INEQUALITIES FOR CONTINUOUS-TIME MODEL FOR SCHEDULING OF CONTINUOUS PROCESSES

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Abstract
This paper presents a mathematical formulation for large-scale continuous process scheduling problem using unit-specific event-based continuous-time representation. The formulation is based on the work of Ierapetritou et al., (1999). The model considers the changeovers for multi-purpose units and tanks, setup constraints for parallel units and tanks, and includes product tanks that satisfy demand of products with intermediate due-date requirements. The main objective of the model is to satisfy the demand by respecting the due-dates and maximize the performance by minimizing changeovers and setup costs. For the large-scale scheduling problem, the resulting formulation is a complex mixed-integer linear programming model that is difficult to solve to global optimality. A set of valid inequalities are proposed that improves the computational performance of the model significantly. Applicability of the proposed valid inequalities is demonstrated by studying a case study.

Keywords
Continuous process, Continuous-time model, Short-term scheduling, Valid inequalities.

Introduction
Much of the work in the area of continuous process manufacturing has been focused on the small-scale scheduling problem but the scheduling problem of large-scale multiproduct and multipurpose continuous plant have received significant less attention. Large-scale scheduling problems arise frequently in chemical industry where the main objective is to assign sequence of tasks to processing units within certain time frame such that demand of each product is satisfied before its due date.

Over the last two decades, different mathematical formulations are proposed for continuous process scheduling on basis of time representation. In discrete-time approaches, the time horizon is divided into a number of fixed time intervals, whereas the continuous-time approaches are based on time slots/events of unknown length. Two types of continuous-time approaches are studied in the literature, where the first is based on a set of events that are used for all tasks and units (global event based models), and the second approach introduces event points based on a task (unit-specific event based models). A couple of excellent reviews can be found in the literature (Floudas and Lin, (2004); Mouret et al., (2010)) Short-term scheduling models based on unit-specific event points have received considerable attention in the literature (Ierapetritou et al., (1999); Giannelos and Georgiadis, (2002); Mendez and Cerd, (2002); Shaik and Floudas, (2007)). Shaik et al., (2009) studied large-scale scheduling models based continuous time approach, where they considered changeovers and proposed a two-level framework to effectively deal with complexity of the medium-term scheduling problem.

In our work we propose a large-scale scheduling model using unit-specific event-based continuous-time representation. The large-scale continuous process

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scheduling problems are highly complex and to reduce the complexity, in our work we present a set of valid inequalities that reduces the complexity and improves the performance of the model.

Proposed model

In this section we present the mathematical formulation for the continuous manufacturing production plant that is an improvement over the model of Ierapetritou et al., (1999). A state-task network (STN) representation introduced by Kondili et al., (1993) is used to describe the plant operations. A detailed description of each variable and parameter used in the model can be found in the nomenclature section.

Variable recipe constraint

Constraints (1-2) enforce that the amount produced or consumed at production unit is restricted by the imposed recipe bounds. Constraint (3) satisfies material balance at each production unit. It states that the total amount of material consumed is equal to the total amount of material produced.

\[
\begin{align*}
\rho_{s,i}^{p,\text{min}} \sum_{j \in I^*} b_{j,i,j,n} & \leq b_{s,i,j,n} \leq \rho_{s,i}^{p,\text{max}} \sum_{j \in I^*} b_{j,i,j,n}, \\
\forall s \in S, i \in I^*, j \in J, n \in N \\
\rho_{s,i}^{c,\text{min}} \sum_{j \in I^*} b_{c_{s,i},j,n} & \leq b_{c_{s,i},j,n} \leq \rho_{s,i}^{c,\text{max}} \sum_{j \in I^*} b_{c_{s,i},j,n}, \\
\forall s \in S, i \in I^*, j \in J, n \in N \\
\sum_{j \in I^*} b_{s,i,j,n} = \sum_{j \in I^*} b_{c_{s,i},j,n}, \forall j \in J, i \in I, n \in N
\end{align*}
\]

Material balance constraints

Constraints (4-5) provide the material balance for the material produced or consumed at each production unit, and constraints (6a-6b) enforce the material balance over the storage tanks.

\[
\begin{align*}
\sum_{j \in J^*} b_{s,i,j,n} = \sum_{k \in K^* \cap K_{s,i}} K_{i,i,\text{in},n} + \sum_{j \in J^* \cap J_{i,i,\text{out}}} J_{i,i,\text{out},n} \\
+ U_{i,i,\text{in}}, \forall s \in S, i \in J^*, j \in J^*, n \in N \\
\sum_{j \in J^*} b_{c_{s,i},j,n} = \sum_{k \in K^* \cap K_{s,i}} K_{i,i,\text{out},n} + \sum_{j \in J^* \cap J_{i,i,\text{out}}} J_{i,i,\text{out},n} \\
+ U_{i,i,\text{in}}, \forall s \in S, j \in J^*, n \in N \\
st_{i,i,\text{in}} = st_{i,i,\text{in}} + \sum_{j \in J^*} K_{i,i,\text{in},n} + R_{i,i,\text{in},n} - \sum_{n \in N} L_{i,i,\text{in},n}, \\
- \sum_{n \in N} L_{i,i,\text{in},n}, \forall s \in S, k \in K^*, n = 1 \\
st_{i,i,\text{out}} = st_{i,i,\text{out}} + \sum_{j \in J^*} K_{i,i,\text{out},n} + R_{i,i,\text{out},n} - \sum_{n \in N} L_{i,i,\text{out},n}, \\
- \sum_{n \in N} L_{i,i,\text{out},n}, \forall s \in S, k \in K^*, i < n \leq N
\end{align*}
\]

Capacity constraints for production units

Constraint (7a) enforces that the material processed by unit \( j \) performing task \( i \) is bounded by the maximum and minimum unloading rate for each unit over the entire time horizon.

\[
R_{i,i,\text{in}}^{\text{min}} (T_{j,i} - T_{s,i}) \leq \sum_{n \in N} b_{i,i,j,n} \leq R_{i,i,\text{in}}^{\text{max}} (T_{j,i} - T_{s,i}), \\
i \in I, j \in J, n \in N
\]

bounds on the total amount of the material processed at each unit over the entire time horizon.

\[
R_{i,i,\text{in}}^{\text{min}} (T_{j,i} - T_{s,i}) \leq \sum_{n \in N} b_{i,i,j,n} \leq R_{i,i,\text{in}}^{\text{max}} (T_{j,i} - T_{s,i}), \\
i \in I, j \in J, n \in N
\]

Capacity constraints for storage tanks

Constraints (9-11) define the binary variables associated with the flow in and out of the tanks. Constraint (12) enforces the maximum capacity of the tank and constraints (13a-13c) define the binary variable associated with material present in the tank.

\[
K_{i,i,\text{in},n} \leq M \cdot \text{in}_{i,i,\text{in},n}, \forall j \in J, k \in K_{i,i,\text{in}}, n \in N \\
K_{i,i,\text{out},n} \leq M \cdot \text{out}_{i,i,\text{out},n}, \forall j \in J, k \in K_{i,i,\text{out}}, n \in N \\
L_{i,i,\text{in},n} \leq M \cdot l_{i,i,\text{in},n}, \forall k \in K_{i,i,\text{in}}, s \in S_{i,i,\text{in}}, o \in O_{i,i,\text{in}}, n \in N
\]

The maximum and minimum unloading rate for product storage tanks must be bounded as specified by constraint (14).

\[
U_{k}^{\text{min}} (T_{o_k,i,\text{in}} - T_{o_k,i,\text{out}}) \leq L_{k,i,\text{in}}, \\
U_{k}^{\text{max}} (T_{o_k,i,\text{in}} - T_{o_k,i,\text{out}}), \forall k \in K_{i,i,\text{in}}, o \in O_{i,i,\text{in}}, n \in N
\]

Demand constraints

The plant has two types of products; type A can be stored in tank and has intermediate due dates and type B that is not stored in tank and whose demand should be satisfied based on the production unit. Constraint (15) guarantees that sufficient amount of product will be available to meet the demand.

\[
D_{s,i} + r_{s,i} + d^g_{s,i} - r^g_{s,i} + \sum_{j \in J} H_{s,i,\text{in},n} + \sum_{j \in J} U_{o_k,i,\text{in}} \leq D_{s,i} + r_{s,i} + d^g_{s,i} + r^g_{s,i}, \forall s \in S, o \in O_{i,i,\text{in}}, n \in N
\]

Due to production capacity limitation, sometimes the demand order of finished product cannot be satisfied within its due date. To obtain a feasible solution positive artificial variables associated with over and under production are introduced and they are penalized in the objective function to minimize quantity giveaway.

Allocation constraints

Constraints (16-18) express the requirements that each multipurpose production unit and storage tank can only
perform one task at any given event point. Constraint (21) restricts that the product tank can satisfy at most one demand order at any event point.

\[
\sum_{i \in J} w_{v,i,n} \leq 1, \quad \forall j \in J, n \in N
\]  
(16)

\[
\sum_{k \in K} y_{v,k,n} \leq 1, \quad \forall k \in K^n, n \in N
\]  
(17)

\[
\sum_{i \in I} l_{i,n,n} \leq 1, \quad \forall k \in K^p, n \in N
\]  
(18)

**Set-up constraints**

Set-up variables are 0-1 continuous variables defined by constraints (19a-19b) for production unit and constraints (20a-20b) for storage tanks. In our work, we include set-up constraints only for parallel units and tanks.

\[
\beta_{j,n} \leq \sum_{k \in K} y_{v,k,n} + \sum_{k \in K} y_{v,k,n}, \quad \forall k \in K^n, n \in N
\]  
(19a)

\[
\beta_{j,n} \geq \sum_{k \in K} y_{v,k,n} - \sum_{k \in K} y_{v,k,n}, \quad \forall k \in K^n, n \in N
\]  
(19b)

\[
\alpha_{j,n} \leq \sum_{i \in I} w_{v,i,n} + \sum_{i \in I} w_{v,i,n}, \quad \forall j \in J^n, n \in N
\]  
(20a)

\[
\alpha_{j,n} \geq \sum_{i \in I} w_{v,i,n} - \sum_{i \in I} w_{v,i,n}, \quad \forall j \in J^n, n \in N
\]  
(20b)

**Changeovers constraints**

Changeover constraints proposed by Shaik et al. (2009) are used in this work. Changeovers variables are 0-1 continuous variables and constraints (21-22) are used to define the changeover variables. Changeovers between modes of operations cause disturbances and additional costs, thus, few changeovers are desired.

\[
X_{c,i,j,n} \leq w_{v,i,n} + \sum_{i \in I} w_{v,i,n} - 1 - \sum_{i \in I} w_{v,i,n}, \quad \forall j \in J, i \in I, i' \in I, i' \neq i, n \in N
\]  
(21a)

\[
X_{c,i,j,n} \geq w_{v,i,n} - \sum_{i \in I} w_{v,i,n} - 1 - \sum_{i \in I} w_{v,i,n}, \quad \forall j \in J, i \in I, i' \in I, i' \neq i, n < N, n' \leq N
\]  
(21b)

\[
\eta_{c,i,k,n} \leq \eta_{c,i,k,n}, \quad \forall k \in K, s \in S, s' \in S, s' \neq s, n < N
\]  
(22a)

\[
\eta_{c,i,k,n} \leq \eta_{c,i,k,n} + \sum_{i \in I} y_{v,i,n} - 1, \quad \forall k \in K, s \in S, s' \in S, s' \neq s, n < N
\]  
(22b)

\[
\eta_{c,i,k,n} \leq \eta_{c,i,k,n} + \sum_{i \in I} y_{v,i,n} - 1, \quad \forall k \in K, s \in S, s' \in S, s' \neq s, n < N, n < N
\]  
(22c)

\[
\eta_{c,i,k,n} \leq \eta_{c,i,k,n} + \sum_{i \in I} y_{v,i,n} - 1, \quad \forall k \in K, s \in S, s' \in S, s' \neq s, n < N, n < N
\]  
(22d)

**Sequence constraints for production units**

Finishing time of any task must be greater than the starting time of that task, as represented by constraint (23).

\[
T_{f,j,k,n} \geq T_{s,j,k,n} + UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(23)

\[
T_{s,j,k,n} \geq T_{f,j,k,n} - UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(24)

\[
T_{f,j,k,n} \geq T_{s,j,k,n} + t_{clean}(2Z_{f,j,k,n} - 1), \quad \forall j \in J, i \in I, i' \in I, i \neq i', n < N
\]  
(25)

Constraints (26a-26d) represent that the two consecutive productions with no storage in between happen at the same time because production units operate as continuous processes.

\[
T_{f,j,k,n} \leq T_{s,j,k,n} + UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(26a)

\[
T_{s,j,k,n} \leq T_{f,j,k,n} - UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(26b)

\[
T_{f,j,k,n} \leq T_{s,j,k,n} + UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(26c)

\[
T_{s,j,k,n} \leq T_{f,j,k,n} - UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall j \in J, n \in N
\]  
(26d)

**Sequence constraints for storage tanks:**

Constraints (27-29) enforce the sequence time requirement for material movement transfer task from one event point to next event point.

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, n \in N
\]  
(27a)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, n < N
\]  
(27b)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, n \in N
\]  
(28a)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, n < N
\]  
(28b)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, o \in O_{s,j,k}, n \in N
\]  
(29a)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, o \in O_{s,j,k}, n < N
\]  
(29b)

Start time sequence constraints for tanks receiving or sending material from/to multiple destinations are given by constraints (30a-30c).

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, j' \in J^n, j' \neq j, n < N
\]  
(30a)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, j \in J^n, j' \in J^n, j' \neq j, n < N
\]  
(30b)

\[
T_{s,j,k,n} \geq T_{s,j,k,n}, \quad \forall k \in K, o \in O_{s,j,k}, o' \in O_{s,j,k}, o' \neq o, n < N
\]  
(30c)

Sequence constraints for material transfer in and out of tanks happening at the same event point is enforced by equations (31-32). The start and finish time of the material transfer must align as to not violate the material balance requirement.

\[
T_{s,j,k,n} \geq T_{s,j,k,n} - UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall k \in K, s \in S, j \in J^n, j' \in J^n, n \in N
\]  
(31a)

\[
T_{s,j,k,n} \geq T_{s,j,k,n} + UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall k \in K, s \in S, j \in J^n, j' \in J^n, n \in N
\]  
(31b)

\[
T_{s,j,k,n} \geq T_{s,j,k,n} - UH(2 - w_{v,j,k,n} - w_{v,j,k,n}), \quad \forall k \in K, s \in S, j \in J^n, j' \in J^n, n \in N
\]  
(31c)
\[ T_{sf,k,n} \leq T_{sf,j,n} + UH \left( 2 - w_{j,i,n} - in_{j,k,n} \right) \]  
\( \forall k \in K, s \in S, j \in J^s, i \in I_n, n \in N \)  

Constraints (33-34) connect material transfer in/out of a tank from one event point to next event point.

Intermediate due dates:

Intermediate due date requirements for group A products, which are stored in product tanks, are given by constraints (37a-37b). To consider demurrage, slack variables are utilized and penalized in the objective function.

All tasks should start and finish before the end of the scheduling time horizon as stated in (38a-38d). The scheduling horizon is bounded as \( H \leq UH \).

Objective function

The objective function (39) is used to maximize the performance and revenue of total production. The performance is represented by the minimization of utilization of units and tanks, start up set-ups, changeovers, demurrage, and under and over production and revenue term is calculated from the sales of the final products. The penalty weights are assigned arbitrary to each term depending on its importance in schedule and note that the different penalty parameters have significant effect on the computational time required to obtain an optimal solution.

\[ z = \sum_{i,j,k,n} c_{i,j,k,n} \cdot w_{i,j,k,n} + \sum_{i,j,k,n} c_{i,j,k,n} \cdot y_{i,j,n} + \sum_{i,j,k,n} c_{i,j,k,n} \cdot \delta_{i,j,k,n} \]  

\( \forall k \in K, s \in S, j \in J^s, i \in I_n, n \in N \)  

Valid Inequalities

Valid inequalities are added in the formulation to improve the computational efficiency of the proposed model. Constraints (40a-40c) enforce that if there is a material flow into/out of the tank at event point \( n \), then the binary variable \( y_{i,j,n} \) is 1.

\[ \sum_{j \in J^s} w_{j,i,n} \leq \sum_{j \in J^s} y_{i,j,n} \quad \forall k \in K, s \in S, n \in N \]  

(40a)

\[ \sum_{j \in J^s} y_{i,j,n} \leq \sum_{j \in J^s} \delta_{i,j,n} \quad \forall k \in K, s \in S, n \in N \]  

(40b)

\[ \sum_{j \in J^s} \delta_{i,j,n} \leq \sum_{j \in J^s} y_{i,j,n} \quad \forall k \in K, s \in S, n \in N \]  

(40c)

Constraints (41a-41b) require material \( s \) flow in/out of tanks to be active at event point \( n \) if the unit \( j \) is processing the material at that event point.
\[
\sum_{k \in K} \sum_{i \in F, j \in S} in_{k,i,j} \geq \sum_{o \in J^*, s \in S^*, n \in N} \sum_{j \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \forall j \in J^*, s \in S^*, n \in N \tag{41a}
\]

\[
\sum_{k \in K} \sum_{i \in F, j \in S} out_{k,i,j} \geq \sum_{o \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \forall j \in J^*, s \in S^*, n \in N \tag{41b}
\]

Constraint (42) enforces the material balance constraint in addition to the constraint presented in equation (3).

\[
\sum_{k \in K} K_{i,j,k,n} + \sum_{j \in J^*, s \in S^*, n \in N} \sum_{j \in J^*, s \in S^*, n \in N} J_{i,j,k,n} + U_{i,j,k,n} = \sum_{k \in K} K_{i,j,k,n} + \sum_{j \in J^*, s \in S^*, n \in N} \sum_{j \in J^*, s \in S^*, n \in N} J_{i,j,k,n} + U_{i,j,k,n} \tag{42}
\]

If two units are consecutive without any storage tank between them, then constraint (43a) imposes the simultaneous operation of these units due to the continuous operation mode. However, this constraint is not imposed on parallel production units that can produce the same type of products. For units that follow or are followed by parallel units, valid inequalities 43b and 43c are included.

\[
\sum_{o \in O} w_{i,j,s,n} = \sum_{o \in O} w_{i,j,s,n} \tag{43a}
\]

\[
\sum_{j \in J^*, s \in S^*, n \in N} \sum_{j \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \geq \sum_{j \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \tag{43b}
\]

\[
\sum_{j \in J^*, s \in S^*, n \in N} \sum_{j \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \geq \sum_{j \in J^*, s \in S^*, n \in N} w_{i,j,s,n} \tag{43c}
\]

The demand order set \( O \) is arranged according to the ascending due date start time. That is, \( t_{\text{start}} \leq t_{\text{start}} \), \( \forall o, o' \neq o, o' > o \). Inequalities 44a are applied to require the demand order \( o \) to be satisfied by product tank \( k \) earlier than order \( o' > o \). Furthermore, if the initial inventory of the products is less than the required total minimum demand orders, then the production should take place before the demand is fulfilled. This requirement is captured by constraint (44b).

Here, \( pr_i = \sum_{o \in O} D_{i,o} - \sum_{k \in K} sto_{i,k} \).

\[
l_{i,o,n} \leq \sum_{k \in K} l_{i,o,n} \tag{44a}
\]

\[
\forall s \in S, k \in K, o \in O, o' \neq o, o' > o, n \in N \]

\[
l_{i,o,n} \leq \sum_{j \in J, s \in S, n \in N} w_{i,j,s,n} \tag{44b}
\]

\[
\forall s \in S, k \in K, o \in O, pr_i > 0, n \in N
\]

**Case Study and Results**

We apply our model to a case study that consists of 14 multipurpose production units that processes 4 different raw materials to produce 18 final products. Of the 14 production units, two are parallel units (R1 and R2). Units (L1-L5) have cleaning downtime impose between certain task mode changeovers. The intermediates that are consumed by unit (M1) are stored in five dedicated tanks and other seven intermediates share three different storage tanks but with restricted allocation to these tanks. Final products (mp1, mp2, and mp3) are stored in three dedicated product. Demand of all eighteen final products is bounded by the maximum and minimum limits. In our problem we consider five demand orders and scheduling time-horizon of 240 hours. The state-task network of the problem is given in Figure 1.

**Figure 1.** STN representation of the plant

The problem is solved on a Dell Precision (Intel® Xeon®TM with CPU 3.20 GHz, 3.19 GHz, and 2 GB memory) running on Windows XP using CPLEX 12.2.0/GAMS 23.6.3. The proposed model requires 4 event points in order to satisfy group A and group B products demand. The objective function corresponds to minimization of costs and maximization of revenue. The optimal objective function is found to be 1982.87 and statistics of model with and without valid inequalities are reported in Table 1.

**Table 1. Model statistics and results**

<table>
<thead>
<tr>
<th>Without Valid-inequalities</th>
<th>With Valid-inequalities</th>
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<tr>
<td>Events</td>
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<td>Continuous Var.</td>
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<td>Constraints</td>
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<td>Iterations</td>
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</tr>
</tbody>
</table>

When the valid inequalities are included in the model, the number of variables remains the same, but the number of constraints and nonzero elements increases. Valid inequalities have no effect on quality of the optimal solution; rather their effect is concentrated in significantly reducing the computational effort needed to find the optimal solution. The CPU time required to reach optimal solution is improved significantly when the valid inequalities are present versus when they are not included in the model. In the problem studied, it is not possible to
find optimal solution within 7 hours; however the model with valid-inequalities obtains the optimal solution in 1783 seconds an improvement of 1 order of magnitude. The optimal objective schedule has minimum changeovers and for units L1-L5, the changeovers are favored between those tasks that have minimum clean-up downtime imposed.

Conclusions

In this paper, we present a scheduling model for large-scale continuous processes using unit-specific event points and continuous-time representation. A set of valid inequalities are proposed that reduces the CPU resolution time by a significant factor for large scale problems. Propose valid inequalities perform best when the problem consists of parallel and sequential units, product tanks, multipurpose units and tanks, and intermediate demand due-dates requirements.

Nomenclature

Indices: n :Tasks, j :Units, o :Orders, s :Tasks
Sets: I_j :Tasks performed in unit j, I^p_j / I^c_j :Tasks produce/consume material s, J :Production units, J^p_j / J^c_j :Units produce/consume material s, J^p_j / J^c_j :Units which are suitable for performing task i, J^p_λ :Parallel production units, J^p_λ / J^c_λ :Units that produce/consume material stored in tank k, J^p_λ :Units that follow unit j (no storage in between), K :Storage tanks, K^p :Parallel tanks, K^p / K^c :Tasks that store material produce/consumed by unit j, K^p / K^c :Tasks that can store final products , K :Tanks that can store material s , N :Event points, O :Demand Orders for Group A products, S :States, S_j / S_λ :Group A/Group B products, S :Materials that can be stored in tank k, S^p / S^c :Materials produced/consumed by task i, S^p / S^c :Materials produced/consumed by unit j
Parameters: D^p_s / D^c_s , r^p / r^c : Demand requirements bounds, \( R^\eta_m / R^p_m \)minimum or maximum rate of production, \( U^m_k / U^p_k \)Minimum or maximum rate of unloading, sto_s :Initial amount of state s in tank k, UH :Available time horizon, V^m :Maximum available storage capacity of storage tank k, \( \rho^\eta_m / \rho^p_m \) :Proportion of state s produced/consumed by task i, Binary Variables: \( w_j \) :Assignment of task i in unit j at event point n, \( in_{jkn} / out_{jkn} \) :flow of s into/out of tank k from unit j at point n, \( l_{km} \) :flow out of tank k to order o at event point n, \( y_{s,i} \) :material is stored in tank k at event point n. Positive Variables: \( b_{j\eta} / b_{j\eta} \) :material produced/consumed by task i in unit j at point n, \( d^p_{\eta} / d^c_{\eta} , r^p / r^c \) :Over and under production, H :Total time horizon, J^F_{\nu j} :Flow from unit j to consecutive unit j’ at point n, J^F_{\nu j} :Flow of material s into/out of tank, L^F_{\nu j} \} U^F_{\nu j} :Flow of product from tank/unit, Rif_{\nu j} \} U^F_{\nu j} :Flow of raw material to tank/unit, sto_{i} :Amount of material present in tank k, T^early \} T^late :Due date violations for order o, T^f_{\nu j} \} T_{\nu j} :Finish/start time for task i in unit j, T^f_{\nu j} \} T_{\nu j} :Finish/start time of flow from tank k for order o at event point n, T^f_{\nu j} \} T_{\nu j} :Finish/start time of flow from unit to tank, T^f_{\nu j} \} T_{\nu j} :Finish/start time of flow from tank to unit, \( \alpha_{\nu j} / \beta_{\nu j} \) :1 if the unit/tank becomes active for very first time at event point n, \( \eta_{\nu j} \) :1 if service changeover in tank k from s at event point n to s’ at later event point, \( x_{\nu j} \) :1 if task at unit j changes from i at event point n to i’ at later event point.

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References


