A GTS Scheduling for Consensus Problems over IEEE 802.15.4 Wireless Networks

Naoki Hayashi¹ and Shigemasa Takai²

Abstract—This paper presents consensus problems over IEEE 802.15.4 wireless networks. The IEEE 802.15.4 protocol defines the Medium Access Control (MAC) and the physical layer (PHY) for Low-Rate Wireless Personal Area Networks (LR-WPANs). The IEEE 802.15.4 MAC protocol supports the beacon-enabled mode that enables real-time communication by allocating Guaranteed Time Slots (GTSs) to a designated node in a network. Despite a number of work on cooperative control, consensus problems over wireless networks remains less well understood. In this paper, we propose a modified version of the standard IEEE 802.15.4 protocol where each agent communicates through a PAN coordinator with a superframe structure. We consider a branch-and-bound GTS scheduling algorithm for non-preemptive communication tasks under the modified IEEE 802.15.4 protocol. The simulation result shows that each agent can successfully communicate with each other and achieve a consensus by the proposed GTS scheduling algorithm.

I. INTRODUCTION

With the wide spread of distributed complex systems, cooperative control of multi-agent systems has emerged in diverse areas of control engineering such as formation control of Unmanned Aerial Vehicles (UAVs), mobile sensor networks, and automated transportation systems [1], [2], [3]. Reaching a consensus is one of the fundamental problems of cooperative control to achieve a group objective of multi-agent systems [4], [5], [6]. The main objective of consensus problems is to guarantee that states of all agents converge to a common value by local information exchanges.

There are a number of related researches on consensus problems. Jadbabaie et al. considered swarming behavior using the nearest neighbor rule and switching information networks [4]. Moreau considered nonlinear consensus dynamics for undirected information exchanges using a set-valued Lyapunov function [7]. Fax and Murray considered cooperative control of vehicle formation with inter-vehicle information exchanges [8]. Olfati-Saber and Murray considered consensus problems under several different situations of information networks [5]. Ren and Beard studied consensus problems over switching and directed information networks with both discrete-time and continuous-time consensus dynamics [6]. Cao et al. considered consensus problems with asynchronous consensus dynamics [9].

In multi-agent systems, the use of wireless networks for local information exchanges has many advantages in terms of flexibility of networks and maintenance costs. Recently, the IEEE 802.15.4 protocol has been widely used for Low-Rate Wireless Personal Area Networks (LR-WPANs) [10]. The IEEE 802.15.4 protocol supports the beacon-enabled mode to provide real-time guarantees of wireless communication. In the beacon-enabled mode, a Personal Area Network coordinator (PAN coordinator) periodically broadcasts beacon signals and defines a superframe structure. Each superframe consists of a Contention Access Period (CAP) and a Contention Free Period (CFP) in its active portion. A PAN coordinator reserves network bandwidth for a specific node by allocating Guaranteed Time Slots (GTSs) of a CFP in each superframe. The GTS based bandwidth allocation is an attractive feature of the IEEE 802.15.4 protocol especially for real-time tasks. There are several researches on control over IEEE 802.15.4 wireless networks. Tiberi et al. considered self-triggered control to consider stability analysis of control loops over IEEE 802.15.4 networks under parameter uncertainty and external disturbances [11]. Araújo et al. proposed energy-efficient self-triggered control over IEEE 802.15.4 wireless sensor networks with a GTS scheduling algorithm [12]. This paper considers consensus problems in multi-agent systems over IEEE 802.15.4 wireless networks. Each agent synchronously updates its state using consensus dynamics with local information exchanges. The local information exchanges are done by the beacon-enabled mode of the IEEE 802.15.4 protocol. A PAN coordinator in the multi-agent system allocates network bandwidth by scheduling GTSs in response to requests of information exchanges from agents. One of the challenges of a GTS scheduling is that GTSs in each superframe are limited to at most 7 slots in the IEEE 802.15.4 standard. Moreover, a task of an information exchange has a timing constraint because consensus can not be achieved when the task misses its deadline. Therefore, in consensus problems over IEEE 802.15.4 wireless networks, an appropriate GTS scheduling algorithm is required to guarantee that all information exchanges are completed within their deadlines under the limited number of GTSs.

In this paper, we propose a modified version of the standard IEEE 802.15.4 protocol where each agent communicates through a PAN coordinator. The modified IEEE 802.15.4 protocol has a superframe structure that consists of several superframes in a period of consensus dynamics to alleviate the communication constraint caused by the limited number of GTSs. With the GTS mechanism of the modified IEEE 802.15.4 protocol, a PAN coordinator cannot interrupt an information exchange of an agent even if the other agent...
requests to hand over GTSs for the more urgent information exchange. In real-time systems, a task that does not allow interruption of execution is called a non-preemptive task [13], [14]. Based on the theory of real-time systems, we propose a non-preemptive GTS scheduling algorithm with a branch-and-bound method. At the beginning of every period of the consensus dynamics, a PAN coordinator schedules GTSs to guarantee that all information exchanges of agents are completed within their deadlines. Though the IEEE 802.15.4 protocol is expected to be a promising technology for low-rate wireless networks, most researches on cooperative control do not consider the effect of information exchanges over IEEE 802.15.4 wireless networks on consensus dynamics. The main contribution of this paper is to propose a real-time scheduling of GTSs to achieve a consensus under the IEEE 802.15.4 protocol specifications.

The rest of the paper is organized as follows: Section II states problem settings of consensus problems over IEEE 802.15.4 wireless networks. In Section III, we propose a GTS scheduling algorithm to achieve a consensus with the IEEE 802.15.4 protocol. In Section IV, we present a numerical example. Finally, we conclude this paper in Section V.

II. PROBLEM SETTINGS

A. Multi-agent Systems

In this section, we review some fundamental facts of graph theory and matrix theory [15], [16]. A directed graph (or a digraph) \( G = (V, E) \) consists of a finite and nonempty node set \( V = \{v_i | i \in \mathcal{I} \} \) and an edge set \( E \subseteq V \times V \), where \( \mathcal{I} = \{1, 2, \ldots, n\} \). Each node \( v_i \) in a digraph \( G \) represents each agent \( i \) and each directed edge \( (v_j, v_i) \) indicates that agent \( i \) receives information from agent \( j \). A directed tree is a digraph whose nodes except a root have exactly one parent. A spanning tree of a digraph is a directed tree with directed edges that connect all nodes from a root. A digraph is said to have a spanning tree if the digraph contains a spanning tree as a subgraph. Let \( G_m(V, E_m) \) be a possible digraph with a node set \( V \) and an edge set \( E_m \) \((m = 1, 2, \ldots, M)\). The union of graphs \( \bigcup_{m=1}^{M} G_m(V, E_m) \) is the digraph with the common node set \( V \) and the union of the edge sets \( \bigcup_{m=1}^{M} E_m \). A matrix \( P = [p_{ij}] \in \mathbb{R}^{n \times n} \) is said to be nonnegative if \( p_{ij} \geq 0 \) for all \( i, j \in \mathcal{I} \). A row stochastic matrix \( P \) is a nonnegative matrix satisfying \( \sum_{j=1}^{n} p_{ij} = 1 \) for all \( i \in \mathcal{I} \).

Let \( \mathbb{Z}_+ \) be the set of nonnegative integers. We consider a multi-agent system with \( n \) agents each of which has the following consensus dynamics:

\[
x_i(t) = \sum_{j=1, j \neq i}^{n} p_{ij}(t)(x_j(t) - x_i(t)),
\]

where \( x_i(t) \) is a state of agent \( i \) at time \( t \), and \( p_{ij}(t) \) has a positive real value if agent \( i \) receives the state \( x_j(t) \) from agent \( j \) at time \( t \), and 0 otherwise \((i, j) \in \mathcal{I})\).

By sampling (1) with a sampling period \( T > 0 \) we have the following discrete-time consensus dynamics:

\[
x_i[k + 1] = x_i[k] + T \sum_{j=1, j \neq i}^{n} p_{ij}[k](x_j[k] - x_i[k]),
\]

where \( x_i[k] = x_i(kT) \), \( p_{ij}[k] = p_{ij}(kT) \), and \( k \in \mathbb{Z}_+ \). In this paper, we define consensus in multi-agent systems as follows:

**Definition 1**: A group of agents achieves a consensus if (3) holds for any \( i, j \in \mathcal{I} \) and for any initial state.

\[
|x_i[k] - x_j[k]| \to 0 \quad \text{as} \quad k \to \infty.
\]

Ren and Beard showed the sufficient condition for consensus using discrete-time consensus dynamics [6].

**Theorem 1**: Each agent \( i \in \mathcal{I} \) has the following discrete-time consensus dynamics:

\[
x_i[k + 1] = \sum_{j=1}^{n} w_{ij}[k]x_j[k],
\]

where \( w_{ij}[k] \in \mathcal{U} \) is the \((i, j)\)-th element of a row stochastic matrix \( W[k] \) with \( w_{ij}[k] > 0 \) and \( \mathcal{U} \) is a set of nonnegative real values that are not larger than 1. Then, agents achieve a consensus by Eq. (4) if there exists an infinite sequence of contiguous, nonempty, and uniformly bounded time intervals \( [k_1, k_2), [k_2, k_3), \ldots \), where the union of digraphs of information exchanges across each interval has a spanning tree \((k_l \in \mathbb{Z}_+, l = 1, 2, \ldots)\).

We can restate the discrete-time consensus dynamics (2) as follows:

\[
x[k + 1] = W[k]x[k],
\]

where \( x[k] = [x_1[k] x_2[k] \cdots x_n[k]]^\top \in \mathbb{R}^n \) and \( W[k] \) is a row stochastic matrix with \( w_{ij}[k] > 0 \) for all \( i, j \in \mathcal{I} \) if the sampling period \( T \) of the discrete-time consensus dynamics (2) satisfies

\[
0 < T < \frac{1}{p^{\sup}(n - 1)},
\]

where

\[
p^{\sup} = \sup_{i,j \in \mathcal{I}, k \geq 0} p_{ij}[k].
\]

**Proof**: From the inequality (6), the \((i,i)\)-th element of the matrix \( W[k] \) is

\[
w_{ii}[k] = 1 - T \sum_{j=1, j \neq i}^{n} p_{ij}[k] > 1 - T \sum_{j=1, j \neq i}^{n} p^{\sup} = 1 - T p^{\sup}(n - 1) > 0.
\]
Then, from the definition of $p_{ij}[k]$, the matrix $W[k]$ is a nonnegative matrix. Moreover, for the sum of the $i$-th row of $W[k]$, we have

$$\sum_{j=1}^{n} w_{ij}[k] = \left(1 - T \sum_{j=1, j \neq i}^{n} p_{ij}[k]\right) + T \sum_{j=1, j \neq i}^{n} p_{ij}[k] = 1.$$  

This shows that the matrix $W[k]$ is a row stochastic matrix with positive diagonal elements.

From Theorem 1 and Lemma 1, we have the following proposition.

**Proposition 1:** A set of agents achieves a consensus with the discrete-time consensus dynamics (2) if the following conditions are satisfied:

(i) The sampling period $T$ of the discrete-time consensus dynamics (2) satisfies the condition (6).

(ii) There exists an infinite sequence of contiguous, nonempty, and uniformly bounded time intervals $[k_1 = 0, k_2], [k_2, k_3], \ldots$, where the union of digraphs of information exchanges across each interval has a spanning tree ($k_\ell \in \mathbb{Z}_+$, $\ell = 1, 2, \ldots$).

The proof of Proposition 1 can be obtained by the similar way as in [17] and is omitted in this paper.

**B. IEEE 802.15.4 Wireless Networks**

In this paper, we consider a star network topology as illustrated in Fig. 1 [10], [11]. The IEEE 802.15.4 MAC protocol provides the beacon-enabled mode where a PAN coordinator periodically broadcasts beacons to all nodes in a network. In the beacon-enabled mode, a PAN coordinator defines a superframe structure to manage communication with nodes as shown in Fig. 2. The length of a superframe is called a beacon interval (BI) and given by

$$\text{BI} = a\text{BaseSuperframeDuration} \times 2^{\text{BO}},$$

where BO is a beacon order ($0 \leq \text{BO} \leq 14$). In the IEEE 802.15.4 standard, $a\text{BaseSuperframeDuration}$ is set to 960 [symbols] (1 [symbol] = 4 [bits]) and this corresponds to 15.36 [ms] when a data rate of the wireless network is 250 [kbps]. A beacon interval is divided into an active portion and an inactive portion. An active portion is a duration where data transmission is carried out while, in an inactive portion, nodes are in a sleep mode. The length of an active portion is called a Superframe Duration (SD) and a SD is divided into 16 equally-sized time slots. A SD is given by

$$\text{SD} = a\text{BaseSuperframeDuration} \times 2^{\text{SO}},$$

where SO is a superframe order ($0 \leq \text{SO} \leq \text{BO} \leq 14$). An active portion consists of a Contention Access Period (CAP) and a Contention Free Period (CFP). In a CAP, data transmission is done by a CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) mechanism. On the other hand, in a CFP, a GTS mechanism is used to reserve network bandwidth for a certain node until the node completes its data transmission. In the standard IEEE 802.15.4 protocol, GTSs in each superframe are limited to at most 7 slots.

**III. NON-PREEMPTIVE GTS SCHEDULING**

This section presents how to schedule GTSs under the prescribed deadlines of information exchanges. In the standard IEEE 802.15.4 protocol of a star network topology, agents must communicate with a PAN coordinator all the time and no information can be sent among agents themselves. However, in consensus problems, each agent should communicate with each other frequently often as shown in Proposition 1. To allow local communications among agents with the limited GTSs, we propose a modified version of the standard IEEE 802.15.4 protocol. The modified IEEE 802.15.4 has the following differences with the standard protocol. Firstly, in the modified IEEE 802.15.4 protocol, each agent communicates with a PAN coordinator only when it needs to send information to other agents. This reduces the number of communication with the PAN coordinator and power consumption by information transmission. Secondly, the modified IEEE 802.15.4 protocol has a superframe structure that consists of several superframes in a period of the consensus dynamics. The multiple superframe structure

![Fig. 1. Star topology of IEEE 802.15.4 with PAN coordinator.](image1)

![Fig. 2. Superframe structure of IEEE 802.15.4.](image2)

![Fig. 3. Communication Task $\tau_{ij}^{\text{comm}}$.](image3)

![Fig. 4. Control Task $\tau_{ij}^{\text{ctrl}}$.](image4)
allows the more GTSs for information exchanges than that of the standard protocol.

Next, we summarize some basic concepts of real-time systems on task parameters used in the proposed GTS scheduling algorithm under the modified IEEE 802.15.4 protocol [13], [14]. A job represents a unit of work that is executed by a system. A task is a set of related jobs that jointly provide a service. If a task releases an infinite sequence of jobs at a constant rate, the task is called periodic.

In the rest of this paper, we assume that there is at least one feasible GTS scheduling algorithm with the given task parameters.

We consider two real-time tasks for each agent \( i \in \mathcal{J} \): the communication task \( \tau_{cmm}^{ij} \) that receives the state \( x_i[j] \) from agent \( j \in \mathcal{J} \) and the control task \( \tau_{ctr}^{ij} \) that updates the state \( x_i[j] \) by Eq. (2). The communication task \( \tau_{cmm}^{ij} \) and the control task \( \tau_{ctr}^{ij} \) are both periodic tasks whose periods are equal to the sampling period \( T \). The communication task \( \tau_{cmm}^{ij} \) is characterized by the following parameters as shown in Fig. 3:

- **Release time** \( r_{cmm}^{ij} \): time at which the \((k+1)\)-st job of \( \tau_{cmm}^{ij} \) is ready for an execution.
- **Start time** \( s_{cmm}^{ij} \): time at which the \((k+1)\)-st job of \( \tau_{cmm}^{ij} \) starts its execution.
- **Phase** \( \phi_{cmm}^{ij} \): start time of the first job of \( \tau_{cmm}^{ij} \).
- **Finishing time** \( f_{cmm}^{ij} \): time at which the \((k+1)\)-st job of \( \tau_{cmm}^{ij} \) finishes its execution.
- **Computation time** \( c_{cmm}^{ij} \): time necessary for an execution of the \((k+1)\)-st job of \( \tau_{cmm}^{ij} \).
- **Absolute deadline** \( d_{cmm}^{ij} \): time instant by which the \((k+1)\)-st job of \( \tau_{cmm}^{ij} \) should be completed.

In Fig. 3, a release time of the communication task is represented by an upward arrow while an absolute deadline is represented by a downward arrow. Similarly, the control task \( \tau_{ctr}^{ij} \) is characterized by a release time \( r_{ctr}^{ij} \), a start time \( s_{ctr}^{ij} \), a phase \( \phi_{ctr} \), a finishing time \( f_{ctr} \), a computation time \( c_{ctr} \), and an absolute deadline \( d_{ctr} \) as shown in Fig. 4.

In consensus problems, each agent has to compute its control task based on local information exchanges. Therefore, we assume that all communication tasks related to agent \( i \) are completed before the control task \( \tau_{ctr}^{ij} \) starts its execution as shown in Fig. 5. This leads to the following assumption.

**Assumption 1:** The absolute deadline \( d_{cmm}^{ij} \) of each agent satisfies

\[
d_{cmm}^{ij} - \alpha T \leq C_{i,\text{worst}}^{\text{ctr}}, \forall i, j \in \mathcal{J}, \forall k \in \mathbb{Z}_+,
\]

where \( C_{i,\text{worst}}^{\text{ctr}} \) is the worst computation time of the control task \( \tau_{ctr}^{ij} \).

In the IEEE 802.15.4 standard, GTSs in each superframe are limited to at most 7 slots. To overcome the limitation of GTSs, we assume the following relation between the sampling period \( T \) and the beacon interval \( BI \) that allows several superframes in a period of consensus dynamics.

**Assumption 2:** The sampling period \( T \) of the discrete-time consensus dynamics (2) and a beacon interval \( BI \) satisfies

\[
T = \alpha BI,
\]

where \( \alpha \) is a positive integer.

Figure 6 shows the relation between the sampling period \( T \) and the beacon interval \( BI \). The parameter \( \alpha \) should be chosen to satisfy the conditions (6) and (7).

We also make an assumption on a computation time of communication tasks.

**Assumption 3:** The computation time \( C_{i,j,k}^{cmm} \) of the communication task \( \tau_{cmm}^{ij} \) satisfies

\[
C_{i,j,k}^{cmm} = \beta_{i,j,k} \times aBaseSuperframeDuration
\]

\[
\forall i \in \mathcal{J}, \forall j \in \mathcal{J}, \forall k \in \mathbb{Z}_+,
\]

where \( \beta_{i,j,k} \) is a positive integer.

A scheduling algorithm may interrupt an execution of a job with a lower priority when a higher priority job is released. After the execution of the higher priority job is completed, the scheduler restarts the execution of the interrupted job. The interruption of an execution of a job is called preemption. A task is called preemptive if its job allows preemption. On the other hand, a non-preemptive task is a task that does not allow preemption and its jobs must be executed without any interruption [13], [14]. In IEEE 802.15.4 wireless networks, a PAN coordinator reserves network bandwidth for a certain agent for its data transmission with a GTS mechanism. A PAN coordinator does not allow any interruption of a communication task that is currently allocated GTSs. Therefore, communication tasks of each agent are non-preemptive tasks. This shows that a non-preemptive GTS scheduling is required to execute communication tasks within their absolute deadlines.

For non-preemptive scheduling problems, the Bratley’s algorithm was proposed to find a feasible scheduling [14]. The Bratley’s algorithm is a non-preemptive scheduling algorithm based on a branch-and-bound method with a search tree as
shown in Fig. 7. Each node of a search tree represents a temporarily scheduled task and the root node is the empty schedule. The objective of the Bratley’s algorithm is to find a feasible scheduling by appropriately pruning a search tree to reduce computational complexity.

Based on the Bratley’s algorithm, we propose a modified branch-and-bound method for a non-preemptive scheduling problem of GTSs over IEEE 802.15.4 wireless networks. In the IEEE 802.15.4 protocol, communication tasks cannot be executed in an inactive portion. To consider the constraint on communication tasks, we introduce a dummy task $\tau_{dum}^\ell$ that is virtually added as an element of a set of communication tasks ($k \in \mathbb{Z}_+, \ell = 1, 2, \ldots, \alpha - 1$) [12]. The dummy task $\tau_{dum}^\ell$ has the following task parameters:

- Release time $r_{dum,k}^{\ell}$: $r_{dum,k}^{\ell} = kT + (\ell - 1)BI$,
- Phase $\phi_{dum}^{\ell}$: $\phi_{dum}^{\ell} = \text{Beacon} + \text{CAP} + \text{CFP}$,
- Computation time $C_{dum,k}^{\ell}$: $C_{dum,k}^{\ell} = \text{Inactive} + \text{Beacon} + \text{CAP}$,
- Absolute deadline $d_{dum,k}^{\ell}$: $d_{dum,k}^{\ell} = r_{dum,k}^{\ell} + \phi_{dum}^{\ell} + C_{dum,k}^{\ell}$,

where $\text{Beacon}$, $\text{CAP}$, $\text{CFP}$, $\text{Inactive}$ are durations of a beacon signal frame, a CAP, a CFP, and an inactive portion, respectively. The proposed branch-and-bound method for a non-preemptive GTS scheduling-abandons a branch of a search tree when

(a) A branch of any node causes a deadline miss;

(b) A feasible schedule is found;

(c) The finishing time $f_{ij,k}^{\text{cmm}}$ of the current communication task $\tau_{ij}^{\text{cmm}}$ satisfies

\[ f_{ij,k}^{\text{cmm}} > kT + \gamma BI + \text{Beacon} + \text{CAP} + \text{CFP}, \]

where $\gamma \in \mathbb{Z}_+$ is the number of branches of the dummy task $\tau_{dum}^\ell$.

The first two rules (a) and (b) are the branch rules of the Bratley’s algorithm that prunes infeasible schedules and stops the search when it finds a feasible schedule. The rule (c) implies that the $(k+1)$-st job of the communication task $\tau_{ij}^{\text{cmm}}$ cannot be scheduled in the $(\gamma + 1)$-st superframe because its finishing time exceeds the duration of the CFP of the $(\gamma + 1)$-st superframe as shown in Fig. 8. In this case, the job should be scheduled in the later superframe. At the beginning of every period of the discrete-time consensus dynamics (2), a PAN coordinator schedules GTSs based on the branch-and-bound rules (a)-(c) to guarantee that all information exchanges are completed within their absolute deadlines.

IV. SIMULATION

In this section, we consider a numerical example with 4 agents ($\mathcal{F} = \{1, 2, 3, 4\}$), where $p^{\text{sup}} = 0.3$. To satisfy the conditions (6) and (7), we consider the superframe structure characterized by Table I. Then we have

\[ \text{BI} = 15.36 \times 2^3 = 491.52 \text{ [ms]}, \]
\[ \text{SD} = 15.36 \times 2^4 = 245.76 \text{ [ms]}, \]
\[ \text{CAP} = 15.36 \times 8 = 122.88 \text{ [ms]}, \]
\[ \text{CFP} = 15.36 \times 7 = 107.52 \text{ [ms]}, \]
\[ T = 2 \times \text{BI} = 983.04 \text{ [ms]}. \]

Note that the sampling period $T$ satisfies the condition (6):

\[ T = 0.98304 \text{ [s]} < \frac{1}{0.3 \times (4 - 1)} \approx 1.11 \text{ [s]}. \]

Figure 9 shows the states of 4 agents. From this figure, we can see that the agents achieve a consensus by the proposed
GTS scheduling algorithm with the modified IEEE 802.15.4 protocol.

In this simulation, the information exchanges and the task parameters in the time interval \([0, T)\) are given as shown in Fig. 10 and Table II, respectively. Figure 11 shows the search tree of the GTS scheduling in \([0, T)\). In Fig. 11, the number on the upper right of each node represents the finishing time when the job of the task corresponding to the node completes its execution. If this number exceeds the absolute deadline of the job of the corresponding task, the job causes a deadline miss. From Fig. 11, we can see that \(\tau_{13}^{\text{comm}}\) and \(\tau_{21}^{\text{comm}}\) are scheduled in the 1-st superframe and \(\tau_{23}^{\text{comm}}\) and \(\tau_{32}^{\text{comm}}\) are scheduled in the 2-nd superframe in \([0, T)\) as shown in Fig. 12.

### V. Conclusion

This paper presented consensus problems over IEEE 802.15.4 wireless networks. We considered a GTS scheduling with a discrete-time consensus dynamics to guarantee that all information exchanges are completed within their absolute deadlines. We proposed the branch-and-bound based non-preemptive GTS scheduling algorithm under the modified IEEE 802.15.4 protocol specifications. Finally, the simulation result showed the effectiveness of the proposed GTS scheduling algorithm.

### References


