Abstract: Control performance assessment, or CPA, is an increasingly vital tool to quantify the performance of industrial control loops. While most of the research and commercial activity in CPA has been applied to linear systems to date, those researchers investigating nonlinear systems fall into one of two groups. The first group focused on the diagnosis of a common specific nonlinearity, namely valve stiction (A. Horch, 1999, M.A.A.S. Choudhury and S.L. Shah and N.F. Thornhill and David S. Shook, 2006, Nina F. Thornhill and Alexander Horch, 2007), while the second group tried to establish the minimum variance performance lower bound (MVPLB) (T.J. Harris and W. Yu, 2007, Yu et al., 2008, 2009, 2010). In this paper, we will continue to investigate CPA for a popular and versatile class of nonlinear model; the Hammerstein-Wiener (HW) model. Since the minimum performance lower bound is hard to establish for nonlinear systems, we propose two new performance indices which can be reliably estimated from the routine closed loop data. These indices can be used in a manner similar to the standard linear CPA performance index. The estimates are obtained by fitting the output data to a nonlinear autoregressive and moving average (NARMA) model. The results of two simulation examples illustrate that the proposed methodology is efficient and accurate for the class of HW models.

Keywords: Control performance assessment, Hammerstein-Wiener model, Static nonlinearity.

1. INTRODUCTION

On a recent site visit to a major dairy plant, we found over 500 control loops were under the maintenance supervision of a single instrument engineer. At that ratio, which from anecdotal evidence is by no means unusual, a realistic period between loop inspections is in the order of years. Consequently it is not surprising that engineers are overwhelmed by the sheer number of loops that need attention on any typical industrial processing plant as noted by control audits (Bialkowski, 1998, Desborough and Miller, 2002). Control performance assessment, or CPA, is a technology to diagnose and maintain operational efficiency of control systems developed in a direct response to address this increasingly important economic problem. CPA is routinely applied in the refining, petrochemicals, pulp and paper and the mineral processing industry as noted by Qin (1998), T.J. Harris (1999), Huang and Shah (1999), M. Jelali (2006), although these and many related publications, are primarily restricted to linear systems.

In practice, industrial control loops invariably include nonlinearities from the control valve, the measurement, or the process itself. The estimates of the minimum variance performance lower bound (MVPLB) and the performance index using the linear CPA techniques may be distorted by these nonlinearities. To deal with this situation, recent research has tried to extend CPA into nonlinear systems. In the case of nonlinear systems, T.J. Harris and W. Yu (2007) superimpose a nonlinear dynamic model to an additive linear or partially nonlinear disturbance. It is shown that a minimum variance feedback invariant exists for a class of nonlinear models and the MVPLB can be estimated from routine operating data. Continuing this idea, estimations of the MVPLBs for the moderate valve stiction cases are proposed by Yu et al. (2008, 2009). These applications are based on one general nonlinear structure: nonlinearity in the dynamics caused by a static...
nonlinearity from manipulated variables plus an additive disturbance which is in form of an ARMA model.

In this paper, we will extend the CPA to block-oriented nonlinear models, Hammerstein-Wiener models (shown in Fig. 1). Nonlinear block-oriented models consist of the interconnection of a linear time invariant (LTI) systems with static, or memoryless, nonlinearities. This class includes Hammerstein models, Wiener models and combinations of the two (Haber and Unbehauen, 1990, Pearson, 1999). Such block-oriented nonlinear descriptions are very useful modelling input/output nonlinearities and have been implemented many industrial processes (i.e. Gomez and Baeyens (2004), Zheng and Zafiriou (2004), Averin (2003), Sung et al. (2008)).

For the linear systems, the MVPLB can be estimated through the Impulse Response Functions (IRF) since there is a direct relationship between the impulse response and the variance. It is not true for nonlinear systems. The general form of MVPLB for the nonlinear systems may be very complex and it is difficult to estimate. In this paper, we proposed two alternative performance indices.

The layout of the paper is as follows. In Section 2, block-oriented nonlinear systems with additive disturbances are introduced. Section 3 describes the first performance index. Section 4 outlines the way to estimate the first performance index and proposed the second performance index. In section 5, two simulations are used to illustrate the proposed methodology. This is followed by a discussion and conclusions highlighting both the limitations and potential of the proposed methods.

2. PROCESS DESCRIPTION

We assume the plant can be adequately modelled by a Hammerstein-Wiener model (shown in Fig. 1) as,

\[ y_t = N_2(x_t) \]
\[ x_t = w_t + d_t \]
\[ w_t = B(q^{-1})A(q^{-1})^{-1}v_t \]
\[ v_t = N_1(u_t) \]

where \( A(q^{-1}) \) and \( B(q^{-1}) \) are polynomials in the backshift operator \( q^{-1} \), and \( b \) is the time delay of the system. \( u_t \) and \( y_t \) are the process input and output respectively; the internal signals \( v_t \), \( w_t \) and \( x_t \) are nonmeasurable. The functions \( N_1 \) and \( N_2 \) represents the static nonlinearities for input and output respectively. The disturbance \( d_t \) is modeled as the output of a linear Autoregressive-Integrated-Moving-Average (ARIMA) filter driven by white noise \( a_t \) with zero mean and variance \( \sigma^2_a \) of the form,

\[ d_t = \frac{\theta(q^{-1})}{\phi(q^{-1})}a_t \]

where \( \nabla \text{def} = (1 - q^{-1}) \) is the difference operator and \( b \) is a non-negative integer, typically less than 2. The polynomials \( \theta(q^{-1}) \) and \( \phi(q^{-1}) \) are monic and stable.

In a process control loop, the HW model can be motivated by considering the input nonlinear block \( N_1 \) such as such as equal percentage valve characteristics, quantisation due to pulse-width modulated controllers and the output nonlinear block \( N_2 \) such as thermocouple or thyristor transducer calibration curves. Mathematically, the HW model includes both Hammerstein and Wiener models as its special cases. The disturbance \( d_t \) is placed before the output static nonlinearity block \( N_2 \), which is different from the usual assumption that the disturbance is additive to the output after \( N_2 \) (see Fig. 2). In some instances this disturbance model is more realistic from a process operation point of view and has been discussed by Zhu (2002), Gomez and Baeyens (2004), Hagenblad et al. (2008). Although, in this HW model we don’t include the measure noise, it will not reduce the contribution of this paper. The reasons are i) in practice, the influence of process disturbances i.e. \( d_t \) in our case, in general, is much greater than the measurement noise; ii) current advances of sensor techniques can provide very good accuracy.

![Fig. 1. Hammerstein-Wiener model for this paper](image1)

![Fig. 2. Hammerstein-Wiener model with a different disturbance structure](image2)

3. PERFORMANCE INDEX FOR HW MODELS

The basis for MVPLB was developed by T.J. Harris (1989) where it was shown that the MVPLB for linear systems could be estimated from routine closed-loop data provided the process delay is known in advance. The underlying theory relies on the development of minimum variance controllers, outlined by Aström (1970) and Box and Jenkins (1970), and the existence of a feedback invariant (T.J. Harris, 1989). The feedback invariant is a dynamic component of the closed-loop system that is not affected by the feedback. In the case of linear systems, the feedback invariant can be easily recovered from a time series description of the closed-loop system. The feedback invariant is then used to estimate the variance of the output that would be achieved if a minimum variance controller were to be implemented.

The derivation of the minimum variance controller with respective to \( x_t \) for a process described by Eqs (2)-(4) is straightforward (Grumble, 2005, T.J. Harris and W. Yu, 2007). The series \( x_{t+b} \) can be written as:

\[ x_{t+b} = \frac{B(q^{-1})}{A(q^{-1})}N_1(u_t) + x_{ss} + d_t \]
\[ = \frac{B(q^{-1})}{A(q^{-1})}N_1(u_t) + d_{t+b|t} + x_{ss} + e_{t+b|t} \]
\[ = x_{t+b|t} + x_{ss} + e_{t+b|t} \]
For linear systems, the performance index is bounded. The 'Delta method' gives the variance approximation as easier to compute. Hence we will use the variance of the output in form of,

$$d_{t+b|t} = \frac{P_b(q^{-1})}{\phi(q^{-1})} a_t$$

(7)

and $e_{t+b|t}$ is the b-step ahead prediction error as,

$$e_{t+b|t} = (1 + \psi_1 q^{-1} + \cdots + \psi_{b-1} q^{-(b+1)}) a_{t+b}$$

(8)

$P_b(q^{-1})$ is a polynomial in the backshift operator obtained by solving the Diophantine equation:

$$\frac{\theta(q^{-1})}{\phi(q^{-1})} = 1 + \psi_1 q^{-1} + \cdots + \psi_{b-1} q^{-(b+1)} + q^{-b} \frac{P_b(q^{-1})}{\phi(q^{-1})}$$

(9)

From Eqn. (6), we can find that the b-step prediction error, $e_{t+b|t}$, is the control invariant. The control signal which results in the minimum achievable variance in the $x_t$ can be obtained by solving the following relation:

$$B(q^{-1}) N_1(ut) + d_{t+b|t} = 0$$

(10)

Therefore, $x_t$ under minimum variance control, $x^{MV}$, will depend on only the most recent $b$ past disturbances,

$$x^{MV} = x^{ss} + e_{t+b|t}$$

(11)

The process output under this MVC is

$$y^* = N_2(x^{MV})$$

(12)

The output $y^*$ is not precisely the minimum variance output, $y^{MV}$, as it is generally not true that $y^{MV} = y^*$. The predictor $y^*$ is often referred to as a naive predictor, (Terasvirta et al., 2005), and has the advantage that it is easier to compute. Hence we will use the variance of $y^*$ as the performance index for HW models

$$\eta = \frac{\sigma_y^2}{\sigma_{y^*}^2}$$

(13)

in place of $\sigma_{y^{MV}}^2$.

Remarks:

- The variance of $e_{t+b|t}$ will never be greater than the variance of $x_{t+b}$.
- For linear systems, the closed-loop setpoint, $y_{sp}$, will not affect the MVLPB. However, it does not hold for the nonlinear systems, since the variances of the $N_2(x^{ss} + e_{t+b|t})$ with the different values of $x^{ss}$ and same $e_{t+b|t}$ will be different.
- For linear systems, the performance index is bounded in the interval $[0,1]$, and this should also hold for our performance index.

To justify this assertion we provide the following approximate expansion. If we assume $E[x^{MV}] = x^{ss}$, then the denominator of the performance index in Eqn. (13) is

$$\sigma_y^2 = \text{var}(y_{t+b}) = \text{var}(N_2(x_{t+b}))$$

$$\approx \text{var}(N_2(x_{t+b} + x^{MV}))$$

(14)

Expanding this last term as a Taylor series following the 'Delta method' gives the variance approximation as

$$\sigma_y^2 \approx \left[ \frac{\partial N_2(x_{t+b})}{\partial x_{t+b} | x^{MV}=x^{ss}} \right]^2 \text{var}(x_{t+b}) + \left[ \frac{\partial N_2(x_{t+b})}{\partial x^{MV} | x^{MV}=x^{ss}} \right]^2 \text{var}(x^{MV})$$

(15)

which shows that $\sigma_y^2$ must always be $\geq \sigma_y^2$, and hence the index is bounded from 0 to 1.

4. ESTIMATING THE PERFORMANCE INDEX

4.1 Identifying the NARMA model

Coupling the HW model given in equations 1–4 with a feedback controller of the form $u_t = \gamma((y_t - y_{sp}), \cdots, (y_{t-1} - y_{sp}))$, we get the closed loop as shown in Fig. 1.

Fig. 3. Closed-loop system

This process is a class of nonlinear systems known as the Nonlinear AutoRegressive and Moving Average (NARMA) model (Leontaritis and Billings, 1985a,b) and has been widely used for identifications of many nonlinear systems. Several methods have been proposed for this purpose such as iterative least-squares, Ding and Chen (2005), Orthogonal Least Squares (OLS) methods (Chen et al., 1989) and Fast Orthogonal Search (FOS) methods (Korenberg, 1988). Use of Artificial Neural Network (ANN) models to approximate the NARMAX models is discussed in (Terasvirta et al., 2005, Chen and Billings, 1992).

Since the disturbance term is generally unmeasured, the identification of this NARMA model will require an iterative approach. The identification procedures will be: i) set the initial sequence $a_2$ by fitting a linear model or setting the $a_2$ to zero, ii) identify the NARMA model, iii) replace the initial sequence $a_2$ by the prediction errors or residuals, vi) repeat the steps ii) and iii) until a certain identification criteria is achieved.

One popular criteria for NARMA model identification is Akaike’s Information Criterion AIC(s) Chen et al. (1989):

$$\text{AIC}(\lambda) = K \ln \hat{\sigma}_y^2 + M - \lambda$$

(16)

where $M$ is the number of the model parameters, $K$ is the number of outputs and $\hat{\sigma}_y^2$ is the residual error. $\lambda$ is a positive value chosen to provide a penalty for model complexity. Using statistical arguments, a value of $\lambda = 4$ is recommended in (Chen et al., 1989, Leontaritis and Billings, 1987).

We use the program nlarx from the MATLAB System Identification Toolbox to estimate the NARMA model. In this function, the dynamic structure and the nonlinearity estimators are the two main design choices. The choice of nonlinearity estimators is very often arbitrary and needs

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The estimation of the performance index in Eqn. (13)

\[ \text{The performance index is strictly bounded in} \ [0, \infty) \]

\[ V = \text{var}(y) \text{controller defined in Eqn. (10), the process output at time} \]

\[ t + b \text{ is equal to,} \]

\[ y_{t+b} = F(y_{sp}, y_{t+b}, a_{t+b}, a_{t+b+1}, 0, \ldots, 0, x^{ss}) \] (18)

which should be equal to \( y^* = N_2(x^{MV}) \) in Eqn. (12).

We can establish the output variance analytically following the definition

\[ \int_{-\infty}^{-\infty} \cdots \int_{-\infty}^{-\infty} (y_{t+b} - y_{t+b})^2 f(a_{t+b}, \ldots, a_{t+b+1}) \, d(a_{t+b}) \ldots d(a_{t+b+1}) \]

(19)

where \( f(.) \) is the pdf function. However given the arbitrary nature of the nonlinear function \( F \) and the high dimension integrals, it will be difficult to obtain the analytical solution from Eqn. (19). A simpler numerical strategy is to use a Monte Carlo method to estimate the variance of \( y_{t+b} \) with

\[ \sigma_y^2 = \frac{1}{m} \sum_{i=1}^{m} (F(y_{sp}, \ldots, y_{sp}, a_{t+b}^i, \ldots, a_{t+b+1}^i, 0, \ldots, 0, x^{ss}) - F(y_{sp}, \ldots, y_{sp}, 0, \ldots, 0, x^{ss}))^2 \]

(20)

The proposed method to estimate the MVPLB can also be applied on the Hammerstein or Wiener models since they are simply special cases of the HW model.

4.3 An alternative performance index for nonlinear systems

We can write the output in Eqn. (17) as \( y_{t+b} = N_2(x_1, x_2) \) where \( x_1 \) includes the disturbances entering the system after time \( t \), and \( x_2 \) includes anything prior to, and including time \( t \). We know that these two groups are uncorrelated. Using an analysis of variance (ANOVA) method, the variance of output, \( y \) can be decomposed as (Harris and Yu, 2010),

\[ \text{var}(y) = V_{x_1} + V_{x_2} + V_{x_1,x_2} \] (21)

where the \( V_{x_i} \), \( i = 1, 2 \) denotes the main effect of \( x_i \) on the \( \text{var}(y) \) and \( V_{x_1,x_2} \) is the interaction contributing to the \( \text{var}(y) \) that is not accounted for the main effects \( V_{x_1} \) and \( V_{x_2} \). Consequently, a suitable performance index is defined as

\[ \eta_1 = \frac{V_{x_1}}{\sigma_y^2} \] (22)

Remarks:

- The term \( V_{x_1} \) depends on the controller used.
- \( V_{x_1} \neq \sigma_y^2 \) again except when the closed loop is controlled by the minimum variance controller with respect to \( x \) defined in Eqn. (10).
- The performance index is strictly bounded in \([0, 1]\). If \( \eta_1 \) reaches 1, it means that the variance of outputs is contributed mostly by the \( x_1 \), so the system controller is close to the minimum variance controller.
- The estimation of the performance index in Eqn. (13) requires one to estimate the MVPLB, \( x^{MV} \), with respect to \( x \) first. It may become very difficult as it is necessary to set \( x_{t+b+1} = 0 \). However as the alternative performance index in Eqn. (22) does not need to estimate \( x^{MV} \), it may be more suitable for nonlinear systems where one does not know the model structure.

5. SIMULATION EXPERIMENTS

The purpose of this section is to demonstrate the proposed method to estimate the MVPLB or alternatively the performance index for a class of nonlinear HW models. In the first example, a Wiener model is used to test the proposed approach. In the second example, a HW model will be used.

5.1 A Wiener model

A pH neutralization system modelled as a Wiener process from Gomez and Baeyens (2004) is adopted as a simulation test. The linear plant is

\[ B(q^{-1}) = \frac{0.0049 - 0.0094q^{-1} + 0.0045q^{-2}}{1 - 2.9160q^{-1} + 2.8339q^{-2} - 0.9179q^{-3}} \] (23)

with time delay \( b = 3 \) controlled using a PI feedback controller

\[ G_c = \frac{0.1 - 0.5q^{-1}}{1 - q^{-1}} \] (24)

This plant is subjected to an additive disturbance of

\[ d_t = \frac{a_t}{1 - 0.8q^{-1}} \] (25)

where \( a_t \) is a sequence of independent and identically distributed Gaussian random variables with zero mean and nominal variance \( \sigma_d^2 = 0.01 \). The static nonlinearity \( N_2 \) is plotted in the right-hand corner of Fig. 4.

Due to the intractable nature of the nonlinearity, a Monte Carlo method is used to estimate the performance of the proposed strategy to estimate the variance, \( \sigma_y^2 \). One thousand observations generated from the Wiener model are passed to the NARMA identification using Matlab function \textbf{nlarx}. Once the NARMA model is identified, the MVPLB and the performance index can be estimated using the method described in Eqn. (20). This procedure is repeated 500 times.

The estimates of the performance index using the linear ARMA and proposed methods are shown in the comparative box plot in Fig. 4. The true value of the performance index for this example is \( \eta = 0.404 \).

5.2 A Hammerstein-Wiener model

In this section, a HW model with linear dynamics

\[ q^{-b}B(q^{-1}) \]

\[ A(q^{-1}) = \frac{q^{-3}(1 - 0.5q^{-1})}{1 - 1.5q^{-1} + 0.7q^{-2}} \] (26)

\[ \text{are simply special cases of the HW model.} \]

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under a feedback control with a PI controller,

\[ G_c = \frac{0.05 - 0.02q^{-1}}{1 - q^{-1}} \] (27)

The first static nonlinearity, \( N_1 \), is a coulombic and viscous friction nonlinearity, \( u = \text{sgn}(u_t)(|u_t| + 0.07) \), where 0.07 is the offset, and the trailing nonlinearity, \( N_2 \), is a third order polynomial, \( y_t = x_t + 0.25x_t^2 + 0.125x_t^3 \). Both \( N_1 \) and \( N_2 \) are inserted in Fig. 5. The same disturbance model structure used in the Wiener example is adopted for this simulation, the only difference being that the variance of \( a_t \), \( \sigma^2_c = 0.05 \). The estimates of the performance index using the proposed method are plotted in Fig. 5.

6. DISCUSSION

From the simulations for Wiener and HW models, we observe that the estimates of the performance index from the linear CPA techniques (using a linear ARMA model to fit the process output) have the significant biases that for our examples are 30–47% deviating from the true values.

Our proposed method reduces this bias down to the range 7–12%.

An obvious question regarding the application of our proposed method is when is it necessary to apply nonlinear, as opposed to the simpler linear, CPA strategies. The answer is simple: one just needs to check the nonlinearity of the output perhaps using the Hinich test proposed by Hinich (1982) and discussed further in Tong (1990), Haber and Keviczky (1999). In a related application, these tests have been used for diagnosing the valve stiction in Choudhury et al. (2004) and estimating MVPLB for the valve stiction problem Yu et al. (2008, 2010).

7. CONCLUSIONS

The contribution of this work is to propose two performance indices for the control loops following the Hammerstein–Wiener (HW) model structure where the additive disturbance enters the system before the second static nonlinearity. The first performance index is based on the prediction error from the naive prediction when the minimum variance control with respect to \( x_t \) is implemented. A second index based on the variance decomposition is also proposed in this paper. Since it is simpler to calculate than the former performance index, it may be used for the more complex nonlinear systems.

This algorithm requires only observable signals and crude estimates of the plant dominant time constants and plant delay. The proposed method does not require one to identify the process (linear dynamic and static nonlinearity) and disturbance structure, but one does need to estimate the closed-loop nonlinear model. The proposed method works well even in the case of non-smooth nonlinearities. Simulation results for a range of differentiable and non-differentiable nonlinearities show that our approach can provide reliable estimates for CPA on the HW nonlinear systems in contrast to simply ignoring the nonlinearity.

For this study, we did not include measurement noise, although it will be considered in the future. Another possible direction based on this paper is to find out the minimum variance controller with respective to the outputs.

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