Observer-Based Controller for Switch-Mode DC-DC Converters

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Abstract—In this paper, an observer-based controller for the basic DC-DC power converters is given. The DC-DC converters are represented in a general form which belong to the class of state affine systems. Passivity Based Control (PBC) is used in order to regulate the output voltage. However, the corresponding feedback law requires the knowledge of the state of the converter, therefore, the estimation problem is solved by designing a Kalman-like observer. The corresponding analysis of stability for the closed-loop system (observer plus control law) is also given. Simulation and experimental results are shown in order to illustrate the performance of the proposed scheme.

Keywords—Switch-Mode DC-DC converters, observer-based controller, nonlinear observers

I. INTRODUCTION

A great number of high-frequency switching regulators built in the power supply industry are using the buck, boost and buck-boost switch-mode converters as the power stage. These converters, as well as other PWM converters, provide voltage and current ratings for loads at a constant switching frequency, see [1], [2], [3] and [4]. In last decade, current-mode control has become very popular as a control strategy for these converters. This scheme employs an inner loop that senses the inductor current and an outer loop that senses the output voltage for feedback purposes.

Current-mode control has gone through considerable development since its conception [5] and various attempts have been made to formulate control strategies for these switching regulators. It is desirable that controls for power converters be robust to noise and to have good dynamic performance. Current-mode control is based on sensing an inductor current and using this signal in place of a triangle carrier. The objective is to maintain the inductor current equal to a reference current. This reference current is generated by an additional control loop that compares the output voltage to a reference voltage [6]. The end results are: (a) a faster transient response, (b) easier-to-design control loop, (c) instant overload protection, and (d) possibility to parallel operation of identical power stages without extra current equalization.

In the conventional peak current-mode technique, switching takes place at the peak of the inductor current, therefore noise sensitivity is relatively high [7]. A current sensor is required, normally a precision resistor of small value. In a practical application, the ideal switch is replaced by a power transistor, therefore, the current sensed through the resistor appears very noisy which may lead to false firing of the power transistor and an incorrect operation of the controller. Also, if a converter is intended to supply a large load range, it must sense and react to currents over the entire range. An approach widely used is average current-mode control [8] which improves the noise robustness by filtering the current signal. Other approach is charge control which integrates the current signal. Other approach widely used is average current-mode control [8] [9] reconstructs the inductor current directly by integrating the inductor voltage. SCM supports direct line and bulk load regulations, however, is highly sensitivity to uncertainty of the inductor and deterioration of the power transistor. In this paper, a new nonlinear observer is given which copes with the above problems.

II. MODELING OF DC-DC CONVERTERS

High-frequency switch-mode converters are power circuits in which the semiconductor devices switch at a rate that is fast compared to the variation of the input and output waveforms. These converters store energy in part of the cycle and deliver it to the output in the remaining part of the cycle. Thus, the resulting output voltage depends on a switching function \( d \). The policy regulating the switch position function \( d \) through a period \( T \) may be specified as follows:

\[
d = \begin{cases} 
1, & \forall t_k \leq \eta T \\
0, & \forall t_k > \eta T 
\end{cases}
\]

where \( t_k \) represents the time in a period; the parameter \( T \) is the fixed sampling period. The dynamic behavior of many classes of power circuits can be analyzed using the notion of averaged-circuit models, which allows to stay close to the circuit techniques. DC-DC converters may have different configurations depending on a required application. For the buck converter, the average output voltage is \( V_0 = E \eta_0 \), for the boost converter \( V_0 = E/(1-\eta_0) \) and for the buck-boost converter \( V_0 = -\eta_0 E/(1-\eta_0) \) where \( E \) is the average input voltage and \( \eta_0 \) the steady state duty ratio. The value \( \eta \) is the duty ratio function, truly acting as an external control
input to the average model of the converter: $0 < \eta < 1$.

In this paper, an observer-based controller scheme is given for the basic converters. It is well known [1], [2] that when the switching frequency is high, it is possible to model the average behavior of these converters. These models are given now.

**Buck converter.** In this converter, see figure (1), the output voltage is lower than the input voltage. The dynamics of this converter can be described by the following differential equations:

$$
\dot{\xi} = \begin{bmatrix}
-\frac{1}{RC} & \frac{1}{C} \\
-\frac{1}{L} & 0
\end{bmatrix} \xi + \begin{bmatrix}
0 \\
\eta E/L
\end{bmatrix},
$$

where $\xi_1$ and $\xi_2$ denote the average output capacitor voltage and the average input current respectively. Figure (2) shows the instantaneous output capacitor voltage and inductor current for this converter.

**Boost converter.** In this converter, see figure (3), the output voltage is higher than the input voltage. The differential equations describing the dynamic behavior for this converter are given by

$$
\dot{\xi} = \begin{bmatrix}
-\frac{1}{RC} & \frac{1-\eta}{C} \\
-\frac{1}{L} & 0
\end{bmatrix} \xi + \begin{bmatrix}
0 \\
E/L
\end{bmatrix},
$$

where $\xi_1$ and $\xi_2$ denote the average output capacitor voltage and the average input current as in the forehead case. Figure (4) shows the instantaneous output capacitor voltage and inductor current respectively.

**Buck-boost converter.** In this converter, see figure (5), the output voltage can be lower or higher than the input voltage depending on the duty ratio. This converter can be described by the following set of differential equations:

$$
\dot{\xi} = \begin{bmatrix}
-\frac{1}{RC} & \frac{1-\eta}{C} \\
0 & \frac{1-\eta}{L}
\end{bmatrix} \xi + \begin{bmatrix}
0 \\
E/L
\end{bmatrix},
$$

where $\xi_1$ and $\xi_2$ denote the average output capacitor voltage and inductor current respectively. Figure (6) shows the instantaneous output capacitor voltage and inductor current where the output voltage takes negative values.

**III. NONLINEAR OBSERVER DESIGN**

In this section, the design of a nonlinear observer for state affine systems is introduced. Systems (1), (2) and (3) can be represented in a more general form given by:

$$
\dot{\xi} = A(\bar{U}(i)(\eta))\xi + \varphi(\bar{U}(i)(\eta)) ,
$$

with:

$$
A(\bar{U}(i)(\eta)) = \begin{bmatrix}
-\frac{1}{RC} & -\frac{U_{i1}(\eta)}{C} \\
\frac{U_{i1}(\eta)}{L} & 0
\end{bmatrix},
$$

$$
\varphi(\bar{U}(i)(\eta)) = \begin{bmatrix}
0 \\
\frac{U_{i1}(\eta)E}{L}
\end{bmatrix},
$$

where:

$U_{i1} = -1; \bar{U}(i) = \eta$ for the buck converter,

$U_{i2} = -(1-\eta); \bar{U}(i) = 1$ for the boost converter,

$U_{i3} = (1-\eta); \bar{U}(i) = \eta$ for the buck-boost converter.
It is well-known that a nonlinear system
\[ \dot{\xi} = f(\xi, \eta) \]
\[ y = h(\xi) \]
is observable if the vector space \( d\mathcal{O} \) has full rank, i.e., \( \text{rank}(d\mathcal{O}) = n \) where \( n \) is the dimension of the system and \( \mathcal{O} \) is given by:
\[ \mathcal{O} = [h \ L_i h \ \cdots \ L_i^{n-1} h]^T. \]

In particular, for systems (2) and (3) the observability can be easily verified by defining \( f(\xi, \eta) = A(U_{i}(\eta))\xi \) and \( h(\xi) = C\xi \). The term \( \varphi(U_{i}(\eta)) \) does not affect the observability property. However, for system (4), the observability property depends on \( \eta \). For \( U_{i}(\eta) = 0; i = 2, 3 \) or \( \eta = 1 \); the system (4) is unobservable, i.e., \( \eta \) is a singular input. In order to guarantee that the observability property is preserved, it is necessary consider \( \eta \) as a regularly persistent input. In that follows \( A(\mathcal{U}) \) denotes \( A(U_{i})(\eta) \).

A further result based on regular persistence is now given.

**Lemma 1:** Assume that the input \( \mathcal{U} \) is regularly persistent for (5), and consider the following Lyapunov differential equation:
\[ \dot{P}(t) = -\theta P(t) - A^T(\mathcal{U})P(t) - P(t)A(\mathcal{U}) + C^T C \]

The regularly persistent inputs represents a class of admissible inputs that allows to observe the state system, thus the observer design is possible.

with \( P(0) > 0 \). Then: \( \exists \theta_0 > 0 \) such that for any symmetric positive definite matrix \( P(0); \forall \theta \geq \theta_0 \)
\[ \exists \alpha > 0, \beta > 0, t_0 > 0 : \forall t > t_0 \]
\[ \alpha I \leq P(t) \leq \beta I \]

**Remark.** This result guarantees the invertibility of matrix \( P \) assuming \( \eta \) is a regularly persistent input. Thus, the gain of the observer is well defined.

For system (4), using a regularly persistent input, the following result can be established:

**Lemma 2:** The system:
\[ \begin{cases}
\dot{\hat{\xi}} = A(\mathcal{U})\hat{\xi} + \varphi(\mathcal{U}) + P^{-1}C^T(y - \hat{\xi}) \\
\dot{\hat{P}} = -\theta \hat{P} - A^T(\mathcal{U})P - PA(\mathcal{U}) + C^T C
\end{cases} \]  

(8)
is an exponential observer for system (4). Furthermore, the estimation error \( e = \xi - \hat{\xi} \) converges exponentially to zero, for \( \theta > 0 \) sufficiently large.

**Proof.** The estimation error dynamics is given by
\[ \dot{e} = \dot{\hat{\xi}} - \hat{\xi} = \{A(\mathcal{U}) - P^{-1}C^T C\}e. \]

Now, from lemma (1), let us choose \( V(e) = e^T Pe = ||e||_P^2 \) as a Lyapunov function. Taking the time derivative of \( V(e) \), it follows that:
\[ \dot{V}(e) = e^T \{A^T(\mathcal{U})P - C^T C + \dot{P} + PA(\mathcal{U}) - C^T C\}e \]
\[ = -\theta e^T Pe - e^T C^T Ce \]
\[ < -\theta e^T Pe = -\theta V(e) \]

This ends the proof.

**IV. PASSIVITY-BASED CONTROLLER DESIGN**

Now a passivity-based controller is designed in order to regulate the output capacitor voltage to a constant value. For more details see [11].

System (2) can be rewritten as:
\[ D_B\dot{\xi} + (1 - \eta)J_B\xi + R_B\xi = \xi_B \]
where
\[
D_B = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}; \quad J_B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\]
\[
R_B = \begin{bmatrix} \frac{1}{\eta} & 0 \\ 0 & 0 \end{bmatrix}; \quad E_B = \begin{bmatrix} 0 & E \end{bmatrix}.
\]

Consider the tracking error variables \( \epsilon := \xi - \xi_d \), where \( \xi_d \) denotes the desired state. Following the passivity-based methodology define the desired energy function \( H_d \) and the total energy function \( H \) of (2), which are given by:
\[
H_d := \frac{1}{2} \epsilon^T D_B \epsilon; \quad H = \frac{1}{2} \xi^T D_B \xi
\]
Employing (9) and \( \epsilon \) the tracking error dynamics is given by
\[
\eta_B \dot{\epsilon} = E_B - (\eta_B \dot{\xi}_d + (1 - \eta) J_B \xi_d + R_B \xi_d) - \epsilon \dot{\xi} \eta_B \epsilon
\]
In order to ensure asymptotic stability a dissipation term is introduced
\[
D_d = \frac{1}{2} \epsilon^T R_{\epsilon} \epsilon = \frac{1}{2} \epsilon^T (R_B + R_{1\beta} \epsilon),
\]
where
\[
R_{1\beta} = \begin{bmatrix} 0 & 0 \\ 0 & R_1 \end{bmatrix}; \quad R_1 > 0.
\]
Substituting (11) into (10), the following result is obtained
\[
\eta_B \dot{\epsilon} + (1 - \eta) J_B \eta_B \epsilon = \Psi
\]
with:
\[
\Psi := E_B - (\eta_B \dot{\xi}_d + (1 - \eta) J_B \xi_d + R_B \xi_d) - R_{1\beta} \epsilon.
\]

The controller that satisfies this condition \(^2\) is given by:
\[
\dot{\eta} = \frac{(1 - \eta)}{C |E + (\xi_d - \xi)| R_1} \left( 1 - \eta \right) I_d
\]
\[
- \frac{E + (\xi_d - \xi)}{R_1} - \frac{\epsilon T}{E} \left( E - (1 - \eta) \xi_1 \right)
\]
for the boost converter. For this case \( \xi_d = I_d \), then it is possible to obtain \( V_d \) using \( I_d \) through \( I_d = \frac{1}{E} V_d \).

Following exactly the same procedure as in the previous case, it is possible to design a control law for the buck converter. Thus, the corresponding control law is given by:
\[
\eta(\xi) = \frac{V_d}{E} - \frac{1}{ER_1} \left( \xi_d - \frac{V_d}{R} \right)
\]
where \( V_d \) denotes the desired output capacitor voltage and \( V_d < E \). Using the same procedure and notation, the control law for the buck-boost converter is given by:
\[
\dot{\eta} = \frac{(1 - \eta)}{C |E + (\xi_d - \xi)| R_1} \left( 1 - \eta \right) I_d
\]
\[
- \frac{E + (\xi_d - \xi)}{R_1} - \frac{\epsilon T}{E} \left( E - (1 - \eta) \xi_1 \right)
\]
\(^2\)This control law provides a feasible regulation alternative based on an indirect output capacitor voltage control \( V_d \), achievable through the regulation of the input current \( I_d \).

V. STABILITY OF THE CLOSED-LOOP SYSTEM

In this section, the asymptotic stability of the nonlinear passivity-based controller (14) (15) or (16) is shown using an exponentially converging nonlinear observer (8).

Considering the observer-based controller given by (17),
\[
\begin{bmatrix}
\dot{\xi} \\
\dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
\css{A(\eta(\hat{\xi}))} + \psi(\eta(\hat{\xi})) & \css{P} \\
\css{\theta P - A^T(\hat{\eta}(\hat{\xi}))} & \css{PA(\hat{\eta}(\hat{\xi})) + CT}
\end{bmatrix}
\]
where \( \eta(\hat{\xi}) = A(U_{\hat{\xi}}(\hat{\xi})) \). Let \( e := \hat{\xi} - \xi \) be the estimation error, thus the dynamics of the resulting observer-based controller can be rewritten as follows:
\[
\begin{bmatrix}
\dot{\xi} \\
\dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
\css{A(\eta(\hat{\xi})) - P^{-1}CT} & \css{e} \\
\css{\theta P - A^T(\hat{\eta}(\hat{\xi}))} & \css{P - PA(\hat{\eta}(\hat{\xi})) + CT}
\end{bmatrix}
\]

where \( \eta(\hat{\xi}) \) is the corresponding control law given by (14), (15) or (16), which depends on the estimated values provided by the observer (8). Now, the following result can established:

**Theorem 3:** Under the assumptions that the nominal controller is globally asymptotically stable and that the state \( \xi \) in (17) remains for positive times in a compact set \( \Omega \) (containing the equilibrium point of the nominal controller) \( \forall \xi(0) \in \Omega \), the whole system (17) is globally asymptotically stable on \( \Omega \times \mathbb{R}^n \times P_+^n \) (i.e., \( \forall \xi(0) \in \Omega, \forall \xi(0) \in \mathbb{R}^n; \forall P(0) > 0 \)).

**Proof:** Observer (8) is such that the estimation error goes to zero (hence it is bounded) and the matrix \( P \), the dynamics given by solution of the Riccati differential equation in (17), is bounded from above and from below in the set of positive-definite matrices, see lemma (1). Hence the whole state \( e = \hat{\xi} - \xi, \hat{\xi}, P \) of (17) remains in a compact set along any trajectory.

Let \( \Lambda = \{(e(t), \xi(t), P(t)), t \geq 0 \} \) be a semitrajectory of the observer-based controller given by (17). This semitrajectory, lying in a compact set as stated above, has a nonempty \( w \)-limit set (the \( w \)-limit of a trajectory is the set of its accumulation points). Let \( [\bar{e}, \hat{\xi}, \hat{P}] \) be an element of the considered \( w \)-limit set of \( \Lambda \). It is clear that when \( e \rightarrow 0 \) implies that \( \bar{e} = 0 \). Let \( \{(0, \xi(t), P(t)), t \geq 0 \} \) be a semitrajectory starting at time \( t = 0 \) from \([0, \xi, \hat{P}] \). Since the \( w \)-limit set is positively invariant, it follows that the semitrajectory \( \{(0, \xi(t), P(t)), t \geq 0 \} \) belongs to the considered \( w \)-limit set of \( \Lambda \). The estimation error is here equal to 0 for this semitrajectory, and using our closed-loop stability assumption, \( \hat{\xi} \) is globally asymptotically stable, i.e., \( \xi(t) \rightarrow \xi^* = \psi(\xi^*) \). So, there are points at which \( e = 0 \) and \( \hat{\xi} = \xi^* \) in the \( w \)-limit set of \( \Lambda \), since it is a closed set. Letting \([0, \xi^*, \hat{P}(t)] \) be an element of the \( w \)-limit set of \( \Lambda \) following the same reasoning: let \( \{(0, \xi^*(t), P(t)), t \geq 0 \} \) be a semitrajectory starting at time \( t = 0 \) from \([0, \xi^*, \hat{P}] \). This semitrajectory belongs to the \( w \)-limit set of \( \Lambda \). The dynamics of \( P(t) \) are given by the differential Riccati equation and...
using the observability of the constant linear system \( (A^* + \varphi^*, C) \), that \( P(t) \) tends to \( P^* \), the unique positive-definite solution of the algebraic Riccati equation.

So, \([0, \xi^*, P^*]\) belongs to the \( w \)-limit set of \( \Lambda \). It follows, under the assumption of (local) asymptotic stability of (17), that \( \Lambda \) enters in a finite time into the basin of attraction of \([0, \xi^*, P^*]\). Hence (17) is globally asymptotically stable on \( \Omega \times \mathbb{R}^n \times P^+_n \).

VI. EXPERIMENTAL RESULTS

A boost converter (2) and the corresponding observer-based controller (14) have been implemented. The converter parameters are given in Table I. The observer-based controller was implemented, only for academic purposes and development, using the dSpace card 1103.

To show the performance of the controller, step changes in the reference voltage were applied. The desired average output voltage was set to be \( V_d = 16\text{V} \) and \( V_d = 20\text{V} \). The performance of the observer-based controller scheme is shown in figures (7), (8). The above figures also show the resulting changes in the duty ratio which is applied to the boost converter. The results show that observer-based controller behaves well. Figure (9) shows the performance of the Kalman-like observer with \( \theta = 10000 \) for estimating the inductor current.

At the end, experimental results of the observer-based controller scheme under a \( R \) variation of 33% are shown. The additional load was connected in parallel to the nominal load using a Power Mosfet. These results are important because load changes occur quite often in these converters. Figure (10) shows the boost behavior in open loop conditions. This result shows that the converter is unable to maintain a constant output voltage due to changes in the output load. Figure (11) shows, under closed-loop conditions, the output voltage behavior when a step change in the reference voltage and load changes were applied.

Thus, the above results show that the proposed observer-based controller scheme allows to operate DC-DC converters even under load variations.

VII. CONCLUSIONS

This paper presents the design of an observer-based controller scheme for a class of state affine systems. This scheme

| \( L \) | 200 \( \mu \)H |
| \( C \) | 100 \( \mu \)F |
| \( R \) | 36Ω |
| \( \eta_0 \) | 0.5 |
| \( E \) | 12V |
| \( f \) | 10KHz |
| \( D1 \) | U1560 |
| \( M \) | IRF510 |

TABLE I
EXPERIMENTAL VALUES

Fig. 7. Boost response with \( V_d = 16\text{V} \). **Top**: Output voltage (axis-y 5 V/div and axis-x 1 s/div), **Bottom**: Duty ratio (axis-y 0.2/div and axis-x 1 s/div)

Fig. 8. Boost response with \( V_d = 20\text{V} \). **Top**: Output voltage (axis-y 5 V/div and axis-x 1 s/div), **Bottom**: Duty ratio (axis-y 0.2/div and axis-x 1 s/div)

Fig. 9. Dash line: estimated current. Solid line: real current. (axis-y 500 mA/div and axis-x 100 \( \mu \)s/div).
is later applied to control switch-mode DC-DC converters. A passivity-based control approach is used to estimate the inductor current to stabilize this class of nonlinear systems. Analytical results confirm the asymptotical stability of the observer-based controller scheme by means of the nonlinear separation principle. Moreover, experimental results in a boost converter illustrate the effectiveness of the proposed scheme under step changes in the reference signal and output load. For a practical application, the observer-based controller can be implemented using a DSP.

REFERENCES