Airborne Technology for Distributed Air Traffic Management

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Abstract — Worldwide air traffic levels are growing at a rate expected to double the current traffic level by 2020. The current technology Air Traffic Control systems are stretched to their limit and are prone to large delays during the peak summer travel season. There is doubt that the current systems can be scaled up to meet the expected demand levels.

Many Air Traffic Management automation systems have been proposed to increase controller capability, and some are in operation. While ATM automation systems will help handle more traffic, it is still doubtful that they can grow to meet the doubling in traffic levels foreseen.

This paper presents an introduction to Distributed ATM using the capability of airborne electronic systems to further relieve the controller workload. An overview of avionics capabilities is presented, followed by a detailed description of five specific examples of airborne capability that can be used to increase airspace capacity, as listed below.

1. An onboard method to control an aircraft to cross a terminal area waypoint at a Required Time of Arrival.
2. A trajectory negotiation process whereby the ground-based ATM system uses the 4D predicted trajectory computed by the aircraft, gives the aircraft RTA constraints to solve traffic conflicts, and contracts the aircraft to stay within a specified tolerance of the predicted 4D trajectory.
3. A formation flight system whereby multiple aircraft can be flown close together and controlled as a single aircraft.
4. An onboard alerting system for closely spaced parallel approaches that can increase the capacity of an airport in instrument meteorological conditions.
5. An onboard collision avoidance system that generates a conflict-free trajectory through complex airspace.

I. INTRODUCTION

There is doubt that the current ATM systems can be scaled up to meet expected demand levels, since further subdividing airspace sectors into smaller pieces to reduce the number of aircraft that a controller is working increases the number of airspace handoffs for each flight, and controllers need a minimum size of airspace to allow room for maneuvering aircraft to avoid conflicts. The procedures for handling flights manually introduce delay by putting aircraft on standard tracks at similar airspeeds so that the controllers can deal with them [47].

Air Traffic Management automation systems, or decision support tools, are being developed and fielded to assist controllers in handling more aircraft and larger airspace per controller station with some success [38]. These systems are typically built upon predicting the trajectories of aircraft and scanning for future conflicts. Early conflict detection is useful, but it is still up to the controller to determine appropriate action and communicate instructions to the aircraft verbally. It is doubtful that these systems can grow to meet the doubling in traffic levels foreseen.

A variety of airborne technologies have the potential to further reduce controller workload by distributing the air traffic management responsibility between the aircraft and ground-based systems, and many concepts have been developed to take advantage of these capabilities to improve how aircraft are controlled. In the simplest concepts the aircraft send accurate current state and future intent data to ground automation systems to improve their tracking and prediction accuracy [5], [9], [18], [29], [62], [65], [66], [67]. In the most complex concepts the aircraft automation systems monitor their own traffic situation and take full responsibility for their own separation assurance [2]. There is also a broad spectrum of integrated air/ground ATM concepts between these two extremes [1], [6], [12], [15], [16], [39], [42], [45], [74].

II. BACKGROUND OVERVIEW

A. Flight Management System (FMS)

Most modern transport aircraft are equipped with a Flight Management System (FMS) that assists the pilot with flight planning and automatically controlling the aircraft. The FMS blends all navigation data on the aircraft from various sources to maintain an accurate status of aircraft position, velocity, and atmospheric conditions. The pilot enters the flight plan into the FMS, including lateral route and vertical requirements, and the FMS predicts the full aircraft trajectory including altitude, speed, fuel, and time. Some FMS systems are capable of optimizing the vertical profile to minimize the cost of fuel consumption and flight time by selecting cruise altitude(s) and climb/cruise/descent airspeeds. The FMS can automatically guide the aircraft to follow the flight plan, sending appropriate commands to the autopilot and autothrottle systems.

B. Automatic Dependent Surveillance-Broadcast (ADS-B)

An ADS-B equipped aircraft broadcasts a message via a digital datalink, which includes position, velocity, and possibly other information. Other ADS-B equipped aircraft or ground systems can receive this information and use it for
distributed air traffic management. References [3], [4], [24]-[29], [52], [53], and [60] provide more information on current and potential use of the ADS-B technology.

A typical ADS-B message may contain one or more reports: either periodic reports (State Vector Report or Mode Status Report), or on-condition reports (Air Referenced Velocity Report, Target State Report, or Trajectory Change Report) [53].

ADS-B technology aims to improve aviation safety by giving pilots and controllers reliable, accurate, real-time information about aviation traffic, even where radar is ineffective or unavailable. ADS-B technology can also be scaled and adapted for use in general aviation and in ground vehicles.

C. Traffic Information Service (TIS-B)

TIS-B is a surveillance service that derives traffic information from ground surveillance sources and broadcasts to equipped aircraft or surface vehicles with the intention of supporting aircraft surveillance applications [28], [56]. TIS-B service is provided where there is both adequate surveillance coverage from ground sensors (Surveillance Coverage Volume) and adequate RF coverage from TIS-B Ground Stations (RF Coverage Volume). An aircraft in the RF Coverage Volume can receive traffic information on targets located in the Surveillance Coverage Volume for which the Ground Station is providing TIS-B messages.

References [10] and [11] propose an extension to TIS-B called TIS-Contract, or TIS-C, for providing traffic information in a distributed fashion.

D. Flight Information Services (FIS-B)

Flight Information Services-Broadcast is an automated datalink system that will provide timely information to pilots, such as weather graphics and text, Special Use Airspace (SUA) information, and Notice to Airmen (NOTAMs). The FIS-B datalink systems use a one-way broadcast protocol. Information flows only from the server to the receiving aircraft without the need for the aircraft to request information from the server, nor to acknowledge receipt.

The goal of FIS-B datalink systems is to provide weather and other non-control flight advisory information to pilots in a way that enhances their awareness of flight conditions and enables better strategic route planning consistent with guidance provided by Federal Aviation Regulations and corporate policy [55].

E. Cockpit Display of Traffic Information (CDTI)

The CDTI is the pilot interface portion of the surveillance system [27]. CDTI functions include a traffic display function, associated control functions, annunciation functions, and alert functions. The traffic display function may present information on a dedicated display device or a shared/multi-function display (MFD) device. Additional information about the traffic such as ground speed, distance, or closure rate may also be shown. Traffic information may be obtained from multiple sources (ADS-B, TIS-B, TCAS).

The operational goals of CDTI applications are to improve the safety and efficiency of flight operations through enhanced surveillance capability [54]-[57]. The CDTI may enhance current air traffic operations by providing enhanced visual acquisition and visual approaches, and in-trail climb and descent in non-radar airspace.

F. Conflict Detection, Prevention, and, Resolution

A conflict is defined as the condition under which an aircraft’s protected airspace zone (PAZ) is violated. The PAZ is a variable sized zone based on legal separation requirements where they exist. For example, a typical PAZ may be a cylinder of 5 nm radius and 2000 ft altitude. A smaller zone, called the collision avoidance zone (CAZ), defines a separation standard between aircraft that represents a risk for a collision. A typical CAZ may have a 0.15 nm radius and 600 ft altitude.

A Conflict Detection (CD) system alerts the pilot that a loss of separation would occur if the aircraft involved maintain their current flight state and intent. One definition of loss of separation is if at some point of time, the minimal distance between these two aircraft, called the Closest Point of Approach (CPA), is less than the PAZ/CAZ limits.

A Conflict Prevention (CP) system provides the pilot with knowledge of potential conflicts. A "potential conflict" exists if a conflict may be created through a change of the ownership state and/or intent.

After a conflict has been detected and an alert issued, the conflict must be resolved. A Conflict Resolution (CR) system provides the pilot with advisories which must be followed by the pilot unless the pilot deems them unsafe or has better information.

A variety of research exists on algorithm development and analysis for conflict detection, prevention, and resolution, both ground and air-based. [13], [21], [34], [37], [40], [41], [46], [49], [51], [75].

G. Autonomous Operations Planner (AOP)

NASA is developing a flight deck decision support tool to support research into autonomous operations in a future distributed air/ground traffic management environment [2], [1], [6], [74]. This interactive real-time decision aid, referred to as the Autonomous Operations Planner (AOP), will enable the flight crew to plan autonomously in the presence of dense traffic and complex flight management constraints. The crew interacts with the AOP to perform trajectory planning that accounts for conflicts with traffic hazards, ownership aircraft performance limitations, system flow constraints that are imposed by the ATSP, airspace constraints, such as severe weather and dynamic SUA, and operator flight goals, such as efficiency and schedule. The AOP assists crew decision-making through presentation of the most relevant information, the accommodation of crew planning preferences, alerting advisories, and automated
negotiations with crews of other aircraft (if needed) and ground-based air traffic controllers.

H. Merging and In-Trail Separation

A merging and self-spacing capability provides a method for aircraft that are maneuvering in a free-flight environment to enter a terminal area in an organized, safe, and efficient fashion. The merging capability provides the ATSP with a means of specifying to an aircraft how it is to merge with other aircraft. Two methods are envisioned: 1) Specification of a ghost aircraft trajectory, with which the aircraft then performs self-spacing; 2) Specification of an RTA at a merge point that is based upon the lead aircraft’s crossing time plus an appropriate time separation.

The self-spacing capability provides a means for the flight crew to maintain a desired spacing (in time) behind a lead aircraft in instrument meteorological conditions. The time separation between the two aircraft is calculated by the ATM automation system. The lead aircraft provides state and trajectory information using ADS-B to assist the trailing aircraft in maintaining the desired separation. The self-spacing system provides the trailing aircraft with guidance cues to the autopilot/autothrottle and the pilot displays. This capability is described in further detail in [22], [23], [33], [35], [44], [48], and [50].

The following five sections describe specific airborne capabilities that the authors have contributed to developing.

III. Time-Based Aircraft Control

While traditional ATC methods separate aircraft by space, several concepts are founded on time-based separation. The simplest time-based systems ensure that aircraft cross chokepoints at specified times. These systems can require no special avionic equipment if the controller “closes the loop” by issuing speed instructions to pilots to keep them on time. But, the main promise of the time-based systems to relieve controller workload is by issuing crossing time clearances directly to the aircraft and allowing the aircraft to adjust speed autonomously to meet the crossing restriction.

Reference [67] presents the results of a study investigating a ground-based ATM system that uses accurate 4D trajectories computed by the FMS to improve the accuracy of the conflict detection, and is representative of several other time-based ATM systems. This method involves negotiating 4D trajectories between the ground controllers and the aircraft FMS prior to departure, and as needed during flight, to ensure that the strategic flight plan remains conflict free. To deal with changing conditions that cause unplanned conflicts, RTA constraints are uplinked to the aircraft, and the FMS replans the trajectory to meet the constraint. While this strategy reduces controller workload by increasing trajectory prediction accuracy, the study found that it raised controller workload by requiring a trajectory negotiation process to deal with effects of remaining uncertainties in the predictions. In essence, this approach only uses the FMS to increase the accuracy of trajectory prediction but still closes the feedback loop through the controller.

Many FMS systems today can already automatically modify the aircraft speed profile during cruise to meet an RTA constraint. However, the FMS RTA systems fielded today can only adjust the aircraft speed profile prior to the top-of-descent point, so they are not useful for controlling the time of arrival in the terminal phase of flight.

Honeywell has developed an advanced RTA system capable of automatically adjusting the speed profile through the descent phase to meet an RTA low in the descent phase [58]. There are two main challenges in accomplishing RTA-in-Descent. First is ensuring that the aircraft has the capability of adjusting the aircraft speed in the descent phase. Since the descent trajectories are typically generated at (or close to) idle throttle setting there is no automatic way the FMS can decelerate. Since it is unlikely that the FMS will be given automatic control of the speed-brakes, the only way to ensure the FMS can decelerate is to plan the trajectory at an off-idle throttle setting, resulting in a more shallow descent path. Since this would result in increased fuel consumption, this would only be used when accurate time control is important.

The second challenge is efficiently computing the speed adjustment required to meet the RTA constraint, brought about by the desire to have the speed adjustment appear to the pilot as a simple change to the descent Mach and CAS profile. The sensitivity of time of arrival to changes in the descent speed profile is not known in current FMSs, leaving only an iterative search to find a feasible speed adjustment. Since each iteration requires a full trajectory prediction to compute the arrival time, this solution is impractical to implement onboard present-day FMSs. Going forward, Honeywell has developed a method to analytically compute the sensitivity of the arrival time to a generic speed adjustment parameter (SAP) that can include any combination of Mach, CAS, or other means of adjusting the speed profile by a single parameter, such as cost index.

The following derivation describes the basic approach used to compute the speed adjustment. First, consider an equation that approximates how an FMS computes the time of arrival in the terminal phase of flight.

\[ \Delta t = \frac{\text{LegDist}}{\text{GndSpd}} + \text{Current Time} \]

The sensitivity of the ETA to a speed adjustment parameter can be found by simple differentiation to be:

\[ \frac{\Delta E T A}{\Delta \text{SAP}} = \frac{\sum_{\text{WPTS}} - \text{LegDist}}{\text{GndSpd}^2} \frac{\Delta \text{GndSpd}}{\Delta \text{SAP}} \]

This sensitivity can be computed by the FMS during the normal trajectory prediction calculations with only a small increase in computations. Once the sensitivity is known, the speed adjustment can be computed by representing the ETA as a Taylor’s series expansion:

\[ E T A(\text{SAP} + \Delta \text{SAP}) \approx E T A(\text{SAP}) + \frac{\Delta E T A}{\Delta \text{SAP}} \Delta \text{SAP} = \text{RTA} \]
and solving for the speed adjustment

$$\Delta SAP = \frac{-Time Error}{\delta ETA}$$

where $Time Error = ETA - RTA$

While the above derivation works well for small time errors, nonlinearities in the equations of motion and performance model merit a higher order solution for large errors using a 2nd order partial derivative. The details of this approach are described in [58].

To ensure that the ground-based conflict probe retains accuracy with the aircraft autonomously changing the speed profile, the 4D trajectory can be periodically down-linked to the ATM system. By placing the RTA constraint(s) at appropriate choke points or aircraft trajectory crossing points, the autonomous speed modifications by the aircraft ensure that the trajectory remains conflict-free with a minimum of controller workload. The resulting aircraft trajectory retains the desired FMS speed profile shape familiar to pilots, while pushing the errors due to the uncertainty into portions of the trajectory that are not critical to the traffic situation.

IV. AIR/GROUND TRAJECTORY NEGOTIATION

In addition to [67] discussed above, several other trajectory negation concepts have been presented in the literature [1] [71] [72]. In these concepts, the ATM system uplinks time constraints to the aircraft, which computes a compliant 4D trajectory and downlinks the trajectory to the ATM system. As discussed in the previous section, using the aircraft FMS to generate an accurate and flyable 4D trajectory and downlinking it to the ATM system for conflict probe has some benefit to reducing controller workload, but further benefit could be realized by transferring additional responsibility to the aircraft to automatically adjust the speed profile to meet the specified time constraints.

As part of the NASA VAMS program, Honeywell and Seagull Technologies developed a detailed trajectory negotiation process that takes advantage of additional airborne capability to further reduce controller workload [61], [63], [64]. This operational concept considers the negotiated 4D trajectory to be a contract between the ATM provider and the aircraft. The aircraft is expected to stay within a specified tolerance of the negotiated trajectory or inform the controller if that cannot be achieved.

Several other references have proposed 4D trajectory contracts, such as [72], however, these studies provide no method for the aircraft to track the contract other than explicitly and continuously controlling the aircraft to track the center of the 4D trajectory bubble. This has several negative aspects. Under realistic wind modeling errors, this tracking would require airspeed deviations on the order of the wind modeling errors, along with associated throttle activity, reduced passenger comfort, increased fuel consumption, and increased engine wear.

Honeywell and Seagull proposed a blended approach that uses the FMS RTA capability to maintain the gross trajectory objectives while monitoring the compliance with the 4D trajectory contract and taking further action to stay within the bubble only when necessary.

It is preferable to fly the aircraft with smooth airspeed commands chosen to achieve the trajectory constraints imposed by ATM. This strategy responds more smoothly to wind modeling errors, essentially attempting to remove time errors before crossing the RTA waypoint, instead of instantaneously. Since this opens up the possibility of leaving the bubble-in-tube contract, the predicted aircraft trajectory must be monitored to ensure that it does not leave the tube contract. If it is projected to leave the contract, the flight crew / FMS has several options available: a) Modify/add trajectory constraints in the vicinity of the predicted violation. E.g., if the aircraft is predicted to leave the front or back of the bubble, an additional RTA could be added to the flight plan. b) Switch aircraft guidance modes to explicitly track the bubble-in-tube contract with a feedback guidance law. c) Reopen trajectory negotiation by down linking the current predicted trajectory that meets the set of trajectory constraints previously uplinked by ATM automation.

V. CIVIL FORMATION FLIGHT

Military transport aircraft such as the C-130 and C-17 regularly fly in formation as an effective way to move large numbers of aircraft safely. Procedures and on-board avionics have been developed to make formation flight possible in all weather conditions. Military aircraft typically fly with less than 1 nm in-trail separation, which is one-half to one-fourth of the spacing standards used today in terminal airspace and substantially less than the spacing in en route and oceanic environments [14], [32], [47].

It is interesting to consider whether the procedures and avionics developed for military formation flight could be adapted for civil formation flight. Civil formation flight, where aircraft take on some self-separation responsibility, could complement other initiatives to reduce or better manage aircraft spacing and alleviate congestion in oceanic, en route or terminal air space. Air traffic controller workload might be reduced, for example, if a controller could provide route deviations to the lead aircraft of a civil formation and all other aircraft would follow the lead.

Military formation flight typically uses on-board avionics to assist the flight crew and provide the capability to fly in low visibility (Figure 1). There is a similarity between the avionics required for military formation flight and those that are available for commercial aircraft. While the military may choose to use specialized communication links and sensing, and typically have a mission computer acting as the flight management system, the avionics functions are essentially the same as those available commercially.
Figure 1. Example Formation Flight System Architecture

- Surveillance sources could include datalinked navigation data from other formation aircraft and/or a direct range and bearing measurement system such as TCAS/Mode S [13].
- A low-bandwidth inter-aircraft datalink to relay navigation data and formation information, such as Automatic Dependent Surveillance-Broadcast [52].
- A flight crew interface that includes a cockpit display of traffic information and deviations from assigned position and the ability to select which formation aircraft to follow and the required relative spacing.
- An optional link to a flight management system or flight control system for autocoupled flight.

There are challenges to implementing civil formation flight. Procedures need to be established to form and dissolve formations, including transferring separation responsibility from the controllers to the flight crew. Military procedures are not directly applicable, since the formation lead controls the formation. For a civil formation, the responsibility of controlling the formation spacing and prescribing formation maneuvers largely will be ceded to ATC. ATC is responsible for aircraft separation and must play an integral role in the creation and dissolution of formations. ATC must permit an aircraft to join a formation and specify the lead aircraft, in-trail spacing, and the lateral spacing. Once an aircraft leaves a formation, ATC must take back controlling authority.

VI. CLOSELY SPACED PARALLEL APPROACHES

Aircraft landing on parallel runways during Visual Meteorological Conditions can operate independently as long as the runways are at least 700 ft apart. Pilots are responsible for separation, and the longitudinal spacing of each aircraft along a runway depends only on the aircraft on that runway, not on the aircraft on the parallel runway. When Instrument Meteorological Conditions are in effect, aircraft can only operate independently on parallel runways separated by at least 4300 ft. With dependent operations, airport landing capacity decreases dramatically. This affects many major airports across the world.

Alerting systems that assist the controllers and/or the pilots have the potential to increase airport capacity substantially by providing arrival rates during Instrument Meteorological Conditions that approach those during Visual Meteorological Conditions. The Precision Runway Monitor (PRM) is a ground-based alerting system based on a high update rate, high accuracy surveillance system combined with special display software and procedures. PRM allows aircraft to fly independent approaches for runways spaced as close as 3400 ft. Minneapolis-St. Paul and St. Louis airports have installed the Precision Runway Monitor and are currently evaluating it.

With Differential Global Positioning System (DGPS) landing systems for precision navigation and aircraft-to-aircraft communications, combined with Automatic Dependent Surveillance-Broadcast (ADS-B), it is possible to develop on-board alerting systems for closely spaced parallel approaches. Rockwell-Collins and United Airlines are collaborating on an airborne alerting system for closely spaced dependent approaches that they plan to demonstrate at the San Francisco airport, where the runway spacing is 750 ft.

Honeywell and NASA Langley Research Center demonstrated in 1999 an airborne alerting system called AILS (Airborne Information for Lateral Spacing) that was designed for independent closely spaced parallel approaches to runways separated by 2500 ft to 4300 ft. [70]. A prototype system was tested at NASA Wallops in
September, 1999, and demonstrated in Minneapolis two months later using the NASA 757 ARIES aircraft and a Honeywell Gulfstream-IV. In the both flights the Honeywell aircraft flew intrusion maneuvers at the NASA 757 and the system alerted the pilot to the threat and instructed the pilot when to perform an emergency escape maneuver.

An airborne alerting system for closely spaced parallel approaches has three key elements—precision navigation, accurate state and approach (intent) information for the other aircraft in the area, and algorithms to alert the pilot to dangerous situations. Precision navigation is provided by the Differential Global Positioning System (DGPS). Differential corrections to the GPS signal are uplinked from a GPS Landing System ground station, such as a Local Area Augmentation System (LAAS).

Each aircraft broadcasts its position, velocity, and approach using ADS-B. The airborne alerting system uses the received ADS-B information from the other aircraft in combination with its own aircraft data to determine if either aircraft is a threat to the other with a specialized alerting algorithm such as AILS. The ADS-B information is also broadcast to the ground so that controllers can stay informed of the status of aircraft on parallel approaches.

The Emergency Escape Maneuver is a fixed procedure consisting of a climbing turn. The procedural EEM may change for different airports or different runways, but it is independent of the blunder characteristics. A procedural EEM simplifies system design since no aircraft-to-aircraft maneuver coordination is required. It also simplifies pilot training, since the EEM is fixed and can be presented on an approach plate in the same way as a missed approach procedure. The major drawback of the procedural EEM is that it has to work for all possible blunder scenarios. Once the pilot performs the EEM, there is nothing more that the alerting system can do to protect the aircraft, since active guidance is not provided. To address this drawback, the system was designed to hand-off to TCAS as soon as possible, but after no less than 5 seconds. TCAS can then provide vertical maneuver coordination, if necessary, as the aircraft performs the EEM.

The AILS alerting algorithms provide multiple levels of alerting for pairs of aircraft that are in parallel approach situations. AILS uses current aircraft states as well as known “intent” information to project ahead for threat determination. This forward projection is based on current positions, velocities, altitudes, turn rates, climb rates, and intended approach path. An alert is raised if one aircraft is likely to penetrate another’s protected region within a specified amount of time.

AILS performs two types of alerting checks: Adjacent ship threat to own ship and Ownship threat to adjacent ship. Each check includes a caution level indicating a potential for alert, and a warning level which represents potential imminent collision.

The AILS’s threat evaluation projects each aircraft’s current states forward a finite amount of time to see if it will penetrate the other aircraft’s protected zone (a linear distance above and below the aircraft), and an elliptical protection area in the horizontal plane. The forward projection assumes that the aircraft will continue turning at the current turn rate, but also that the aircraft may roll level at any time. This concept is known as the AILS “fan” and represents an added level of safety check and conservatism.

VII. AIRBORNE COLLISION AVOIDANCE

This section examines the use of Laplace’s equation for airborne collision avoidance for UAVs. While not immediately applicable to commercial airspace conflict prevention, this method illustrates advanced avionics capabilities in development that may be useful in the future. This technique constructs paths, r(t), through a 3D domain by assigning a potential value of v(r)=0 for r on any boundaries or obstacle, and a potential of v(r)=1 for r on the goal region. Then Laplace’s equation is solved in the interior of the 3D region, guaranteeing no local minima in the interior of the domain, leaving a global maximum of v(r)=1 for r on the goal region, and global minima of v(r)=0 for r on any boundaries or obstacle. A path from any initial point, r(0), to the goal, is made by following the gradient of the potential, v.

Previous applications of Laplacian path-planning include [7], [8], [43], and [69]. An analogy for paths obtained by Laplace’s equation is to apply a voltage of 0 to all boundary and obstacle locations, a voltage of 1 to goal region, fill interior region with a conductor, then electrons will follow paths from anywhere in the interior to the goal region.

Laplace’s equation sets the divergence of a potential to zero in the interior of a domain. Solutions of Laplace’s equation are harmonic functions, which have no local minima in the interior of their domain.

Numerical solutions of Laplace’s equation are obtained by gridding the domain, then iteratively setting the potential at each interior point, equal to the average of its nearest neighbors. By varying the grid size (halving or doubling cell length at each step) from the crudest that stills leaves paths between obstacles, to the finest that is required for smooth paths, the iteration can be made to converge in a time proportional to the number, N, of cells in the finest grid. The solution on crude grids is cheap, and is used to initialize the solution on finer grids. This multigrid technique is described in [68] and is applied to robotic path planning in [43].

Textbook convergence proofs for empty domains, give the total number of computations to be e*5N, where N is the number of cells in the finest grid, and c = 5 is some small number of iterations at each grid size. That convergence speed relies on being able to set the crudest grid cell size equal to the entire domain for the crudest solution. However, we have found that in a domain with obstacles, the number of needed iterations is c*(path length)/(path width), since path-width between obstacles, limits the largest cell size of the crudest grid that still preserves the topology of the computed paths. With largest grid cell size equal to path width, the number of grid cells along the path is (path length)/(path width). The iterative process of setting a cell’s
potential equal to the average of its neighbor’s potentials propagates a nonzero solution value a distance of one more grid cell along the path each iteration. So it takes \((\text{path length})/(\text{path width})\) iterations for a nonzero solution to propagate along the entire path length, when crudest grid cell size is equal to path width. After \(c \times (\text{path length})/(\text{path width})\) iterations, with \(c \approx 5\), the iteration converges on the crudest grid. Using iterations on each finer grid size results in the bulk of the work being done by \(c \times (\text{path length})/(\text{path width})\) iterations on the finest grid size, for a total number of operations of approximately \(c \times (\text{path length})/(\text{path width}) \times N\), where \(N\) is the total number of cells in the finest grid.

An example region of size 128x128x16 is shown in Figure 2. The low ceiling in the front half of the region forces the UAV to fly between buildings, while in the rear of the region, the high ceiling allows UAV to fly over obstacle buildings. This example took 6 seconds in Matlab, on a 1.8 GHz Pentium.

**Figure 2. UAV obstacle avoidance path with ceiling constraint**

UAV to fly between buildings, while in the rear of the region, the high ceiling allows UAV to fly over obstacle buildings. This example took 6 seconds in Matlab, on a 1.8 GHz Pentium.

**REFERENCES**


