Automatic Feedforward Tuning for PID Control Loops

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Abstract—In this paper we propose a method for the automatic design of the feedforward compensator for PID control loop where the process is affected by a disturbance. The parameters of the compensator are automatically computed after the disturbance is rejected for the first time by a simple PID controller which can be roughly tuned. Further, a technique to assess the performance of the compensator is also proposed. Simulation results show the effectiveness of the methodology.

Index Terms—PID control, feedforward control, disturbance rejection, tuning, performance assessment.

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are the most employed controllers in industry owing to the cost/benefit ratio they are able to provide. In fact, in spite their relative simplicity, they are capable to provide a satisfactory performance for a wide range of processes. Further, many tuning rules [1] have been devised and automatic tuning methodologies are also available to make their design easier and therefore more suitable for an industrial point of view.

It has also to be recognized that the great success of PID controllers is also due to the implementation of those additional functionalities (set-point weighting, anti-windup, etc.) that allow the user to improve the performance in practical cases [2]. Among these additional functionalities, the use of a feedforward action plays a major role in improving the performance (when a sufficiently accurate model of the system can be estimated) both for the set-point following [3] and for the load disturbance rejection task [4], [5]. In the latter case, which is the one considered in this paper, the disturbance must be measurable. For an overview of feedforward methodologies in the context of PID control, see [6] and references therein contained.

In this paper we present an automatic tuning procedure for the automatic tuning of a feedforward block for the compensation of disturbances. By following the same rationale employed in [7], [8], [9], [10], the method consists in estimating the disturbance transfer function by evaluating a closed-loop disturbance step response (where only the, possibly roughly tuned, feedback PID controller is employed). It is therefore possible to use routine data which is obviously preferable in industrial settings [12], no one (at least to the authors’ knowledge) has still been devised for feedforward controllers for disturbance compensation.

The proposed methodology can be used together with the automatic tuning methodology for the feedback PID controller proposed in [7] (which is briefly reviewed in Section III) in order to provide an overall control system automatic design.

The paper is organized as follows. In Section 2 the initial control scheme is described. In Section 3 the methodology for the estimation of the parameters is proposed. The design technique and the performance assessment methodology are presented in Section 4. Simulation results are given in Section 5. Finally, conclusions are drawn in Section 7.

II. INITIAL CONTROL SCHEME

We consider the unity-feedback control system of Figure 1 where the self-regulating process $P$ with an overdamped open-loop step response has the transfer function

$$P(s) = \frac{ke^{-sL_0}}{q(s)}; \quad q(s) = \prod_{j}(T_js + 1). \quad (1)$$

As an estimated model of the process (1), the following first-order-plus-dead-time (FOPDT) transfer function can be considered:

$$\hat{P}(s) = \frac{ke^{-sL}}{Ts + 1}. \quad (2)$$

It is worth recalling that this simple model may come from a model reduction technique like the so called “half-rule” [13] which states that the largest neglected (denominator) time constant is distributed evenly to the effective dead time and the smallest retained time constant. Thus, we have

$$T = T_1 + \frac{T_2}{2}, \quad L = L_0 + \frac{T_2}{2} + \sum_{j \geq 3} T_j. \quad (3)$$

Thus, the value of the sum of lags and delay of the model is the same one as that of the real process, i.e.,

$$T_0 := \sum_{j} T_j + L_0 = T + L. \quad (4)$$

Hence, $T_0$ is a relevant process parameter that is worth estimating for the purpose of retuning the PID controller, as it will be shown in the following sections. Note also
the presence of positive zeros is not considered explicitly
in (1); however, the associated time constants can be simply
added to the dead time of the process [13]. The feedback
controller is a PID controller whose transfer function is in
series (“interacting”) form:
\[
C(s) = K_p \left( \frac{T_i s + 1}{T_i s} \right) (T_d s + 1).
\] (5)
where \(K_p\) is the proportional gain, \(T_i\) is the integral time
constant and \(T_d\) is the derivative time constant.
The series form has been chosen for the sake of simplicity,
however the use of other forms is straightforward by suitably
applying translation formulas to determine the values of the
parameters [2]. A filter on the derivative action is also applied
for reducing the actuator wear caused by the unavoidable
process variable measurement noise. The filter time constant
is usually selected in order to filter the high-frequency noise
and, at the same time, in order for the cutoff frequency to be
higher than the system bandwidth [14]. For this reason, the
filter will be neglected hereafter without loss of generality.
The controller transfer function can therefore be rewritten as
\[
C(s) = \frac{K_p}{T_i s} c(s), \quad c(s) = (T_i s + 1)(T_d s + 1).
\] (6)
Finally, by following a reasoning similar to that applied for
the process model, the (measurable) process disturbance can be represented by the following model:
\[
G(s) = \frac{\mu e^{-s \theta_0}}{g(s)}, \quad g(s) = \prod_j (1 + s \tau_j)
\] (7)
which, by applying again the “half-rule”, can be eventually reduced to a FOPDT transfer function
\[
\hat{G}(s) = \frac{\mu e^{-s \theta}}{\tau s + 1}
\] (8)
where the value of the sum of lags and delay of the model
is denoted as
\[
\tau_0 = \sum_j \tau_j + \theta_0 = \tau + \theta.
\] (9)

It is worth noting that the classical load disturbance mod-
ing, where the disturbance is added to the control variable,
is a specific case of the scheme of Figure 1 with \(G = P\).

III. ESTIMATION OF THE PARAMETERS

The parameters of the process transfer function (2) can be obtained by applying the method already proposed an
discussed in [7] and also applied in an industrial application
as reported in [8].
In particular, by evaluating a closed-loop response when a
step signal of amplitude \(A_s\) is applied to the set-point \(r\)
(and the PID controller has been previously tuned, possibly
roughly), the sum of the lags and of the dead time of the
process can be obtained by calculating the integral of \(v(t)\),
defined as
\[
v(t) = ku(t) - y(t),
\] (10)
where \(u\) is the manipulated variable and \(y\) is the process variable. In fact, it can be shown that [7]
\[
T_0 = \frac{1}{A_s} \lim_{t \to +\infty} \int_0^t v(\xi) d\xi
\] (11)
It appears that the steady-state value of \(v(t)\) does not depend
on the PID parameters, but only on the process parameters
(inded, this is a nice feature of the method as it can be
applied with any controller parameters).
Further, by means of a simple application of the final value
theorem, the process gain can be determined as
\[
k = A_s \frac{T_i}{K_p \int_0^\infty e(t) dt}
\] (12)
where \(e = r - y\) is the control error. Note that both the value
of the gain and of sum of the lags and of the dead time of the
process are determined by considering the integral of signals
and therefore the method is inherently robust to the
measurement noise. Finally, the apparent dead time \(L\) of the
process can be evaluated by considering the time interval
from the application of the step signal to the set-point and the
time instant when the process output attains the 2% of
the new set-point value \(A_s\), namely, when the condition
\(|r - y| > 0.02 A_s\) occurs. Actually, from a practical point
of view, in order to cope with the measurement noise, a
simple sensible solution is to define a noise band \(NB\) [15]
(whose amplitude should be equal to the amplitude of the
measurement noise) and to rewrite the condition as \(y > NB\).
Once the apparent dead time has been estimated, the time
constant of the process can be simply calculated as
\[
T = T_0 - L.
\] (13)
Based on the determined process model, the PID controller
can be retuned by applying anyone of the tuning rules
proposed in the literature [1], depending on the control
requirements. In the simulation results given in Section V,
the tuning procedure proposed in [7] (based on the SIMC
tuning rules presented in [13]) have been employed.
We now address the problem of estimating the disturbance
transfer function \(G(s)\) (see (8)) for which a new methodology
has to be applied. Consider again the closed-loop control
scheme of Figure 1 where the PID controller \(C\) has been
already (possibly roughly) tuned. The measurable step dis-
turbance \(d\) of amplitude \(A_d\) is then assumed to affect the
process output at the time $t_0$ (note that if the disturbance signal is not a step, its dynamics can be included in any case in $G(s)$). By considering the final value of the integral of the control error, it can be easily be proven that:

$$
\lim_{t \to +\infty} \int_{t_0}^{t} e(\xi)d\xi = -\frac{\mu T_i}{kK_p} A_d.
$$

(14)

Thus, the gain of $G(s)$ can be determined easily as

$$
\mu = -\frac{kK_p}{A_d T_i} \int_{t_0}^{\infty} e(t)dt.
$$

(15)

The determination of the dynamic parameters of $G(s)$ is less trivial, although still based on the final value theorem. The following variable $w$ is considered:

$$
w = ku + \mu d - y
$$

$$
= -\frac{kC(s)G(s)}{1 + C(s)P(s)} d(s) + \mu d(s) - \frac{G(s)}{1 + C(s)P(s)} d(s)
$$

$$
= -\frac{kC(s)G(s) + \mu (1 + C(s)P(s)) - G(s)}{1 + C(s)P(s)} d(s)
$$

$$
= \frac{sT_i q(s)}{sT_i q(s) + kK_p c(s)e^{-sL_o}} \left(-\frac{\mu kK_p c(s)e^{-s\theta_0}}{sT_i g(s)} + \mu \right)
$$

$$
\frac{\mu kK_p c(s)e^{-sL_o}}{sT_i g(s)} - \frac{\mu e^{-s\theta_0}}{g(s)}
$$

$$
d(s)
$$

(16)

Thus, by evaluating the final value of the integral of $w$ when $d(t)$ is a step signal of amplitude $A_d$ (namely, $d(s) = A_d/s$), we can write:

$$
\lim_{t \to +\infty} \int_{t_0}^{t} w(\xi)d\xi = \lim_{s \to 0} \frac{W(s) A_d}{s}
$$

$$
= \frac{A_d T_i}{kK_p} \left(\lim_{s \to 0} \mu kK_p \frac{-q(s)e^{-s\theta_0} + g(s)e^{-sL_o}}{sT_i q(s)g(s)} + \mu - \mu\right)
$$

$$
= \mu A_d \lim_{s \to 0} \left(g(s)e^{-sL_o-\theta_0} - \frac{g(s)}{g(s)}\right)
$$

$$
= \mu A_d \lim_{s \to 0} \left(e^{-s(L_o-\theta_0)} - 1 + \frac{1 - \frac{g(s)}{g(s)}}{s}\right)
$$

$$
= \mu A_d \left(\theta_0 - L_o + \sum_j \tau_j - \sum_j T_j\right) = \mu A_d (\theta_0 - T_0)
$$

(17)

Thus, the sum of all the lags of $G(s)$ can be obtained as

$$
\tau_0 = T_0 + \frac{1}{\mu A_d} \int_{t_0}^{\infty} w(\xi)d\xi
$$

(18)

Note that also in this case the estimation of the process parameters is based on the integral of signals and therefore the method is inherently robust to the measurement noise. Further, the process parameters are obtained independently on the values of the PID parameters, because the estimation is based on steady-state values of the variables. Then, as in the process parameters estimation procedure, the apparent dead time $\theta$ of the system can be evaluated by considering the time interval from the occurrence of the load disturbance and the time instant when the process output attains the 2% of the set-point value, namely, when the condition $|\tilde{y} - y| > 0.02\mu A_d$ occurs, where $\tilde{y}$ is the current steady state value of the process variable (the noise band concept can be employed in order to cope with measurement noise). Then, the time constant $\tau$ of transfer function (8) can be simply obtained as

$$
\tau = \tau_0 - \theta.
$$

(19)

IV. FEEDFORWARD DESIGN AND PERFORMANCE ASSESSMENT

In order to compensate effectively for the disturbance, the classical ideal feedforward compensator

$$
H(s) = -\frac{\tilde{G}(s)}{P(s)}
$$

(20)

is added to the control scheme, which therefore is modified as shown in Figure 2. This means that, by assuming that the FOPDT transfer functions (2) and (8) are accurate models both for the process and for the disturbance, the following compensator can be added in the control strategy [16]:

$$
H(s) = -\frac{\mu T_s + 1}{k \tau s + 1} e^{-\alpha s}
$$

(21)

where the dead time term $\alpha$ is chosen as

$$
\alpha = \max (0, \theta - L)
$$

(22)

as the causality of the feedforward block has to be ensured. Obviously, the ideal feedforward compensation (that is, the disturbance effect is completely rejected) is possible only if $\theta > L$. On the other hand, when $\theta < L$ the delay in $H(s)$ is set equal to zero and some effect on the process variable can not be avoided. This case is further analyzed hereafter. By assuming that the action of $C(s)$ can be neglected during the disturbance response transient, which is therefore assumed to be executed mainly by $H(s)$ (note that, in practice, the presence of $C$ is unavoidable because only the feedback action can ensure a zero steady-state error because of the presence of unavoidable modelling uncertainties), the open-loop behaviour of the process variable $y$ is given by (note that $\alpha = 0$)

$$
Y(s) = \tilde{G}(s) A_d + H(s) \tilde{P}(s) A_d
$$

$$
= \left(\frac{\mu e^{-s\theta}}{1 + s\tau} - \frac{\mu e^{-sL}}{1 + s\tau}\right) A_d.
$$

(23)

Hence, by assuming that $\tilde{P}(s)$ and $\tilde{G}(s)$ are perfect models, a reference value for the Integrated Absolute Error can be computed. In fact, under these hypotheses, the control error will have the following expression (with null initial conditions):

$$
e(t) =
\begin{cases} 
0 & \text{if } t_0 \leq t \leq \theta \\
-\mu \left(1 - e^{-\frac{t - \theta}{\tau}}\right) A_d & \text{if } \theta \leq t \leq L \\
-\mu \left(1 - e^{-\frac{t - L}{\tau}}\right) A_d + \mu \left(1 - e^{-\frac{t - L}{\tau}}\right) A_d & \text{if } t \geq L
\end{cases}
$$

(24)
Thus, by integrating (24) we can state that the integrated absolute error results in
\[
I A E = \int_{t_0}^{\infty} |e(t)| \, dt = \mu A d (L - \theta). \tag{25}
\]
Note that, as the error function has always the same sign during the transient, the same result can be obtained also by integrating expression (23) and by subsequently applying the final value theorem to the resulting expression.

In spite of the simplifying assumptions, this value can be used as a reference to evaluate the performance of a feedforward compensator block. Indeed, the integrated absolute error improves from \( I A E = 9.998 \) (obtained with the initial disturbance rejection performance index (DRPI) can be employed to represent the desired performance of the PID controller and with the use of the feedforward compensator block. Indeed, the integrated absolute error results can be compared (see Section I). In particular, the following Disturbance Rejection Performance Index (DRPI) can be introduced:
\[
D R P I = \frac{\mu A d (L - \theta)}{\int_{t_0}^{\infty} |e(t)| \, dt} \quad L > \theta. \tag{26}
\]
In the ideal case it should be \( DRPI = 1 \), but taking into account the simplifying assumptions made previously and by evaluating many simulation results, the compensator performance can be said to be satisfactory if the value of the index is greater than a given threshold equal to 0.5. Note that considering a threshold value smaller than 0.5, a performance index greater than one might also result. In any case, the user can select the threshold value depending on the tightness of the required performance. Actually, by taking into account the uncertainties that are unavoidably introduced in the estimation of \( L \) and \( \theta \) (and the fact that the FOPDT model considered can be an approximation of the real one), a performance index greater that one might also result. In any case, the Disturbance Rejection Performance Index depends only on the difference of the estimated dead time parameters \( L \) and \( \theta \) and, as the estimation technique is the same for both the dead times, the difference between the two should be closer to its actual value.

The parameter estimation technique can be employed also for the design of the feedforward block \( H \) according to the methodology proposed recently in [11] which takes into account also the role of the feedback controller and which reduces the high frequency gain of the compensator. In this context, the feedforward block has to be set as:
\[
\tilde{H}(s) = -k_{ff} \frac{T_s + 1}{T_p s + 1} e^{-\alpha s} \tag{27}
\]
where
\[
T_p = \begin{cases}
\tau & \text{if } L \leq \theta \\
\frac{L - \theta}{\theta} & \text{if } 0 < L - \theta \leq 1.7(\tau - \theta) \\
0 & \text{if } L - \theta > 1.7(\tau - \theta)
\end{cases} \tag{28}
\]
and
\[
k_{ff} = \begin{cases}
\frac{\mu}{k} - \frac{k_p}{k e} \mu (T_p - \tau) & \text{if } \theta \geq L \\
\frac{\mu}{k} - \frac{k_p}{k e} \mu (T_p - \tau - \theta + L) & \text{if } \theta < L \tag{29}
\end{cases}
\]

V. SIMULATION RESULTS

In all the following simulation examples we set the amplitude of the set-point and load disturbance step signals as \( A_s = A_d = 0.5 \). After having evaluated a set-point step response and a disturbance step response with an initial tuning of the PID controller, the parameters of the process and disturbance transfer function have been estimated. Then, the method developed in [7] has been employed to retune the PID controller and the method presented in Section IV has been used to design the feedforward compensator. Another experiment has been subsequently performed to verify the effectiveness of the methodology.

A. Example 1

As a first example the following systems are considered:
\[
P(s) = \frac{e^{-2s}}{(1 + 10s)(1 + s)^2} \tag{30}
\]
\[
G(s) = \frac{e^{-4s}}{1 + 20s}
\]

The initial PID parameters are \( K_p = 0.5, T_i = 10 \) and \( T_d = 1 \). The corresponding step responses are shown in Figure 3 as dotted line (the set-point and disturbance step responses are shown in two different plots for the sake of clarity and both the process and control variable are shown). The parameters obtained by employing the estimation procedure (see (11), (12), (13), (15), (18) and (19)) are \( k = 1, T_0 = 14, L = 3.07, T = 10.93, \mu = 1, \tau_0 = 23.99, \theta = 4.43, \tau = 19.56 \).

Then the retuning algorithm gives \( K_p = 1.125, T_i = 6.907 \) and \( T_d = 4.022 \) while the feedforward compensator transfer function results in
\[
H(s) = \frac{1 + 10.93 s}{1 + 19.56 s} e^{-1.36 s} \tag{31}
\]

Figure 3 shows the corresponding results where it is evident that the control performance improves after the retuning of the PID controller and with the use of the feedforward compensator block. Indeed, the integrated absolute error improves from \( I A E = 9.998 \) (obtained with the initial
tuning) to $IAE = 3.389$ for the set-point response and from $IAE = 9.966$ to $IAE = 0.272$ for the disturbance rejection task. Note that by employing the method proposed in [11], as it is $\theta > L$, we obtain the same results (see (29) where $\tau = T_p$).

B. Example 2

The following models are now considered:

\[
P(s) = \frac{e^{-2s}}{(1+s)^3} \quad \frac{e^{-4s}}{(1+0.5s)}
\]

\[
G(s) = \frac{1}{1+10s}(1+s)^2
\]

the initial PID parameters are $K_p = 0.8$, $T_i = 3$ and $T_d = 1$. Correspondingly, the parameters obtained by using the estimation procedure are $k = 1$, $T_0 = 5.002$, $L = 2.26$, $T = 2.742$, $\mu = 1$, $\tau_0 = 4.504$, $\theta = 4.03$ and $\tau = 0.474$. Then the retuning algorithm gives $K_p = 0.340$, $T_i = 1.539$ and $T_d = 1.203$ while the feedforward compensator transfer function results in

\[
H(s) = \frac{1 + 2.742s}{1 + 0.474s}e^{-1.77s}
\]

Figure 4 shows the performance improvement that results from the retuning of the PID controller and from the use of the feedforward compensator block. Indeed, the integrated absolute errors are $IAE = 2.575$ and $IAE = 2.509$ with the initial tuning and $IAE = 2.507$ and $IAE = 0.998$ after the application of the proposed method for the set-point and disturbance response respectively. As in Example 1, the method proposed in [11] gives the same result.

C. Example 3

As a third example, another third-order process is considered, but in this case the disturbance dead time is smaller than the process dead time:

\[
P(s) = \frac{e^{-4s}}{(1 + 10s)(1+s)^2}
\]

\[
G(s) = \frac{e^{-s}}{1+5s}
\]

The initial PID parameters are again $K_p = 0.5$, $T_i = 10$ and $T_d = 1$. After the evaluation of the set-point and disturbance step responses, the parameters obtained are $k = 1$, $T_0 = 16$, $L = 5.07$, $T = 10.93$, $\mu = 1$, $\tau_0 = 6.0$, $\theta = 1.13$ and $\tau = 4.87$. Then the retuning algorithm gives $K_p = 0.653$, $T_i = 6.630$ and $T_d = 4.360$, while the compensator transfer function results in

\[
H(s) = \frac{1 + 10.97s}{1 + 4.82s}
\]

Note that no delay has been included in the feedforward action (namely, $\alpha = 0$) and the ideal compensation can not be achieved. It is therefore sensible to calculate the performance index which results $DRPI = 0.528$, compared with a value $DRPI = 0.197$ obtained by the feedback controller only. Figure 5 shows the performance improvement that results from the retuning and the feedforward compensator block. The values of the integrated absolute errors are $IAE = 9.991$ and $IAE = 9.979$ with the initial tuning and $IAE = 5.539$ and $IAE = 3.158$ after the application of the proposed method for the set-point and disturbance response respectively. The results obtained from the application of the method described in [11] are also shown. The feedforward compensator results to be

\[
H(s) = -0.83\frac{1 + 11.0s}{1 + 2.51s}
\]

The corresponding performance index (although it has been devised for the standard feedforward compensator) improves to $DRPI = 0.832 \ (IAE = 2.004)$. Obviously, for set-point following task the result is exactly the same.
VI. CONCLUSIONS

In this paper we have proposed a methodology for the closed loop identification of the (FOPDT) transfer function through which a process disturbance affects the controlled variable of a PID control system. The main parameters of the transfer function are automatically determined by integral computations which are robust with respect to the measurement noise. It has to be remarked that the procedure can be applied to data available during routine operations and that the results do not depend on the employed PID parameters. As a result, a classical feedforward compensator can be automatically designed and included in the control strategy. When the ideal compensation is not possible (i.e. when the process transfer function has an apparent dead time greater than the one of the disturbance transfer function) a performance index has been introduced. A recently proposed technique for improving the performance by retuning the compensator can also be employed.

The effectiveness of the overall methodology has been demonstrated by simulation results.

REFERENCES