A mode-switching path planner for UAV-assisted search and rescue

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Abstract—Unmanned aerial vehicles (UAVs) can assist in U.S. Coast Guard maritime search and rescue missions by flying in formation with a manned helicopter while using infrared cameras to search the water for targets. The presence of constant wind is treated as a moving point in the helicopter’s location, velocity, and acceleration [7]. It will also include the command station for the autopilot system, and a laptop computer to run the estimation and control. The tracking algorithm must send feasible trajectories to the UAV autopilot in the form of waypoint commands. Given a feasible trajectory of waypoints, the autopilot will control the UAV’s nonlinear dynamics, justifying the use of a kinematic model for the combined autopilot and UAV system, and the phrasing of the helicopter tracking control problem as path planning.

B. Assumptions on mission and hardware

The hardware for UAV-assisted search and rescue will consist of a number of UAVs, an unmodified Coast Guard helicopter, and the UAV central controller, to be located on the helicopter but not interfaced with onboard systems. This means that the entire system will be self-contained, and can be used with any available helicopter at short notice. The UAVs will have minimal onboard equipment, limited to cameras and a waypoint-following autopilot such as the Piccolo system by Cloud Cap Technology [5]. The UAVs’ cruising speed will determine the nominal speed of the search pattern. The central controller will include a global positioning system receiver for Kalman Filter estimation of the helicopter’s location, velocity, and acceleration [7]. It will also include the command station for the autopilot system, and a laptop computer to run the estimation and control. The tracking algorithm must send feasible trajectories to the UAV autopilot in the form of waypoint commands. Given a feasible trajectory of waypoints, the autopilot will control the UAV’s nonlinear dynamics, justifying the use of a kinematic model for the combined autopilot and UAV system, and the phrasing of the helicopter tracking control problem as path planning.

C. Other work

Path planning in two dimensions for a vehicle with turn rate constraints is a well-studied problem, with much development based on Dubins’ result [1]. The search and rescue tracking problem differs from Dubins’ result in that factors such as collision avoidance and ground coverage must be considered. Also, the goal is to track a moving rather than fixed target. The adaptation of Dubins’ result to tracking a moving target is addressed in [6], where a fixed point in the presence of constant wind is treated as a moving point in the wind-fixed frame. Optimal trajectory tracking using a Dubins model is also discussed in [8]. Previous work specific to the UAV-assisted search and rescue problem addresses tracking of helicopter altitude changes and the effects of redundant sensor coverage due to formation spacing [3]. The contents of this paper are presented in more depth in [4].
D. Paper structure

In section II the helicopter trajectory and the form of the control will be detailed. In section III a specific control will be developed. Analysis of safety and convergence properties, along with simulation results, will be presented in section IV, and conclusions and future work in section V.

II. PROBLEM FORMULATION

The tracking algorithm will use Kalman Filter estimation of the helicopter’s position, velocity, and acceleration to calculate UAV waypoint commands. A waypoint is fixed distance \( d \) from the helicopter when its velocity and heading are constant. The distance \( d \) between the vehicles’ parallel tracks is a function of their altitude \( A \) and angle of sensor sweep \( \beta \), such that sensor coverage at ground-level is maintained with the desired percent redundancy \( f \).

\[
d = 2(1 - f)A \tan \left( \frac{\beta}{2} \right)
\]  

A helicopter following the expanding square search pattern will be modeled as flying at either a constant velocity \( V \) or with constant magnitude acceleration \( a \). The pattern consists of straight line segments of increasing length and velocity \( V \), alternated with right turns. A turn is modeled as a constant deceleration to zero velocity, an instantaneous turn, and a constant acceleration back to velocity \( V \). The Kalman Filter will detect the deceleration that begins a turn, as well as estimate \( V \) and \( a \) to predict the helicopter’s position throughout. Turns cannot be simply predicted based on the size of the search pattern due to the uncertainty introduced by the human pilot.

The turn maneuver begins when the helicopter begins to decelerate. At this time, the UAV is located a distance \( L \) behind the helicopter, measured along-track as shown in fig. 2. \( L(k) \) is defined as this distance, measured at the beginning of the turn maneuver with index \( k \) in a series. The end of the turn maneuver can be defined in space by choosing a finish line perpendicular to the helicopter path at which the helicopter and UAV have constant parallel headings and speeds.

The progression of \( \{L(k)\} \) is determined only by the UAV’s path during the series of turn maneuvers because the UAV’s and helicopter’s relative positions are fixed when not turning. The UAV will complete the turn maneuver in \( T_u \) seconds, and the helicopter in \( T_h \) seconds. \( L(k+1) \) can now be written as a function of \( T_u \) and \( T_h \), where \( T_u \) will depend on \( L(k) \) and the UAV’s path. Any variation from a vehicle’s expected route or velocity that takes place between turn maneuvers will appear as a disturbance, \( \Delta(k) \).

\[
L(k+1) = (T_u - T_h)V + \Delta(k)
\]

The series \( \{L(k)\} \) parameterizes the UAV’s tracking error through a series of turns, evaluating the two-dimensional continuous time tracking problem in terms of the single discretely evolving variable, \( L \). A path planning law can now be evaluated based on the resulting convergence properties of \( \{L(k)\} \) and the sensor coverage provided throughout the turn. During a corner maneuver, the control law takes the form of a set of path planning rules, \( \{T_i\} \). A particular path rule \( T_i \) is implemented when \( L(k) \) falls within the associated domain, \( D_i \), and determines \( L(k+1) \) according to the progression function \( F_i \). Implementation of path rule \( T_i \) results in a certain UAV path length, and corresponding time \( T_u \). Therefore \( T_i \) determines \( L(k+1) \) as in equation (2), or more generally as follows.

\[
L(k+1) = F_i(L(k)) + \Delta(k), \quad L(k) \in D_i
\]

The interaction of the set of path rules can be represented as a finite automaton with discrete states \( \{T_i\} \) and invariants \( \{D_i\} \). A transition from \( T_i \) to \( T_j \) is enabled when \( L(k) \in T_i \) and \( F_i(L(k)) \in T_j \). A transition is made at each discrete time step, and because \( \{D_i\} \) partitions the space of \( L(k) \), exactly one transition will be enabled and the execution will be deterministic.

III. SOLUTION: A SET OF FOUR PATH RULES

The following four path planning rules form a solution to the search and rescue tracking problem. They guarantee convergence of \( \{L(k)\} \) and safe separation between the UAV and helicopter, and are formulated to provide ground coverage. Convergence is based on a particular (convergent) path rule that causes the series \( \{L(k)\} \) to converge geometrically to zero after the execution enters that path rule’s domain. The other path planning rules ensure that for any initial condition, the execution enters the domain of the convergent path rule in a small number of steps.
of the convergent path rule. This prevents the existence of a

Fig. 3. Cutting the corner \((T_1)\) reduces large lag.

The helicopter’s new path instead of immediately as it would

Fig. 4. The fixed curve \((T_2)\) causes constant reduction in lag and provides

better sensor coverage than the convergent rule would over this domain.

1) Cut Corner Path: For large values of \(L(k)\), the UAV
can round off the inside of the corner, resulting in \(L(k+1)\)
greatly reduced (fig. 3). The domain is calculated geometri-
cally such that \(L(k+1) \geq \min D_3\), where \(D_3\) is the domain
of the convergent path rule. This prevents the existence of a
cycle in the transition graph.

\[
D_1 = \left\{ L(k) : \frac{V^2}{2a} + 2d + \left(2 - \frac{\pi}{2}\right) R \right\}
\]

\[
F_1 = L(k) - \frac{V^2}{a} - 2d + \left(\frac{\pi}{2} - 2\right) R
\]

2) Fixed Curve Path: The fixed curve path rule is a
variation of the convergent rule, where the path is modified
to keep the UAV closer to the helicopter for better ground
coverage. It can be seen in the following section that the
second arc in the convergent path rule moves farther from the
helicopter’s track as \(L(k)\) increases, which would eventually
result in a coverage gap for large \(L(k)\). Instead, for \(L(k)\)
equal to \(L_0\), the arcs are fixed, resulting in constants \(y_0\) and
\(\theta_0\) for the fixed curve path rule. When \(L(k)\) is greater than
\(L_0\), the UAV begins to turn the corner a distance \(L_0\) behind
the helicopter’s new path instead of immediately as it would
in path rule three.

\[
D_2 = \left\{ L(k) : L_0 \leq L(k) \leq \frac{V^2}{2a} + 2d + \left(2 - \frac{\pi}{2}\right) R \right\}
\]

\[
F_2 = (c-1)L_0 + L(k)
\]

\[
L_0 = \frac{1}{c+1} \left[ \left(\frac{5\pi}{2} - 4\right) R - \frac{2V^2}{a} \right]
\]

\(c\) chosen by designer in \(T_3\)

3) Convergent Path: The convergent path consists of a
half-circle, a straight segment, and two tangent arcs (fig. 5).
The angle \(\theta\) decreases as the second arc is shifted farther
from the UAV’s desired track. This path rule is defined on a
domain in which the UAV starts the turn on the near side of
the helicopter’s new path and angle \(\theta\) will be positive, i.e.,
the path consists of just one intermediate straight segment.
\(\theta(k)\) is a function of the length \(y(k)\) of the straight segment,
which is selected so that \(\{L(k)\}\) converges geometrically
to zero (6) at a rate \(c\) selected by the designer.

\[
L(k + 1) = c L(k), \quad c \in (0, 1)
\]

\[
D_3 = \left\{ L(k) : \frac{\sqrt{2a}}{2} < L(k) < \frac{1}{c+1} \left[ \left(\frac{5\pi}{2} - 4\right) R - \frac{2V^2}{\pi} \right] \right\}
\]

\[
L(k + 1) = \left(\frac{5\pi}{2} - 2\theta(k) - 3 - 2\cos \theta(k)\right) R + y(k) - d - \frac{3V^2}{2a} \quad (7)
\]

\[
\theta(k) = \arcsin \left( \frac{R + d - L(k) - y(k) - \frac{V^2}{2a}}{2R} \right)
\]

Eqn. (7) cannot be solved explicitly for the desired \(y(k)\) that
will produce \(L(k+1)\) as in 6. However, by fixing \(L(k)\), \(L(k+1)\)
can be approximated very accurately as a linear function
of \(y(k)\) and replaced by its first order Taylor expansion. The
function \(L(k+1)\) is linearized at the point \((y_0, L(k))\). The
selection of \(y_0\) can be made a priori because its effect is
linear over the entire range.

\[
L(k + 1) = f(L(k), y(k)) \quad (8)
\]

\[
f(L(k), y(k)) \approx f(L(k), y_0) + \frac{\partial f}{\partial y} \bigg|_{y=y_0} (y - y_0)
\]

\[
\frac{\partial f}{\partial y} \bigg|_{y=y_0} = 1 + \left(1 - \sin \theta_0\right) \frac{1 - \left(\frac{R + d - L(k) - \sqrt{2a}}{2R} - y_0\right)^2}{2R} \quad (9)
\]

The distance \(y(k)\) can now be selected based on the approx-
imation of its relation with \(L(k+1)\).

\[
y(k) = y_0 + \frac{c L(k) - f(L(k), y_0)}{\frac{\partial f}{\partial y} \bigg|_{y=y_0}}
\]
4) **Outside Loop Path:** If the UAV begins the turn maneuver ahead of the helicopter’s new path, as shown in fig. 6, its safe path is to loop back behind the helicopter. This leads to large $L(k+1)$, causing the UAV to lag behind the helicopter in the next turn.

$$D_4 = \left\{ L(k) : L(k) < \frac{V^2}{2a} \right\}$$

$$L(k+1) = \frac{3\pi R - 3\sqrt{2V^2 - d} - d}{2} + \left( L(k) + \frac{V^2}{2a} - d + R \right)^2$$

**IV. Analysis and Simulation**

Assuming that the helicopter executes the square spiral search pattern, and that the UAV perfectly tracks the path assigned by the active path rule, the progression of $\{L(k)\}$ can be modeled as in (3) without accounting for the system’s continuous-time dynamics. This type of simulation was used to verify the analysis of convergence properties. A more accurate simulation can be produced using a non-linear six degree of freedom UAV model controlled by a Piccolo autopilot. This is referred to as a hardware in the loop (HIL) simulation because the autopilot hardware is in the control loop. In this case, the Kalman Filter and mode-switching path planner are implemented using C++ and send waypoint commands to the Piccolo autopilot. This environment is very high fidelity because only the UAV’s flight dynamics are simulated; all other UAV systems are as they would be in application.

The following parameter values were used for all simulations. The UAV dynamics $V$ and $R$ are estimated from the SigRascal experimental platform at the University of California, Berkeley [5], which will be used for future field testing. The separation $d$ is calculated using (1) and an altitude suggested in U.S. Coast Guard documentation [2]. This altitude also ensures that a human-sized target will occupy a sufficient number of pixels on an application-typical infrared camera. The autopilot system receives GPS updates at 1 Hz.

$$V = 20 \frac{m}{s} \quad \text{UAV and helicopter nominal velocity}$$

$$R = 100 \ m \quad \text{UAV minimum turn radius}$$

$$a = 2.5 \frac{m}{s^2} \quad \text{helicopter acceleration}$$

$$A = 150 \ m \quad \text{altitude}$$

$$d = 250 \ m \quad \text{desired separation between UAV and helicopter}$$

The mathematical properties of the four path planning rules can be used to show that the aircraft will maintain a safe separation distance and that the helicopter tracking error will converge to zero. These analytical properties are then compared to the results of HIL simulation.

**A. Collision avoidance conditions**

Each of the path rules proposed in the previous section provides a safe path for the UAV. Path rule one is not addressed because the helicopter and UAV do not cross paths. Under path rule two, the UAV begins its first turn at distance $L_o$ from the helicopter’s initial position. If $L_o$ satisfies (11), the UAV will not cross the helicopter’s path.

$$L_o > R - \frac{V^2}{2a} \quad (11)$$

Substituting for $L_o$, (5) leads to the following condition for the safety of this trajectory. If this is not satisfied, the UAV will cross the helicopter’s path, and there is a chance of collision. Recall that $c$ was chosen by the designer as the convergence rate of path rule three, and is between zero and one.

$$\frac{V^2}{2aR} > \frac{5\pi - 5 - c}{3 - c} \quad (12)$$

Following path rule three, the UAV immediately begins turning in an arc that may cross the helicopter’s path twice, as shown in fig. 7. The intersection points are located at angles $\alpha$ and $\pi - \alpha$ along the arc, and the UAV and helicopter pass through the points with separations of $T_1$ and $T_2$ seconds. The worst case time differences for the two vehicles passing through these points are as follows.

$$\min T_1 = \frac{3V}{2a} + \frac{d + R - R\pi}{V} + \frac{R}{\sqrt{0 \leq \alpha \leq \pi/2}} \min \{ \cos \alpha + \alpha \}$$

$$\min T_2 = \frac{3V}{2a} + \frac{d + R}{V} - \frac{R}{\sqrt{0 \leq \alpha \leq \pi/2}} \max \{ \cos \alpha + \alpha \}$$
The designer will likely specify some minimum clearance time $T_d$ to ensure a factor of safety, resulting in the following conservative requirement on $V$, $a$, and $R$ after substituting for the appropriate limits. This requirement and (12) are both satisfied over a range of parameters including the simulation values.

$$\frac{3V}{2a} + \frac{d - R}{V} > T_d$$  \hspace{1cm} (13)

Although the UAV’s and helicopter’s paths intersect once under rule four, the helicopter passes through the intersection point almost immediately and the UAV passes through it only after completing a minimum turn radius loop (fig. 6), so there is no danger of collision.

B. Convergence properties

Recall that after \{\{L(k)\}\} converges to zero, the UAV will begin each corner at its desired location ($L = 0$) if there is no disturbance. We show that this will occur for any initial condition $L(1)$, and that the required number of corners before convergence is a bounded function of $L(1)$.

The convergence guarantee of the combination of path rules is based on the convergent rule $T_3$, which causes \{\{L(k)\}\} to converge geometrically to zero. Thus, showing that all trajectories enter $D_3$ is equivalent to proving convergence for the system. Because in the absence of disturbances, $L(k+1)$ depends only on $L(k)$, the range $R_i$ of possible $L(k+1)$ resulting from each path rule can easily be determined. $F_1$, $F_2$, and $F_3$ are all linear in $L(k)$ with positive slope, and $F_4$ decreases monotonically with $L(k)$ over its range. Therefore, the extrema of the ranges occur at the extrema of the domains of each path rule. Comparing the extrema of the ranges to the extrema of the domains allows the possible transitions between path rules to be enumerated as shown in fig. 8.

It can be seen from fig. 8 that all paths either lead to the convergent domain or remain in a cycle on $T_1$ or $T_2$. Therefore, by showing that the cycles on $T_1$ and $T_2$ can occur only a finite number of times, we show that \{\{L(k)\}\} enters $D_3$ in finite time. This is easily shown by the fact that for each of $T_1$ and $T_2$, $L(k+1)$ decreases by a positive constant $C_i$. $D_1$ and $D_2$ are both lower bounded, so the maximum number of executions ($N_i$) of either loop is bounded.

$$N_1 \leq (L(1) - \min(D_1))/C_1$$  \hspace{1cm} (14)

$$N_2 \leq (\max D_2 - \min(D_2))/C_2$$

Thus, for any initial condition, \{\{L(k)\}\} reaches $D_3$ in fewer than $3 + N_1 + N_2$ steps, and then converges exponentially to zero in the absence of disturbance. When the UAV or helicopter is allowed to deviate from its expected trajectory and introduce a disturbance, convergence can be ensured to a limited extent. Because the disturbance is not filtered through plant or controller dynamics, it cannot be directly attenuated. Therefore, a constant disturbance will result in steady state error. However, disturbances will not be amplified, and any single large disturbance has the same effect as restarting the system from a new initial condition, so convergence properties hold.

C. Simulation Results

Fig. 9 shows the helicopter and UAV paths from a typical HIL simulation. The percent sensor coverage of the search area is measured by plotting appropriately sized sensor footprints over the UAV’s and helicopter’s paths.

A single helicopter perfectly tracking a space filling curve can provide perfect sensor coverage of the ground. UAV assistance decreases the required density of the helicopter’s space-filling curve, and therefore allows faster search rates. However, when the UAV varies from the ideal space filling curve, gaps in sensor coverage may result. This effect also depends on the degree of redundancy in the formation spacing, represented by $f$ in (1). Increasing the redundancy reduces both the gaps in sensor coverage and the search speed. This tradeoff is examined for simulations with 0, 5, and 10 percent redundancy in the formation spacing and the results are shown in table I.

The sensor coverage results all fall within a very small range, meaning that it is difficult to conclude differences be-
corrected before the UA V begins its maneuver, and so has no effect in this case. However, in some cases when \( L(k) \) is small and the UA V’s and helicopter’s paths intersect, this estimation error leads to unacceptable clearance distance between the two vehicles.

The markers on the UAV’s path show the commanded endpoints of the arc. The marker at the beginning of the arc shifts when the estimate of the helicopter’s corner is corrected. Note that the UAV does not achieve the necessary turn rate to reach the commanded end point. This is due to delay in beginning the turn and overestimation of the UAV’s maximum turn rate. As a result, the UAV overshoots its desired path. This type of overshoot also leads to near-collisions in some cases.

The UAV’s failure to track the desired trajectory added 300 to 550 meters to its path length in HIL simulations. As a result, the transition diagram in fig. 8 is no longer correct, and the tracking error does not converge. In the simulation shown in fig. 9, the UAV alternates between corners of type one and two. The convergence properties developed previously depend on a transition to path rule three. The additional 300 to 500 meters in path length cause a transition back to path rule one instead, thereby creating a cycle between path rules one and two that causes \( \{ L(k) \} \) to remain in the range of approximately 400 to 1000 meters.

V. CONCLUSIONS AND FUTURE WORK

UAV-assisted search and rescue in the existing Coast Guard framework requires a tracking and formation flight algorithm that provides reliable ground coverage and ensures the safety of all aircraft. By assuming that an onboard autopilot is capable of tracking a feasible waypoint path, this problem is reduced from controller design to path planning. In this case we consider the path planning for a single UAV flying on the inside of a spiral pattern. The solution consists of a set of four path planning rules, which combine to provide safety and convergence guarantees.

The path rules were selected to keep the UAV as close as possible to its nominal trajectory within the constraints of vehicle kinematics, collision avoidance, and steady state tracking. In hardware in the loop simulation, unmodeled delays and estimation error caused the UAV to deviate from the desired trajectories. These deviations cause unpredictable path lengths, thereby negating the path-dependent guarantees of collision avoidance and tracking error convergence. These issues may be resolvable by using a more powerful trajectory tracking controller rather than parameterizing the desired path using a small number of waypoints.

The complete solution to UAV-assisted search and rescue must also include high-level control for tasks such as formation reconfiguration, recognition and tracking of detected objects. An independent collision avoidance control should also be included.

REFERENCES