Passivity-based Control of Visual Feedback Systems with Dynamic Movable Camera Configuration

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Abstract—This paper deals with control of visual feedback systems with a dynamic movable camera configuration. This configuration consists of a robot manipulator and a camera that is attached to the end-effector of another robot manipulator. Firstly the brief summary of the dynamic visual feedback system with an eye-in-hand configuration is given with the fundamental representation of a relative rigid body motion. Secondly we construct the visual feedback system with a dynamic movable camera configuration by combining the hand control error system. Next, we derive the passivity of the dynamic visual feedback system. Based on the passivity, stability and $L_2$-gain performance analysis are discussed. Finally the validity of the proposed control law can be confirmed by comparing the experimental results.

I. INTRODUCTION

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. Visual information is undoubtedly suited to recognize unknown surroundings. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1][2].

In classical visual servoing, many practical methods are reported by two well known approaches with two camera configurations, i.e., position-based visual feedback control and image-based one with an eye-in-hand configuration or a fixed camera one (see, e.g. [1]). Specifically, several approaches which originate from the classical visual servoing have been proposed to overcome the drawbacks which are found from an experimental point of view. In [3], the 2 1/2-D visual servoing which includes the advantages of both position-based and image-based visual servoing is proposed in order to guarantee the robustness with respect to calibration errors. The partitioned visual servoing is considered to remain all features in the image [4]. Recently, the switching approach among position-based visual servoing and backward motion is investigated for dealing with the field of view problem [5]. However, classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. This assumption is invalid for high speed tasks, while it holds for kinematic control problems.

Kelly et al. [6] considered a simple image-based controller for dynamic visual feedback system in the three dimensional(3D) workspace under the assumption that the objects’ depths are known. Zergeroglu et al. developed an adaptive control law for the position tracking and the camera calibration problems of the dynamics visual feedback system with parametric uncertainties in [7]. Cowan et al. [8] addressed the problems of the field of view for the 3D dynamic visual feedback system by using the navigation functions. Although the good solutions to the set-point problems are reported in those papers, few results have been obtained for the tracking problems of the moving target object in the full 3D dynamic visual feedback system. Additionally, most of the previous works are discussed for the camera configurations in detail, while the position-based visual feedback control and the image-based one are developed in some issues [3][4][5].

In this paper, we discuss the visual feedback control for the target tracking problem with a dynamic movable camera configuration as in Fig. 1. This configuration consists of a robot manipulator (we call a work manipulator) and a camera that is attached to the end-effector of another robot manipulator.
Thus, the whole system has five coordinate frames to represent the change of the principle axes of a frame works \[9\],[10],[11],[12]. The validity of the proposed control be interpreted as the special case of this system. Then, we control the camera manipulator in order to enlarge the field of view. The visual feedback system with a dynamic movable camera configuration preserves the passivity of the system is obviously to control the work manipulator, we also can derive that the visual feedback system with a dynamic movable camera configuration preserves the passivity of the visual feedback system which is obtained in our previous works \[9\],[10],[11],[12].

Throughout this paper, we use the notation \(\xi b_{\theta ab} \in \mathbb{R}^{3 \times 3}\) to represent the change of the principle axes of a frame \(\Sigma_b\) relative to a frame \(\Sigma_a\). The notation ‘\(\Lambda\)’ (wedge) is the skew-symmetric operator such that \(\hat{\xi} \theta = \xi \times \theta\) for the vector cross-product \(\times\) and any vector \(\theta \in \mathbb{R}^3\). The notation ‘\(\Lambda\)’ (vee) denotes the inverse operator to ‘\(\Lambda\)’; i.e., \(so(3) \rightarrow \mathbb{R}^3\), \(\xi_{ab} \in \mathbb{R}^3\) specifies the direction of rotation and \(\theta_{ab} \in \mathbb{R}\) is the angle of rotation. Here \(\xi_{ab}\) denotes \(\theta_{ab}\) for the simplicity of notation. We use the \(4 \times 4\) matrix

\[
g_{ab} = \begin{bmatrix} e^{\xi \theta_{ab}} & p_{ab} & 0 \\ 0 & 1 & \end{bmatrix}
\]

as the homogeneous representation of \(g_{ab} = (p_{ab}, e^{\xi \theta_{ab}}) \in SE(3)\) which is the description of the configuration of a frame \(\Sigma_b\) relative to a frame \(\Sigma_a\). The adjoint transformation associated with \(g_{ab}\) is denoted by \(Ad(\xi_{ab})\) \[13\]. Let us define the vector form of the rotation matrix as \(e_R(\xi_{ab}) := \text{sk}(e^{\xi \theta_{ab}})\) where \(\text{sk}(e^{\xi \theta_{ab}})\) denotes \(\frac{1}{2}(e^{\xi \theta_{ab}} - e^{-\xi \theta_{ab}})\).

**II. Dynamic Passivity-based Visual Feedback System with an Eye-in-hand Configuration**

**A. Fundamental Representation for Visual Feedback System**

In this paper, we consider the visual feedback system with a dynamic movable camera configuration as shown in Fig. 1. Thus, the whole system has five coordinate frames which consist of a world (base of the camera manipulator) frame \(\Sigma_w\), a camera (end-effector of the camera manipulator) frame \(\Sigma_c\), a base frame of the work manipulator \(\Sigma_{co}\), a hand (end-effector of the work manipulator) frame \(\Sigma_{h}\) and a target object frame \(\Sigma_o\) as in Fig. 1. Then, \(g_{co}\) denotes the rigid body motion from \(\Sigma_w\) to \(\Sigma_{co}\). Similarly, the rigid body motions \(g_{wz}, g_{wh}\) and \(g_{wo}\), and the relative rigid body motions \(g_{ch}, g_{co}, g_{hz}\) and \(g_{ho}\) are represented, respectively, as shown in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motions \(g_{co}\) and \(g_{ho}\) to given references \(g_{cd}\) and \(g_{hd}\), respectively. Our goal is to determine the camera’s motion and the hand’s motion using the visual information for this purpose.

Firstly, we construct the dynamic visual feedback system with an eye-in-hand configuration in order to enlarge the field of view. The visual feedback system with an eye-in-hand configuration uses only the camera manipulator which has three coordinate frames, i.e. the world frame \(\Sigma_w\), the camera frame \(\Sigma_{co}\) and the target object frame \(\Sigma_o\). Besides, control objective is to bring \(g_{co}\) to \(g_{cd}\).

The relative rigid body motion from \(\Sigma_{co}\) to \(\Sigma_o\) can be led by using the composition rule for rigid body transformations \([13]\), Chap. 2, pp. 37, eq. (2.24)) as follows

\[
g_{co} = g_{wc}g_{wo}.
\]

The fundamental representation of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in \([13]\). Now, we define the body velocity of the camera relative to the world frame \(\Sigma_w\) as \(V_{wc}^b = \left[v_{wc}, \omega_{wc}\right]^T\), where \(v_{wc}\) and \(\omega_{wc}\) represent the velocity of the origin and the angular velocity from \(\Sigma_w\) to \(\Sigma_{co}\), respectively (\([13]\) Chap. 2, eq. (2.55)).

Differentiating (2) with respect to time, the fundamental representation of the relative rigid body motion \(g_{co}\) is described as follows \([9]\).

\[
V_{co}^b = -Ad_{g_{co}^{-1}}V_{wc}^b + V_{wco}^b
\]

where \(V_{wco}^b\) is the body velocity of the target object relative to \(\Sigma_w\). Roughly speaking, the relative rigid body motion \(g_{co}\) will be derived from the difference between the camera velocity \(V_{wc}^b\) and the target object velocity \(V_{wco}^b\).

**B. Estimation Error and Camera Control Error Systems**

Here the brief summary of our prior work in \([9]\) is given. The visual information \(f(\hat{g}_{co})\) which includes the relative rigid body motion can be exploited, while the relative rigid body motion \(g_{co}\) can not be obtained directly. In order to bring the actual relative rigid body motion \(g_{co}\) to a given reference \(g_{cd}\) in Fig. 1, we consider the control and estimation problems in the visual feedback system. Firstly, we shall consider the following model which just comes from the fundamental representation (3).

\[
\dot{V}_{co}^b = -Ad_{g_{co}^{-1}}V_{wc}^b + u_e
\]

where \(\hat{g}_{co}\) and \(\dot{V}_{co}^b\) are the estimated value of the relative rigid body motion and the estimated body velocity from \(\Sigma_{co}\) to \(\Sigma_o\), respectively. \(u_e\) is the input in order to converge the estimated value to the actual relative rigid body motion. Next, the estimation error of the relative rigid body motion from \(\Sigma_{co}\) to \(\Sigma_o\), i.e. the error between \(g_{co}\) and \(\hat{g}_{co}\), is defined as

\[
ge_{ee} = \hat{g}_{co}^{-1}g_{co}.
\]

which is called the estimation error. Using the notation \(e_R(\xi_{ee})\), the vector of the estimation error is given by \(e_{ee} := [p_{ee}^T \xi_{ee}^T (e^{\xi \theta_{ee}})]^T\). Note that \(e_{ee} = 0\) iff \(p_{ee} = 0\) and \(e^{\xi \theta_{ee}} = I_3\). Then, the estimation error vector \(e_{ee}\) can be obtained by using image information \(f(\hat{g}_{co})\). The estimation error system is represented by

\[
\dot{V}_{ee}^b = -Ad_{g_{cd}^{-1}}u_e + V_{wco}^b
\]

Similarly, we define the error between \(g_{cd}\) and \(\hat{g}_{cd}\), which is called the camera control error, as follows

\[
ge_{ec} = g_{cd}^{-1}\hat{g}_{cd}.
\]
The vector of the camera control error is defined as \( e_c := [p_{eC}^T \ T_0^T(\hat{\Theta}_{be})]^T \). The camera control error system is described by
\[
V_{ec}^b = -\text{Ad}_c(\hat{g}_{ec}) V_{wc}^b + u_c - \text{Ad}_c(\hat{g}_{ec}) V_{cd}^b
\]
where \( V_{cd}^b \) is the desired body velocity of the relative rigid body motion \( g_{ec} \).

C. Dynamic Passivity-based Visual Feedback System with an Eye-in-hand Configuration

The manipulator dynamics of the camera manipulator (we call the camera manipulator dynamics) can be written as
\[
M_c(q_c) \ddot{\hat{q}}_c + C_c(q_c, \dot{q}_c) \dot{\hat{q}}_c + g_c(q_c) = \tau_c + \tau_cd
\]
where \( M_c \in \mathbb{R}^{n_c \times n_c} \) is the inertia matrix, \( C_c \in \mathbb{R}^{n_c \times n_c} \) is the Coriolis matrix, \( g_c \in \mathbb{R}^{n_c} \) is the gravity terms, and \( q_c, \dot{q}_c \) and \( \ddot{q}_c \) are the joint angles, velocities and accelerations of the camera manipulator, respectively. \( \tau_c \) is the vector of the input torques and \( \tau_cd \) represents a disturbance input. The body velocity of the camera is given by
\[
V_{bc}^b = J_{cb}(q_c) \dot{\hat{q}}_c
\]
where \( J_{cb}(q_c) \) is the manipulator body Jacobian [13]. We define the reference of the joint velocities as \( \dot{\hat{q}}_{cd} := J_{cb}^T(q_c) u_{cd} \) where \( u_{cd} \) represents the desired body velocity of the camera. Thus, \( V_{bc}^b \) in (8) should be replaced by \( u_{cd} \).

Let us define the error vector with respect to the joint velocities of the camera manipulator as \( \xi_c := \dot{\hat{q}}_c - \dot{\hat{q}}_{cd} \). Now, we consider the passivity-based dynamic visual feedback control law as follows
\[
\tau_c = M_c(q_c) \ddot{\hat{q}}_{cd} + C_c(q_c, \dot{\hat{q}}_c) \dot{\hat{q}}_{cd} + g_c(q_c) + J_{cb}^T(q_c) \text{Ad}_c^T(\hat{g}_{cd}) \dot{\hat{e}}_c + u_{\xi_c}.
\]
(11)

The new input \( u_{\xi_c} \) is to be determined in order to achieve the control objectives.

Using (6) and (8)–(11), the visual feedback system with the camera manipulator dynamics (we call the dynamic visual feedback system with an eye-in-hand configuration) can be derived as follows
\[
\begin{bmatrix}
\dot{\xi}_c \\
\dot{V}_{bc}^b \\
\dot{V}_{ec}^b
\end{bmatrix} = \begin{bmatrix}
-M_c^{-1}C_c \xi_c + M_c^{-1}J_{cb}^T \text{Ad}_c^T(\hat{g}_{cd}) e_c \\
-\text{Ad}_c(\hat{g}_{cd}) J_{cb} \xi_c \\
-M_c^{-1} 0 0 \\
0 -\text{Ad}_c(\hat{g}_{cd}) 0 \\
0 0 -\text{Ad}_c(\hat{g}_{cd})
\end{bmatrix}\begin{bmatrix}
\xi_c \\
V_{bc}^b \\
V_{ec}^b
\end{bmatrix} + \begin{bmatrix}
0 \\
\text{Ad}_c(\hat{g}_{cd}) V_{cd}^b + M_c^{-1} 0 0 \\
0 0 0 \\
0 0 I
\end{bmatrix}\begin{bmatrix}
\tau_cd \\
\dot{V}_{wc}^b \\
\dot{V}_{cd}^b
\end{bmatrix}.
\]
(12)

The details are omitted due to space limitations, dynamic visual feedback system with an eye-in-hand configuration (12) is passive from the input to a appropriate output (Lemma 2).

III. PASSIVITY-BASED VISUAL FEEDBACK CONTROL WITH A DYNAMIC MOVABLE CAMERA CONFIGURATION

A. Hand Control Error System

In this section, the work manipulator is considered in addition to the camera manipulator, as depicted in Fig. 1. In other words, we determine the hand’s motion of the work manipulator (we call only the hand) to bring the actual relative rigid body motion \( g_{ho} \) to a given reference \( g_{bd} \), in addition to bring \( g_{co} \) to \( g_{cd} \).

Because \( g_{co} \) can not be obtained directly, we represent the relative rigid body motion from \( \Sigma_h \) to \( \Sigma_o \) with the estimated one \( \hat{g}_{co} \) as
\[
\hat{g}_{ho} = g_{ch}^{-1} \hat{g}_{co}.
\]
(13)

Here \( g_{ch} = g_{wc}^{-1} g_{wz} g_{zh} \) can be obtained directly, because the rigid body motions \( g_{wc}, g_{wz} \) and \( g_{wz} \) is known by the angle of the manipulator and the structure of the system. It is supposed that the relative rigid body motion from \( \Sigma_c \) to \( \Sigma_h \), i.e. \( g_{ch} \), can be measured exactly. Since the problem of the camera calibration in our approach is treated in [11], we will not consider the error of the camera calibration in this paper.

Here we define the hand control error between the estimated value \( \hat{g}_{ho} \) and the reference of the relative rigid body motion \( g_{bd} \) as
\[
g_{eh} = g_{bd}^{-1} \hat{g}_{ho}.
\]
(14)

Using the notation \( e_R(\hat{\Theta}_{eh}) \), the vector of the hand control error is defined as \( e_h := [\hat{p}_{eh}^T \ T_0^T(\hat{\Theta}_{eh})]^T \).

Similarly to (6) and (8), the hand control error system can be obtained as
\[
V_{eh}^b = -\text{Ad}_c(\hat{g}_{eh}) V_{wh}^b + u_e - \text{Ad}_c(\hat{g}_{eh}) V_{hd}^b
\]
(15)

where \( V_{hd}^b \) is the desired body velocity of the relative rigid body motion \( g_{ho} \).

B. Passivity-based Visual Feedback System with a Dynamic Movable Camera Configuration

Similarly the camera manipulator dynamics, the manipulator dynamics of the work manipulator (we call the work manipulator dynamics) can be written as
\[
M_h(q_h) \ddot{\hat{q}}_h + C_h(q_h, \dot{\hat{q}}_h) \dot{\hat{q}}_h + g_h(q_h) = \tau_h + \tau_{hd}
\]
(16)

where \( M_h \in \mathbb{R}^{n_h \times n_h} \) is the inertia matrix, \( C_h \in \mathbb{R}^{n_h \times n_h} \) is the Coriolis matrix, \( g_h \in \mathbb{R}^{n_h} \) is the gravity terms, and \( q_h, \dot{\hat{q}}_h \) are the joint angles, velocities and accelerations of the work manipulator, respectively. \( \tau_h \) is the vector of the input torques and \( \tau_{hd} \) represents a disturbance input. Using \( V_{zh}^b = V_{wh}^b \), the body velocity of the hand is given by
\[
V_{wh}^b = J_{hb}(q_h) \dot{\hat{q}}_h
\]
(17)

where \( J_{hb}(q_h) \) is the manipulator body Jacobian of the work manipulator. We define the reference of the joint velocities as \( \dot{\hat{q}}_{hd} := J_{hb}^T(q_h) u_{hd} \) where \( u_{hd} \) represents the desired body velocity of the hand.

In addition, we define the error vector with respect to the joint velocities of the hand manipulator as \( \xi_h := \dot{\hat{q}}_h - \dot{\hat{q}}_{hd} \). The passivity–based dynamic visual feedback control law is proposed as follows
\[
\tau_h = M_h(q_h) \dot{\hat{q}}_{hd} + C_h(q_h, \dot{\hat{q}}_h) \dot{\hat{q}}_{hd} + g_h(q_h) + J_{hb}^T(q_h) \text{Ad}_c^T(\hat{g}_{cd}) e_h + u_{\xi_h}.
\]
(18)

Using (12), (15)–(18), the visual feedback system with the camera and the work manipulator dynamics (we call
the visual feedback system with a dynamic movable camera configuration, or the dynamic visual feedback system for short) can be derived as follows

\[
\begin{pmatrix}
\dot{\xi}_c \\
\dot{\xi}_h \\
V_{\xi c} \\
V_{\xi h}
\end{pmatrix} =
\begin{pmatrix}
-M_{c}^{-1}C_c\xi_c + M_{c}^{-1}J_{eh}^TAd_{(g_{cd})}^{-1}e_c \\
-M_{h}^{-1}C_h\xi_h + M_{h}^{-1}J_{hh}^TAd_{(g_{hd})}^{-1}e_h \\
-Ad_{(g_{cd})}^{-1}\xi_e \\
-Ad_{(g_{hd})}^{-1}\xi_e
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
M_{c}^{-1} & 0 & 0 & 0 \\
0 & M_{h}^{-1} & -Ad_{(g_{cd})} & I \\
0 & 0 & 0 & Ad_{(g_{cd})}^{-1} \\
0 & 0 & 0 & -Ad_{(g_{hd})}^{-1}
\end{pmatrix}
\begin{pmatrix}
u_c \\
u_h \\
e_c \\
e_h
\end{pmatrix}
\]

where \( \nu := [u_{\xi c}^T u_{\xi h}^T (u_{cd} + Ad_{(g_{cd})}V_{ed}^b)^T u_{e c}^T (u_{cd} + Ad_{(g_{cd})}V_{ed}^b)]^T \). We define the disturbance of the dynamic visual feedback system as \( w := [r_{cd}^T r_{hd}^T (V_{we}^b)]^T \).

Before constructing the dynamic visual feedback control law, we derive an important lemma.

**Lemma 1:** If \( w = 0 \), then the dynamic visual feedback system (19) satisfies

\[
\int_0^T \nu^T v dt \geq -\beta, \quad \forall T > 0
\]

where \( \nu \) is defined as

\[
\nu := [v_{\xi c} v_{\xi h} v_{\xi e} v_{\xi c} v_{\xi h} v_{\xi e} v_{\xi c} v_{\xi h} v_{\xi c} v_{\xi h} v_{\xi c} v_{\xi h}]
\]

and \( \beta \) is a positive scalar.

Due to space limitations, the proof is only sketched. By using the following positive definite function, the proof can be completed.

\[
V = \frac{1}{2} \dot{\xi}^T M_c \xi_c + \frac{1}{2} \dot{\xi}^T M_h \xi_h + E(g_{cd}) + E(g_{cd}) + E(g_{cd}).
\]

where \( E(g) := \frac{1}{2} ||p||^2 + \phi(e^T e) \) and \( \phi(e^T e) := \frac{1}{2} \text{tr}(I - e^T e) \) which is the error function of the rotation matrix (see e.g. [14]).

Lemma 1 would suggest that the dynamic visual feedback system is passive from the input \( u \) to the output \( \nu \) as in the definition in [15].

**Remark 1:** If the camera velocity \( V_{wc}^b = 0 \), the desired camera velocity \( u_{cd} = 0 \), the relative rigid body motion \( g_{wc} = I \), and the camera control error \( e_c \) and the camera manipulator is not considered, then the dynamic visual feedback system (19) represents the dynamic visual feedback system with a fixed camera configuration [10]. From section II, it is obvious that the dynamic visual feedback system with an eye-in-hand configuration is included. Thus, the dynamic visual feedback system with an eye-in-hand configuration and a fixed camera one are regarded as the special cases of the system (19). This is one of main contributions of this work.

**C. Stability Analysis for Dynamic Visual Feedback System**

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

\[
u = -K \nu = -K Nx, \quad K := \text{diag}\{K_{\xi c}, K_{\xi h}, K_{e c}, K_{e h}\}
\]

(22)

where \( K_{\xi c} := \text{diag}\{k_{\xi c,1}, \ldots, k_{\xi c,n_c}\} \) and \( K_{\xi h} := \text{diag}\{k_{\xi h,1}, \ldots, k_{\xi h,n_h}\}\) denote the positive gain matrix for each joint axis of the camera manipulator and work one, respectively. \( K_{e c} := \text{diag}\{k_{e c,1}, \ldots, k_{e c}\} \), \( K_{e h} := \text{diag}\{k_{e h,1}, \ldots, k_{e h}\} \) are the positive gain matrices of \( x \) and \( y \) axes of the translation and the rotation for the camera control error, the estimation one and the hand control one, respectively. The result with respect to asymptotic stability of the proposed control input (22) can be established as follows.

**Theorem 1:** If \( w = 0 \), then the equilibrium point \( x = 0 \) for the closed-loop system (19) and (22) is asymptotic stable.

The proof is omitted here due to space limitations. Theorem 1 can be proved using the energy function (21) as a Lyapunov function. It is interesting to note that stability analysis is based on the passivity as described in (20).

**D. \( L_2 \)-gain Performance Analysis for Dynamic Visual Feedback System**

Based on the dissipative systems theory, we consider \( L_2 \)-gain performance analysis for the dynamic visual feedback system (19) in one of the typical problems, i.e. the disturbance attenuation problem. Here, we reconstruct the dynamic visual feedback system as the generalized plant as follows.

\[
\begin{pmatrix}
\dot{\xi}_h \\
V_{\xi h}
\end{pmatrix} =
\begin{pmatrix}
-M_{h}^{-1}C_h\xi_h + M_{h}^{-1}J_{hh}^TAd_{(g_{hd})}^{-1}e_h \\
-Ad_{(g_{cd})}^{-1}\xi_e
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
M_{h}^{-1} & 0 & 0 & 0 \\
0 & M_{c}^{-1} & 0 & 0 \\
0 & 0 & 0 & -Ad_{(g_{hd})} \\
0 & 0 & 0 & -Ad_{(g_{hd})}^{-1}
\end{pmatrix}
\begin{pmatrix}
u_h \\
\xi_c \\
\xi_e \\
\xi_e
\end{pmatrix}
\]

where \( u_{\xi h} := [u_{\xi c}^T u_{\xi h}^T (Ad_{(g_{cd})}u_{cd} + V_{ed}^b)^T (Ad_{(g_{cd})}u_{cd} + V_{ed}^b)]^T \).

For the generalized plant of the dynamic visual feedback system in (23), we consider the following input

\[
u_L = -K_L N_L x_L
\]

(24)

where \( K_L := \text{diag}\{K_{\xi h}, K_{\xi c}, K_{h}, K_{e c}, K_{e h}\}\)

\[
N_L := \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 \\
0 & 0 & -I & 0 & 0 \\
0 & 0 & 0 & Ad_{(e)} & Ad_{(e)}
\end{pmatrix}, \quad x_L := \begin{pmatrix}
\xi_h \\
\xi_c \\
\xi_e \\
\xi_e
\end{pmatrix}
\]

In order to derive simple gain conditions, we redefine \( K_{e c} = k_{e c} I \) and \( K_{e h} = k_{e h} I \) where \( k_{e c} \) and \( k_{e h} \) are positive scalars.
**Theorem 2:** Given a positive scalar $\gamma$ and consider the control input (24) with the gains $K_\xi_h$, $K_\xi_c$, $K_h$, $k_c$ and $k_e$ such that the following inequalities (25)–(29) are satisfied, then the closed-loop system (23) and (24) has $L_2$-gain $\leq \gamma$

\[
\begin{align*}
K_\xi_h - \frac{1}{\gamma^2} I - \frac{1}{4} I & > 0 \quad (25) \\
K_\xi_c - \frac{1}{4\gamma^2} I - \frac{1}{2} I & > 0 \quad (26) \\
K_h - \frac{1}{\gamma^2} I - (2k_e - 1) I & > 0 \quad (27) \\
k_c - \frac{1}{2} - l & > 0 \quad (28) \\
k_e - \frac{1}{2\gamma^2} - \frac{1}{2} & > 0 \quad (29)
\end{align*}
\]

where

\[ l = \frac{k_e(\gamma^2 + 1)}{\gamma^2 (2k_e - 1) - 1} \quad (30) \]

**Proof:** Differentiating the positive definite function $V$ defined in (21) along the trajectory of the closed-loop system, it can be verified that the inequality

\[
\dot{V} + \frac{1}{2} \|x_L\|^2 - \frac{\gamma^2}{2} \|w\|^2 \\
= -N_L^T K_h N_L x_L + \frac{1}{2}\gamma^2 W \|x_L\|^2 + \frac{1}{2}\|x_L\|^2 \\
= -x_L^T P x_L \leq 0 \quad (31)
\]

holds if $P := N_L^T K_h N_L - \frac{1}{\gamma^2} W - \frac{1}{2} I$ is positive semi-definite, where $W := \text{diag}(\bar{I}, I, 0, 0, I)$. Integrating (31) from 0 to $T$ and noticing $V(T) \geq 0$, we have

\[
\int_0^T \|x_L\|^2 dt \leq \gamma^2 \int_0^T \|w\|^2 dt + 2V(0), \quad \forall T > 0. \quad (32)
\]

From the Schur complement,

\[
P = \begin{bmatrix}
K_\xi_h - \frac{1}{\gamma^2} I - \frac{1}{2} I & 0 & 0 \\
0 & K_\xi_c - \frac{1}{4\gamma^2} I - \frac{1}{2} I & 0 \\
0 & 0 & K_h + k_e I - \frac{1}{2} I \\
k_c Ad_{(\xi_{\theta ec})} A d_{(\xi_{\theta ec})} & 0 & 0 \\
0 & k_e Ad_{(\xi_{\theta ec})} A d_{(\xi_{\theta ec})} & 0 \\
-k_c Ad_{(\xi_{\theta ec})} A d_{(\xi_{\theta ec})} & -k_e Ad_{(\xi_{\theta ec})} A d_{(\xi_{\theta ec})} & \left( k_e - \frac{1}{2\gamma^2} - \frac{1}{2} \right) I
\end{bmatrix} > 0 \quad (33)
\]

can be modified as the conditions (25)–(29).

The conditions (25)–(29) can be regarded as an extension of the ones for the disturbance attenuation of the robot motion control which are described in Proposition 3.1 [16], although these are only sufficient conditions.

The $L_2$-gain performance analysis of the dynamic visual feedback system is discussed via the dissipative systems theory. In $H_\infty$-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed $L_2$-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the dynamic visual feedback systems.

**IV. EXPERIMENTAL CASE STUDY**

The experimental results on the same kind of two degree-of-freedom manipulators are shown in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. The experimental arms are depicted in Fig. 3. The left side is the camera manipulator and the right side is the work one. These manipulators are controlled by a digital signal processor (DSP) from dSPACE Inc., which utilizes a powerPC 750 running at 480 MHz. Control problem is written in MATLAB and SIMULINK, and implemented on the DSP using the Real-Time Workshop and dSPACE Software which includes ControlDesk, Real-Time Interface and so on. A Sony XC-HR57 camera is attached at the tip of the camera manipulator (see Fig. 3). The video signals are acquired by a frame graver board PicPort-Stereo-H4D and a image processing software HALCON.

Due to space limitations, we present experimental results for the $L_2$-gain performance analysis in the case of a moving target object. Specifically, we compare the performance in the case of a dynamic movable camera system and a fixed camera system discussed in [10].

The target object which is projected on the display, has four feature points and moves for $t = 9.6$ [s] along a straight line ($0 \leq t < 4$) and a "Figure 8" motion ($4 \leq t \leq 9.6$) as depicted in Fig.3 [10]. The experiment is carried out with the initial condition $x = 0$, $p_{uc} = [0.4732 0.1 0]^T$, $\xi_{\theta uc} = [0 0 0]^T$, $p_{uz} = [0 0 1.16]^T$, $\xi_{\theta uz} = [0 0 0]^T$, $p_{wh} = [0.4732 0.1 1.16]^T$, $\xi_{\theta wh} = [0 0 0]^T$.

The objective of the visual feedback control is to bring the actual relative rigid body motions $g_{cd}$ and $g_{hd}$ to given references $g_{cd}$ and $g_{hd}$, respectively. We use the references of the relative rigid body motions as constant values, i.e. $p_{cd} = [0 0 -0.84|^T$, $\xi_{\theta wd} = [0 0 0]^T$, $p_{hd} = [0 0 -2]^T$, $\xi_{\theta hd} = [0 0 0]^T$, $V_{cd} = 0$, $V_{hd} = 0$.

Here, we show a design procedure in order to assign the gains for the disturbance attenuation problem of the dynamic visual feedback system.

Step 1: The hand control gain $K_h$ and the camera control gain $k_c$ are suitably selected.

Step 2: The estimation gain $k_e$ satisfying the conditions (27)–(29) is decided.

Step 3: The hand velocity gain $K_\xi_h$ and the camera velocity gain $K_\xi_c$ satisfying the conditions (25) and (26) are chosen for a given $\gamma$.

Based on the design procedure, the following gains were selected in order to confirm the adequacy of the $L_2$-gain
This paper dealt with the control of visual feedback systems with a dynamic movable camera configuration. Moreover, this system has been included the dynamic visual feedback system with an eye-in-hand configuration [9] and a fixed camera configuration [10]. Then, we derived that the dynamic visual feedback system preserves the passivity of the visual feedback system which is obtained in our previous works [9][10][11][12]. Stability and $L_2$-gain performance analysis for the dynamic visual feedback system have been discussed based on passivity with the energy function. The validity of the proposed control law was confirmed by comparing the experimental results. In our future work, we have to consider that the reference velocities $V_{ed}$ and $V_{hd}$ will play a role in the trajectory planning of the dynamic visual feedback system.

REFERENCES