheating.

B. Problem Configuration

The system to be controlled is shown in Fig. 2. It is a series connection consisting of a cooling coil and a heating coil. A constant air mass flow of \( \dot{m}_a = 1.5 \, \text{kg/s} \) is assumed. The air is dehumidified by the cooling coil (cooling below the dew point temperature) and reheated by the heating coil to the desired temperature. The power of the cooling coil and the
heating coil can be adjusted by valves. For the cooling coil the water mass flow is varied, whereas for the heating coil the mixing ratio between return water and hot water is varied. The valve positions are the actuating signals \( u_1 \) and \( u_2 \). The control variables are the temperature \( \vartheta_{\text{in}} \) and the relative humidity \( \varphi_{\text{in}} \) of the outlet air. The inlet air temperature \( \vartheta_{\text{in}} \) as well as the water supply temperature are regarded as measurable disturbances. The delay times from the inputs to the outputs are considered in the proposed control strategy.

III. METHOD

A. Local Linear Neuro Fuzzy Model (LLNFM)

To model the system described above, the so-called Local Linear Model Tree (LOLIMOT) algorithm [17] is used. Based on measurements, the essential feature of this method is to split the input and output range of the system into \( M \) partitions and to identify one local linear model for each generated partition of the form

\[
y_k = w_0 + \sum_{i=1}^{n} \left( w_i^y y_{k-i} + \sum_{j=1}^{m} w_{j,i}^y u_{j,k-i} \right)
\]

where \( m \) is the number of inputs and \( n \) is the order of the local model. The constant coefficients \( w \) describe the local model and are identified by the least squares algorithm. The outputs of the local linear models can be interpolated by basis functions \( \Phi_l \left( u^* \right) \) to a common output

\[
\hat{y}_k = \sum_{l=1}^{M} \left( w_{0,l} + \sum_{i=1}^{n} \left( w_{i,l}^y y_{k-i} + \sum_{j=1}^{m} w_{j,i,l}^y u_{j,k-i} \right) \right) \Phi_l \left( u_{l,k}^* \right)
\]

with

\[
u_{l,k}^* = \left[ u_{1,k-1} \ldots u_{1,k-n} \ldots u_{m,k-1} \ldots u_{m,k-n}, y_{k-1} \ldots y_{k-n} \right]^T
\]

The basis functions \( \Phi_l \left( u^* \right) \) are chosen as [17]:

\[
\Phi_l \left( u^* \right) = \exp \left( -\frac{1}{2} \sum_{i=1}^{n} \frac{(y_{k-i}-c_{il})^2}{\sigma_{il}^2} \right) + \ldots + \frac{\sum_{j=1}^{m} \left( u_{j,k-i} - c_{ij}^l \right)^2}{(\sigma_{ij}^l)^2}
\]

where \( c \) describes the center of each partition and \( \sigma \) is equal to half of the corresponding partition size.

![Fig. 1. Test plant](image-url)
This LLNFM allows the simulation of a nonlinear dynamic process by a set of local linear models. For the implementation of an offline identification, there are several tools available (e.g. [23], [24] and [25]). According to the configuration described in Section II-B the system to be modeled is considered as a multi-input-multi-output (MIMO) system with four inputs and two outputs. The inputs are the valve position of the cooling coil \( u_1 \), the valve position of the heating coil \( u_2 \), the supply water temperature \( d_1 \) and the inlet air temperature \( d_2 \) = \( \theta_a^o \). As already mentioned above, the inputs \( u_1 \) and \( u_2 \) represent the manipulated variables of the system and the inputs \( d_1 \) and \( d_2 \) are considered as measurable disturbances. The temperature \( \theta_a^o = y_1 \) and the humidity \( \varphi^o = y_2 \) of the supply air are considered as the outputs of the model. For the identification, all input-output delays are removed and the “delay free” system is identified. For each output a separate neuro fuzzy model is generated.

### B. Control

In the field of MIMO control, the model predictive control algorithm (MPC) is well established [7], [26]–[28]. In this work a linear MPC is implemented. Based on a discrete time state space model\(^1\) of the form

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k \\
y_k &= Cx_k
\end{align*}
\]

with \( u_k = u_{k-1} + \Delta u_k \), the output \( y \) is predicted over the prediction horizon \( n_p \). The predicted output signals \( y_{k+1}, \ldots, y_{k+n_p} \) are collected in the vector

\[
\tilde{y}_T = [y_{k+1} \ y_{k+2} \ldots \ y_{k+n_p}].
\]

The same is true for the reference signal, the manipulated signal and the manipulated signal rate:

\[
\begin{align*}
\tilde{y}_r &= [y_{r,k+1} \ y_{r,k+2} \ldots \ y_{r,k+n_p}] \\
\tilde{u} &= [u_k \ u_{k+1} \ldots \ u_{k+n_p-1}] \\
\Delta \tilde{u} &= [\Delta u_k \ \Delta u_{k+1} \ldots \ \Delta u_{k+n_p-1}]
\end{align*}
\]

The rate \( \Delta \tilde{u} \) of the manipulated signal \( \tilde{u} \) is optimized over the control horizon \( n_c \) based on the optimization problem

\[
\begin{align*}
\min_{\Delta \tilde{u}} & \quad (\tilde{e}^T Q \tilde{e} + \Delta \tilde{u}^T R \Delta \tilde{u}) \\
s.t. & \quad \tilde{u} \leq \tilde{u}_{\text{max}} \\
& \quad \tilde{u} \geq \tilde{u}_{\text{min}} \\
& \quad |\Delta \tilde{u}| \leq \Delta \tilde{u}_{\text{max}}.
\end{align*}
\]  

Thereby the variable \( \tilde{e} \) contains the error between the predicted output \( \bar{y} \) and the reference signal \( y_r \). Additionally, the difference between the model output \( y_k \) and the measured output \( y_{m,k} \)

\[
\Delta y_k = y_k - y_{m,k}
\]

\(^1\)Other formulations based on transfer function and/or step response, e.g. GPC, DMC [27], [28], might be more efficient, however for future stability and robustness analysis the state space formulation was selected.

In the field of MIMO control, the model predictive control (MPC) is well established [7], [26]–[28]. In this work a linear MPC is implemented. Based on a discrete time state space model of the form

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k \\
y_k &= Cx_k
\end{align*}
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with \( u_k = u_{k-1} + \Delta u_k \), the output \( y \) is predicted over the prediction horizon \( n_p \). The predicted output signals \( y_{k+1}, \ldots, y_{k+n_p} \) are collected in the vector

\[
\tilde{y}_T = [y_{k+1} \ y_{k+2} \ldots \ y_{k+n_p}].
\]

The same is true for the reference signal, the manipulated signal and the manipulated signal rate:

\[
\begin{align*}
\tilde{y}_r &= [y_{r,k+1} \ y_{r,k+2} \ldots \ y_{r,k+n_p}] \\
\tilde{u} &= [u_k \ u_{k+1} \ldots \ u_{k+n_p-1}] \\
\Delta \tilde{u} &= [\Delta u_k \ \Delta u_{k+1} \ldots \ \Delta u_{k+n_p-1}]
\end{align*}
\]

The rate \( \Delta \tilde{u} \) of the manipulated signal \( \tilde{u} \) is optimized over the control horizon \( n_c \) based on the optimization problem

\[
\begin{align*}
\min_{\Delta \tilde{u}} & \quad (\tilde{e}^T Q \tilde{e} + \Delta \tilde{u}^T R \Delta \tilde{u}) \\
s.t. & \quad \tilde{u} \leq \tilde{u}_{\text{max}} \\
& \quad \tilde{u} \geq \tilde{u}_{\text{min}} \\
& \quad |\Delta \tilde{u}| \leq \Delta \tilde{u}_{\text{max}}.
\end{align*}
\]

Thereby the variable \( \tilde{e} \) contains the error between the predicted output \( \bar{y} \) and the reference signal \( y_r \). Additionally, the difference between the model output \( y_k \) and the measured output \( y_{m,k} \)

\[
\Delta y_k = y_k - y_{m,k}
\]

is considered in the prediction, i.e.

\[
\bar{e} = \bar{y} - y_r + \Delta \bar{y}.
\]

In \( \Delta \bar{y} \), the future differences are assumed to be constant such that

\[
\Delta \bar{y} = \begin{bmatrix}
\Delta y_{k+1} \\
\Delta y_{k+2} \\
\vdots \\
\Delta y_{k+n_p}
\end{bmatrix} = \begin{bmatrix}
\Delta y_k \\
\Delta y_{k+1} \\
\vdots \\
\Delta y_{k+n_p}
\end{bmatrix}. 
\]

The matrices \( Q \) and \( R \) represent weighting factors of the errors \( \bar{e} \) and the signal rates \( \Delta \tilde{u} \). As the future trend of the reference signal is not known in many applications, the reference signal \( y_r \) is assumed to be constant along the prediction horizon \( n_p \). The manipulated signal \( \bar{u} \) is assumed to be constant for \( k \geq n_c \) as well.

Usually, the linear MPC is based on one linear model describing the dynamics nearby one operating point. To improve the performance of the MPC for the whole operating range a combination of linear MPC and a LLNFM is discussed in the following section.

### C. Communication between MPC and LLNFM

To improve the performance of the linear MPC a new state space model based on the identified LLNFM will be generated at each time step \( k \) (see Fig. 3). This state space generation is discussed in the following. As an example, a LLNFM of first order for a system with one input and one output is discussed in the following. As an example, a LLNFM of first order for a system with one input and one output is discussed in the following.
output is considered:
\[
y_k = \sum_{i=1}^{M} (w_d^i u_k + w_u^i u_{k-1} + w_y^i y_{k-1}) \Phi (\tilde{u}_k),
\]
with \( \tilde{u}_k = (u_{k-1}, y_{k-1}) \). For a better handling of the original equation in (13) at each time step \( k \) the parameters \( w_d^i, w_u^i \) and \( w_y^i \) of the M local models are combined to one parameter as shown in (14).
\[
a_k = - \sum_{i=1}^{M} w_y^i \Phi_i (\tilde{u}_k) \]
\[
b_k = \sum_{i=1}^{M} w_u^i \Phi_i (\tilde{u}_k) \]
\[
w_k = \sum_{i=1}^{M} w_d^i \Phi_i (\tilde{u}_k)
\]
So (13) reduces to
\[
y_k = w_k + b_k u_{k-1} - a_k y_{k-1}.
\]
To generate the state space model from the LLNFM in (15), the output \( y \) at time \( k \) will be assumed to be the state \( x_k \). To consider the offset \( w_k \) of the LLNFM, the state space model will be extended by an additional disturbance input \( d_k = 1 \).

Based on these assumptions, the state space model has the form
\[
x_{k+1} = a_k x_k + b_k u_k + w_k d_k
\]
\[
y_k = x_k.
\]
Since the parameters \( a_{k+1}, b_{k+1} \) and \( w_{k+1} \) for calculating the state \( x_{k+1} \) are not yet known at time step \( k \), they will be approximated by the currently known values \( a_k, b_k \) and \( w_k \) leading to the state space model
\[
x_{k+1} = a_k x_k + b_k u_k + w_k d_k
\]
\[
y_k = x_k.
\]

The input-output delays are considered by augmenting the state vector. Based on this state space model (17), the linear MPC mentioned in Section III-B was designed for the system configuration explained in Section II-B and verified on the pilot plant described in Section II-A.

IV. APPLICATION

A. Identification

For the identification of the LLNFM parameters of the pilot plant, the training data shown in Fig. 4 was generated by measurements. The diagram on the top shows the excitations for the input signals \( u_1 \) and \( u_2 \) which are selected to excite the process dynamics in different operating points. The second diagram shows the temperature \( T_1 \) of the hot water supply and \( T_2 \) shows the inlet air temperature. Based on this test data LLNFMs with 10, 20 and 30 first order local models are generated for each output. The various LLNFMs are compared by the squared error. To find a tradeoff between computational effort and the model quality, the LLNFM with 20 partitions is chosen. To verify the generated models, a comparison of the model outputs and the corresponding measurements is shown in Fig. 5. For verification, validation data was generated from another measurement. To confirm the need for multiple models, the outputs of the LLNFM’s are additionally compared to one linear model.

V. RESULTS

The Figures 6 to 10 show the results using the proposed control strategy. Different steps of the air temperature and the relative air humidity are presented. The control strategy is compared to a MIMO linear MPC based on one linear model and to a conventional PI strategy consisting of two single-input-single-output (SISO) PI-controllers. The PI-controllers are designed very conservatively to ensure proper operation in the whole operating range and they are processed with a sampling time of 10 milliseconds. The model of the linear MPC is linearized around 24°C and 50%. The sample time \( T_s \) and the parameters \( Q, R, n_p \) and \( n_c \) are set to
\[
T_s = 10s, \quad Q = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]
\[
n_p = 100, \quad n_c = 5
\]
for both linear MPC and LLNFM-based MPC. The mentioned controller parameters were found empirically via
In the conditioning of laboratories and/or test benches, it is particularly important that setpoints for temperature and humidity can be adjusted quickly and accurately. Fig. 6 shows a small step of the air temperature from 25°C to 23°C at a constant relative humidity of 50%. The corresponding actuating signals are shown in Fig. 7. For the considered scenario the MPC-LLNFM combination outperforms the reference methods. The displacement of the relative humidity due to the temperature step is adjusted accordingly. Especially at a higher setpoint step from 23°C/50% to 30°C/30% (see Fig. 8) the performance can be increased by using a LLNFM. Due to the deviation from the operating point (24°C/50%), the use of the linear MPC and the PI controller methods leads to poor dynamic behaviour in both temperature and humidity. Steps of the relative humidity setpoint shown in Fig. 9 and Fig. 10 can be tracked as well. Thereby the temperature remains almost constant using the MPC with a LLNFM.
The proposed control strategy shows significant improvements in comparison to conventional methods. The performance of the linear MPC can be substantially increased by the combination with LLNFM. The objective to extend the operating range of the linear MPC using a LLNFM is achieved. From the implementation point of view no additional effort is required. The LLNFM provides the required information of a complex nonlinear dynamic process in a relatively simple way. This property makes the combination of MPC and LLNFM attractive for the use in industrial control units. For future work the control of the whole plant will be investigated. Additional actuating signals like the control units. For future work the control of the whole plant will be considered.

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