Observer-based optimal control of dry clutch engagement

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Abstract—Optimal control of the engagement of a dry clutch is examined paying particular attention to the driver’s comfort. Due to the strong constraints and large errors of the torque control for internal combustion engines, only the normal force on the clutch disks is considered as controlled input. The resulting analytically derived controller is used in an observer-based trajectory tracking system whose performances have been tested on a highly realistic nonlinear model yielding very good results.

I. INTRODUCTION

The engagement control of automotive dry clutch is getting attention from automotive industry as a mean of enhancing the comfort of manual transmission (MT) passenger cars. The increasing torque of modern engines coupled with the stiffness reduction of the driveline make very hard for the driver to meet standard comfort requirements with a manually operated clutch, particularly in the standing start scenario where the energy dissipated in the clutch is maximal. The introduction of an automated manual transmission (AMT) or, eventually, a clutch-by-wire system is seen as a mean of easing the driver task and, thus, enhancing his satisfaction about the car.

The control of powertrain systems is an ample and well-established research field covering thermic, chemical, mechanical and electric systems. The driveline alone has been the subject of different studies concerning, for example, optimal shift strategies, hybrid vehicle management, engine torque estimation, dry clutch controlled sliding and engagement control.

Literature on dry clutch engagement control for AMT transmissions is quite ample and many different approaches have been proposed: quantitative feedback theory [1], fuzzy control [2] [3], model predictive control [4] and decoupling control [5]. Also optimal control of a dry clutch has been studied in some detail [6] and [7].

Our previous article [8] introduced a finite time, optimal control based engagement strategy employing the clutch torque as the only controlled input. This limitation is motivated by the poor performances of the torque control for internal combustion engines under dynamic load particularly for the low rotation speeds found during a standing start scenario. The present paper extends the dynamic model to include a Dual Mass Flywheel (DMFW), analyses the

1. In order to assure a smooth transition of the vehicle speed from zero to a minimal stabilised speed several coupling devices are used; on manual transmission or automated manual transmission vehicles a dry clutch, first introduced in the automotive field by Karl Benz in 1885 for his Benz-Velo, is used.

2. A dry clutch behaviour can be thought as composed by two running modes; a sliding mode where the torque is generated by the friction of the clutch disk against the flywheel and the pressure plate, and a non-sliding mode where the clutch behaves as a simple connecting link. During the sliding phase the clutch torque \( \Gamma_c \) is proportional to the normal force \( F_n \) exerted by the washer spring on the friction surfaces. The amount of normal force is controlled by acting on the washer spring’s end either through an hydraulic system connected to the clutch pedal or an hydraulic actuator, in the case of manual transmissions or automated manual transmission vehicles respectively.

During a standing start the normal force is gradually increased from zero to a maximum value whose amount gives the sportiness of the manoeuvre. Under the clutch torque action the driveline, and thus the vehicle, accelerate; since \( \Gamma_c \) is usually higher than the engine torque \( \Gamma_e \) the engine slows down and the clutch sliding speed is gradually reduced to zero. Once the sliding phase over, the clutch acts as a linking element; \( \Gamma_e \) equals \( \Gamma_c \) minus the engine inertial reaction torque. This sudden torque change excites the driveline generating highly uncomfortable longitudinal oscillations. Fig. 1 shows a starting stand simulation with subsequent driveline oscillations.

Usually an experienced driver can avoid, or at least minimise, the occurrence of this situation but the concurrent increase in engine torque output and reduction of the transmission stiffness, due to tougher NVH standards, make the

\(^1\) for robustness reasons the idle speed of an engine is slightly higher than the actual minimum speed.

\(^2\) Actually inside the clutch disk there is spring-damper system designed to filter out the engine acyclicity. Since its working frequency is sensibly higher than the one involved in driveline oscillations, its presence will be ignored in our analysis.
developing oscillation-preventing control strategies becomes from a classical manual transmission the importance of the automated manual transmission is an inexpensive upgrade both measured on the engine side and the driveline since the driveline poles are dominant when the

The driveline reduced model, valid during the clutch phase, is a simple four state linear system shown in Fig. 1 whose dynamic equations are:

\[
\begin{align*}
J_e \dot{\omega}_e &= \Gamma_c - \gamma F_n \\
J_g \dot{\omega}_g &= \Gamma_c - k_t \theta - \beta_t (\omega_g - \omega_v) \\
J_v \dot{\omega}_v &= k_t \theta + \beta_t (\omega_g - \omega_v) \\
\dot{\theta} &= \omega_g - \omega_v
\end{align*}
\]

where \(\omega_e, \omega_g, \omega_v\) are respectively the engine, gearbox and vehicle revolution speeds, \(J_e\) is the engine inertia, \(J_g\) and \(J_v\) are respectively the gearbox inertia and the equivalent vehicle inertia both measured on the engine side of the gearbox, \(\theta\) the transmission torsion, \(k_t\) and \(\beta_t\) the transmission stiffness and friction coefficients, \(\Gamma_c\), the engine torque, \(F_n\) the normal force on the clutch friction surfaces and \(\gamma = 2\mu_d R_c\) where \(\mu_d\) is the dynamic clutch friction coefficient and \(R_c\) is the mean radius of the clutch disks. This model is sufficiently simple to be used for designing the controller while still capturing the main dynamic of the the driveline since the driveline poles are dominant when the first gear is selected.

III. REDUCED MODEL FOR CONTROL

The driveline reduced model, valid during the clutch sliding phase, is a simple four state linear system shown in Fig. 1 whose dynamic equations are:

\[
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J_e \dot{\omega}_e &= \Gamma_c - \gamma F_n \\
J_g \dot{\omega}_g &= \Gamma_c - k_t \theta - \beta_t (\omega_g - \omega_v) \\
J_v \dot{\omega}_v &= k_t \theta + \beta_t (\omega_g - \omega_v) \\
\dot{\theta} &= \omega_g - \omega_v
\end{align*}
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IV. CONTROLLER DESIGN

A. Control objectives

The control objective is reaching the clutch’s engagement, i.e. \(\omega_e = \omega_g\), while assuring the comfort and minimising the dissipated energy and actuator activity. A finite-time optimal control approach with prescribed final states has been chosen since it assures the engagement to take place in a limited time window. The optimal controller is designed for the simplified system (1) with \(\Gamma_c\) as controlled input and \(\Gamma_c\) as known non-controlled input.

Oscillations induced in the powertrain by the sudden change of torque due to the end of the sliding phase are an important element of the engagement comfort. In order to suppress, or at least reduce, these oscillations it is necessary that the torque \(\Gamma_c(t_\Delta)\) transmitted by the clutch just before the engagement instant \(t_\Delta\) equals the engine torque \(\Gamma_c(t_\Delta^-)\) minus the engine inertia reaction torque. Expressing this equivalence in terms of the speeds at the sides of the clutch gives the so called no-lurch condition [5] which requires that \(\dot{\omega}_e(t_\Delta^-) - \dot{\omega}_g(t_\Delta^-) = 0\). This condition does not guarantee the driveline equilibrium after the engagement; numerical simulations, in fact, show that even when this condition is met the surplus elastic energy in the transmission can cause residual oscillations.

Ideally after the engagement all the powertrain elements should rotate with same speed and acceleration:

\[
\begin{align*}
\dot{\omega}_e &= \dot{\omega}_g = \omega_v \\
\ddot{\omega}_e &= \ddot{\omega}_g = \omega_v = \frac{\Gamma_c}{J_e + J_g + J_v}
\end{align*}
\]

Defining \(z_1 = \omega_e - \omega_g\) and \(z_2 = \omega_g - \omega_v\), through simple algebraic manipulation the simplified system (1) can be written as:

\[
\begin{align*}
\dot{z}_1 &= \frac{J_e + J_g}{J_e + J_g + J_v} z_2 + \frac{k_t}{J_e + J_g + J_v} \frac{J_v}{J_v + J_g} \Gamma_c - \frac{J_v}{J_v + J_g} \frac{J_{11}}{J_{11} + J_{12}} \Gamma_c \\
\dot{z}_2 &= -\frac{J_v}{J_v + J_g} z_2 - \frac{k_t}{J_e + J_g + J_v} \frac{J_v}{J_v + J_g} \Gamma_c + \frac{J_v}{J_v + J_g} \frac{J_{11}}{J_{11} + J_{12}} \Gamma_c \\
\dot{\theta} &= z_2
\end{align*}
\]

where

\[
\begin{align*}
J_{11} &= \frac{J_e J_g}{J_e + J_g} \\
J_{12} &= \frac{J_g J_v}{J_g + J_v}
\end{align*}
\]

The final boundary conditions that assure a perfect equilibrium at the end of the engagement for this model are:

\[
\begin{align*}
z_1(t_f) &= 0 \\
z_2(t_f) &= 0
\end{align*}
\]

\[
\theta(t_f) = \frac{1}{\beta_t} \frac{J_e \Gamma_c(t_f)}{J_e + J_g + J_v} \\
\Gamma_c(t_f) = \frac{(J_e + J_g) \Gamma_c(t_f)}{J_e + J_g + J_v}
\]

Due to the physical structure of the clutch the main limitation of the actuator is the slew rate of the normal force \(F_n\). Adding the extra state equation \(\dot{\Gamma}_c = u\) to the simplified system and weighting the new input \(u\) allows to limit this aspect of the actuator activity since \(\Gamma_c = \gamma F_n\).

Finally, in order to increase the driver’s comfort, the peak clutch torque during the engagement should be as low as possible while still archiving the specified final conditions.

B. Optimisation problem

All the control objectives defined in the previous subsection define the following optimisation problem: finding \(u(t)\) on \(T = [t_0, t_f]\) such that minimises the following quadratic value function:

\[
J = \frac{1}{2} \int_{t_0}^{t_f} \left[ z^T Q z + u^T R u \right] dt
\]
under the constraint:

\[ \dot{z} = A_z z + B_{z1} \Gamma_e + B_{z2} u \]  

where:

\[ A_z = \begin{bmatrix} 0 & -\frac{\partial}{\partial t} & \frac{1}{J_z} & -\frac{1}{J_z} \\ 0 & \frac{1}{J_z} & -\frac{1}{J_z} & \frac{1}{J_z} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{z1} = \begin{bmatrix} \frac{1}{J_z} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{z2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \]

with prescribed initial and final states:

\[ z_1(t_0) = z_{10}, \quad z_2(t_0) = z_{20}, \quad \theta(t_0) = \theta_0, \quad \Gamma_e(t_0) = 0 \]

\[ z_1(t_f) = z_{1f}, \quad z_2(t_f) = z_{2f}, \quad \theta(t_f) = \theta_f, \quad \Gamma_e(t_f) = \Gamma_{ef} \]

under the assumption that \( \Gamma_e \) is a measured non-controllable input. Even dropping the hard final state constraints in favour of a heavy weighting of the final states in the cost function does not allow the use of a LQ controller since (4) is not stabilisable under the same assumptions made in subsection IV-C for \( \Gamma_e \).

The more general differential analysis theory [9] defines the optimal input \( u(t) \) as:

\[ u = -\lambda_4 \]

where \( \lambda_4 \) is defined by the following Two Point Boundary Value Problem (TPBVP):

\[ \dot{x} = A_L x + B_L \Gamma_e \]  

where \( \Gamma_e \) is a known non-controllable input and:

\[ x = \begin{bmatrix} z_1 & z_2 & \theta & \Gamma_e & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}^T \]

\[ A_L = \begin{bmatrix} A_z & -B_{z2}R^{-1}B_{z2}^T \\ -Q & -A_z^T \end{bmatrix}, \quad B_L = \begin{bmatrix} B_{y1} \\ 0 \end{bmatrix} \]

with the following boundary conditions:

\[ z_1(t_0) = z_{10}, \quad z_1(t_f) = z_{1f}, \quad z_2(t_0) = z_{20}, \quad z_2(t_f) = z_{2f}, \quad \theta(t_0) = \theta_0, \quad \theta(t_f) = \theta_f, \quad \Gamma_e(t_0) = \Gamma_{e0}, \quad \Gamma_e(t_f) = \Gamma_{ef} \]

C. Analytic solution

Solving the TPBVP implies finding \( \lambda_1(t_0) \ldots \lambda_4(t_0) \) that satisfy the boundary conditions in \( t_f \) effectively transforming (5) in an initial value problem (IVP). Standard TPBVP numerical resolution methods, such as the shooting method, are not an interesting option for online implementation of the controller due to their calculation cost.

Assuming \( \Gamma_e \) constant\(^3\) over the interval \( T, (5) \) can be written as an homogeneous linear system with a non-controllable constant state \( \Gamma_e \):

\[ \dot{x} = A_L x + B_L \Gamma_e \Rightarrow \dot{\chi} = \Phi_L \chi(t_0) \]

where:

\[ \chi = \begin{bmatrix} z_1 & z_2 & \theta & \Gamma_e & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \Gamma_e \end{bmatrix}^T \]

Since the system is linear:

\[ \chi(t_f) = e^{A_L t_f} \chi(t_0) = \Phi_{tf} \chi(t_0) \]

Defining:

\[ \chi(t_0) = \begin{bmatrix} z_{10} & z_{20} & \theta_0 & \Gamma_{e0} & \lambda_{10} & \lambda_{20} & \lambda_{30} & \lambda_{40} & \Gamma_{e0} \end{bmatrix}^T \]

\[ \chi(t_f) = \begin{bmatrix} z_{1f} & z_{2f} & \theta_f & \Gamma_{ef} & \lambda_{1f} & \lambda_{2f} & \lambda_{3f} & \lambda_{4f} & \Gamma_{ef} \end{bmatrix}^T \]

we get:

\[ \begin{bmatrix} z_f \\ \theta_f \\ \Gamma_{ef} \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \lambda_f \\ \Gamma_e \end{bmatrix} \]

whose first line defines the linear system:

\[ \varphi_{12} \lambda_0 = z_f - \varphi_{11} z_{0f} - \varphi_{13} \Gamma_e \]

which, since \( \varphi_{12} \) is invertible, defines \( \lambda_0 \) as a function of \( z_0, z_f \) and \( \Gamma_e \). Once this initial value is known the feedback controller is defined by an opportune partition of \( A_L \):

\[ \dot{\lambda} = A_L \lambda + B_\lambda \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \]

\[ u = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix} \lambda \]

D. Resulting control law

The clutch torque:

\[ \Gamma_c(t) = \int_{t_0}^{t_f} u(\tau) \, d\tau \quad \forall t \in [t_0, t_f] \]

where \( u(\tau) \) is defined by the dynamic feedback:

\[ \dot{\lambda} = A_L \lambda + B_\lambda \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \]

\[ u = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix} \lambda \]

is the optimal, finite time, clutch engagement control relative to the weight function:

\[ J = \int_{t_0}^{t_f} \left[ a z_1^2(t) + b z_2^2(t) + c \Gamma_e^2 + u^2(t) \right] \, dt \]

for the system (1) under the assumption that \( \Gamma_e \) is a known constant or linearly time dependant input. The initial state of the controller are defined by the a linear combination of the initial and final states:

\[ \lambda_0 = \varphi_{12}^{-1} (z_f - \varphi_{11} z_{0f} - \varphi_{13} \Gamma_e) \]

\(^3\)The same line of reasoning holds for \( \Gamma_e \) linearly time-dependant.
E. Controller robustness analysis

The analytic solution of the TPBVB has been obtained under the assumption that the engine torque is constant or linearly time dependent. As pointed out in the introduction the torque control under dynamic change for low revolution speeds is quite inaccurate thus the robustness of the proposed controller has to be investigated.

Decomposing the perturbed engine torque \( \Gamma_e \) in its nominal component \( \bar{\Gamma}_e \) and the perturbation \( \tilde{\Gamma}_e \) we have, by linearity:

\[
\begin{align*}
\dot{x}(t) &= \bar{x}(t) + \tilde{x}(t) \\
\dot{\bar{x}} &= A_L \bar{x} + B_L \bar{\Gamma}_e \\
\dot{\tilde{x}} &= A_L \tilde{x} + B_L \tilde{\Gamma}_e
\end{align*}
\]

Since the minimum target that has to be assured is \( z_1(t_f) \approx 0 \) our analysis will be limited to this quantity:

\[
\begin{align*}
z_1(t_f) &= \bar{z}_1(t_f) + \tilde{z}_1(t_f) = \bar{z}_1(t_f) \\
&= C_L e^{A_L(t_f-t_0)} + \int_{t_0}^{t_f} C_L e^{A_L(t-f\tau)} B_L \bar{\Gamma}_e(t) \, d\tau \\
&= \left( \int_{t_0}^{t_f} C_L e^{A_L(t-f\tau)} B_L \right) \bar{\Gamma}_e(t_f) \\
&= \Phi(\bar{\Gamma}_e)
\end{align*}
\]

where:

\[
C_L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

and the initial error \( \tilde{x}(t_0) \) has been assumed equal to zero.

Defining \( \bar{\Gamma}_e, \Phi(\bar{\Gamma}_e) : \mathbb{G} \rightarrow \mathbb{R} \) can be seen as a linear operator from the \( \mathbb{G} \) space to the real space. The induced \( L_2 \) norm on \( \Phi \) is:

\[
\| \Phi(\bar{\Gamma}_e) \|_2^2 = \sup_{\bar{\Gamma}_e} \| \Phi(\bar{\Gamma}_e) \|_2^2 \\
= \left( \int_{t_0}^{t_f} C_L e^{A_L(t-f\tau)} B_L \right) \sup_{\bar{\Gamma}_e} \left( \int_{t_0}^{t_f} F_2(\tau) \, d\tau \right)^2
\]

Since a zero crossing \( \bar{\Gamma}_e \) cannot yield an extremum, we can assume without loss of generality that either \( \bar{\Gamma}_e \geq 0 \) \( \forall t \) or \( \bar{\Gamma}_e \leq 0 \) \( \forall t \); applying the Chebyshev integral inequality for \( \bar{\Gamma}_e \geq 0 \) (a symmetrical line of reasoning holds in the opposite case) we get:

\[
\| \Phi(\bar{\Gamma}_e) \|_2^2 \leq (t_f - t_0) \int_{t_0}^{t_f} \bar{\Gamma}_e^2(\tau) \, d\tau
\]

(7)

Since (7) becomes an equality for a constant \( \bar{\Gamma}_e \), the norm of the \( \Phi \) operator is:

\[
\| \Phi(\bar{\Gamma}_e) \|_2^2 = (t_f - t_0) \left( \int_{t_0}^{t_f} C_L e^{A_L(t-f\tau)} B_L \right) \sup_{\bar{\Gamma}_e} \left( \int_{t_0}^{t_f} F_2(\tau) \, d\tau \right)^2.
\]

Since \( A_L \) presents unstable poles this norm increases exponentially with time and has high values even for short \( t_f - t_0 \) intervals.\( \| \Phi(\bar{\Gamma}_e) \|_2 \approx 1.54 \) for a mid-sized car with \( a = b = c = 1 \) and \( t_f = 1 \) show a strong sensibility to perturbations on \( \bar{\Gamma}_e \).

F. Trajectory tracking

In order to assure the engagement in spite of the engine torque perturbations the overall control structure has been modified.

The optimal control, which, as seen in the previous section, is highly sensitive to the perturbations, has been connected to a linear unperturbed model in order to generate a target optimal trajectory \( y^* \) in the \( z \) state space and an optimal clutch torque \( \bar{\Gamma}_c^* \). A feed-forward/output-feedback trajectory tracking is then employed to drive the perturbed system.

The final structure, including the MIMO adaptive observer introduced in the next section, is shown in Fig. 3.

The \( \Phi \) operator norm using this control scheme can be shown, by simple algebraic manipulation, being equal to:

\[
\| \Phi(\bar{\Gamma}_e) \|_2^2 = (t_f - t_0) \left( \int_{t_0}^{t_f} C_{z_1}(t_f - t_0) \right)^2
\]

where:

\[
C_{z_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_{z_1} = \begin{bmatrix} 1 & 0 \end{bmatrix}
\]

With a correct choice of the feedback matrix \( K_{\bar{\Gamma}_e} \), the \( \Phi \) norm can be made arbitrary small.

V. OBSERVER DESIGN

In a production AMT vehicle only \( \omega_c, \omega_g, \omega_v \) and the position \( x_c \) of the clutch hydraulic actuator are directly measured. the engine torque \( \Gamma_e \) is estimated by the engine control unit and the actual normal force \( F_n \) exerted by the pressure plate on the clutch disk can be obtained by \( x_c \) under the assumption that the washer spring characteristic is known. Since \( \bar{\Gamma}_c = \gamma F_n \), an estimation of the friction coefficient \( \gamma = 2 \mu_d R_c \) is needed in order to apply a the optimal control output.

The system (1) can be written as:

\[
\begin{align*}
\dot{x} &= Ax + Bu(t) + \gamma B_2 F_n(t) \\
\dot{\gamma} &= 0 \\
y &= Cx
\end{align*}
\]

(8)

where \( A, B_1, B_2, C \) are constant matrices, \( F_n \) the normal force on the clutch disks, \( \gamma \) the friction coefficient and \( \omega_c, \omega_g, \omega_v \) the measured outputs \( y \).

Literature on adaptive observers for LTV SISO systems is quite ample; [10] extends these observers to the class of MIMO LTV systems of whom (8) is a particular case.

Using the results of [10], under the following sufficient hypotheses:

- \( \gamma = 2 \mu_d R_c \) constant

Due to the heating and wear effects on the contact surfaces the friction coefficient \( \mu_d \) varies but can be assumed constant for a single engagement event.
• exists a matrix \( \mathbf{K} \) | the system \( \dot{\eta} = (\mathbf{A} - \mathbf{KC})\eta \) is stable; i.e. the couple \((\mathbf{A}, \mathbf{C})\) is detectable.
• \( F_n(t) \) is persistently exciting, i.e. \( \exists \delta, T > 0 \) such that the following inequality is satisfied:

\[
\int_t^{t+\delta T} \mathbf{Y}^T(\tau)\mathbf{C}^T\mathbf{Y}(\tau)d\tau \geq \delta I \quad \forall t
\]

where

\[
\dot{\mathbf{Y}} = [\mathbf{A} - \mathbf{KC}]\mathbf{Y} + \mathbf{B}_2 F_n
\]

the system:

\[
\begin{align*}
\dot{x} &= \mathbf{A}x + \mathbf{B}_1 u + \hat{\gamma} \mathbf{B}_2 F_n + [\mathbf{K} + \mathbf{YB}_2 \mathbf{Y}^T\mathbf{C}^T] F_n [y - \mathbf{C}\hat{x}] \\
\dot{\gamma} &= \mathbf{B}_2 \mathbf{Y}^T \mathbf{C} [y - \mathbf{C}\hat{x}] F_n
\end{align*}
\]

is a global exponential adaptive observer for (8).

Since the couple \((\mathbf{A} - \mathbf{KC}, \mathbf{B}_2)\) is controllable and \((\mathbf{A} - \mathbf{KC}, \mathbf{C})\) observable the persistent excitation condition is easily verified with for \( F_n(t) \geq 0 \); a detailed justification is given in the appendix. Having that the simplified model is valid only during the sliding phase, we can also assume that \( \omega_e \neq \omega_d \) without loss of generality. Physically this implies that the driveline state and the friction coefficient can only be observed when \( \Gamma_c \neq 0 \), i.e. when the optimal control is active.

VI. NUMERICAL RESULTS

A. Simulation model

In order to obtain more realistic simulations a highly detailed nonlinear powertrain model has been used. This model, first introduced in our previous paper [8], has been improved including a nonlinear stiffness Dual Mass Flywheel (DMFW).

The dynamic equations describing the powertrain model, whose simplified scheme is shown in Fig. 4, are:

\[
\begin{align*}
J_e \dot{\omega}_e &= \Gamma_e - k_{dm}(\theta_{dm})\dot{\theta}_{dm} - \beta_{dm}(\omega_e - \omega_{dm}) \\
J_{dm} \dot{\omega}_{dm} &= k_{dm}(\theta_{dm})\dot{\theta}_{dm} + \beta_{dm}(\omega_e - \omega_{dm}) - \Gamma_e \\
J_g \dot{\omega}_g &= \Gamma_g - \Gamma_d / \alpha \\
J_{tl} \dot{\omega}_tl &= \Gamma_d / 2 - k_{tl}\theta_{tl} - \beta_{tl}(\omega_{tl} - \omega_{ul}) \\
J_e \dot{\omega}_tr &= \Gamma_d / 2 - k_{tr}\theta_{tr} - \beta_{tr}(\omega_{tr} - \omega_{wr}) \\
J_{ul} \dot{\omega}_{ul} &= k_{tl}\theta_{tl} + \beta_{ul}(\omega_{tl} - \omega_{ul}) - R_w F_{xul} \\
J_{wr} \dot{\omega}_{wr} &= k_{tr}\theta_{tr} + \beta_{wr}(\omega_{tr} - \omega_{wr}) - R_w F_{xwr} \\
\dot{\theta}_{dm} &= \omega_e - \omega_{dm} \\
\dot{\theta}_{tl} &= \omega_{tl} - \omega_{ul} \\
\dot{\theta}_{tr} &= \omega_{tr} - \omega_{wr} \\
M \ddot{v} &= F_{xul} + F_{xwr}
\end{align*}
\]

where \( \omega_e \) is the engine rotational speed; \( \omega_d \) the DMFW secondary mass speed; \( \omega_g \) the gearbox speed; \( \omega_{tr} \) and \( \omega_{tl} \) the right and left transmission speeds; \( \omega_{wr} \) and \( \omega_{uw} \) right and left wheel speeds; \( v \) the vehicle speed; \( \theta_{dm} \) the DMFW internal torsion; \( \theta_{tr} \) and \( \theta_{tl} \) the right and left transmission torsion; \( \Gamma_e \) the engine torque; \( \Gamma_c \) the clutch torque and \( \Gamma_d \) the differential torque; \( F_{xtr} \) and \( F_{xul} \) the right and left longitudinal wheel friction forces; \( k_{dm}(\theta_{dm}) \) the nonlinear DMFW stiffness coefficient; \( k_{tr} \) and \( k_{tl} \) the right and left transmission stiffness coefficients; \( \beta_{tr} \) and \( \beta_{tl} \) the right and left transmission damping coefficients; \( R_w \) the wheel radius and \( \alpha \) the gearbox reduction.

The clutch torque \( \Gamma_c \) generation, defined by a normal force controlled LuGre model, is modeled through a nonlinear spring-damper dynamic system whose internal state is analogous to the bristle friction in the bristle friction model. This solution assures a simple continuous, albeit nonlinear, alternative to the linear piecewise switching model usually proposed in literature. Detailed explication of the model and its parameters is can be found in [11].

Right and left tire longitudinal friction forces are defined by a similar averaged lumped LuGre model [12] and the differential torque \( \Gamma_d \) is defined by imposing \( \omega_g = 1/2(\omega_{tr} + \omega_{tl}) \).

B. Simulation results

The complete control system has been tested on the nonlinear simulation model using actual car parameters (Renault Megane II 2.0 petrol engine) simulating a standing start scenario. In order to verify the robustness of the proposed control law worst case \( \pm 20\% \) constant offsets on the engine torque have been introduced together with an initial estimation error of the friction coefficient of \( \pm 100\% \); numerical results confirm very good perturbation rejection. In Fig. 5 a standing start simulation using the complete control system is shown, together with a non-stabilised optimal control and nominal run for comparing purposes. Since in this case the estimated friction coefficient is just half of the actual one \(( -100\% \) estimation error) the open loop optimal control generates a clutch torque twice the needed value causing a strong excitation of the driveline and a serious drop in \( \omega_e \) which would probably lead to an engine shutdown on a real car. The proposed control law, thanks to the concurrent action of the MIMO observer and trajectory tracking, on the other hand, assures a comfortable engagement despite the serious perturbations.

It's worth pointing out that since the trajectory stabilisation is done in the \( z \) space state the final revolution speeds of the driveline and vehicle acceleration are different from the ones obtained on the nominal run while still satisfying the constraints imposed on the final state.

VII. CONCLUSIONS

The engagement of a dry clutch for an automated manual transmission vehicle under realistic control and resources constraints has been considered. The finite-time optimal control problem obtained from this specifications has been addressed using the dynamic lagrangian method with analytical solution of the resulting Two Point Boundary Value Problem. As a result of the robustness analysis of the optimal control against engine torque perturbations a trajectory stabilisation feedback has been added. The control law thus obtained has the structure of a dynamic feedback with initial states defined by a linear combination of initial and target driveline states. An exponentially globally convergent nonlinear adaptive observer is used to estimate the clutch friction coefficient and the driveline state. The proposed complete controller structure has been tested using a novel, highly realistic,
A nonlinear model on the standing start scenario with a worst case perturbation on the engine torque with very good results.

The resulting engagement control system will soon be tested on a specially equipped AMT production vehicle.

APPENDIX

PERSISTENT EXCITATION CONDITION

Lemma 1: Given the controllable and observable linear system:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

\(y(\tau) = 0 \quad \forall \tau \in (0, t)\) implies \(u(\tau) = 0\) on the same interval.

Proof: \(y(t) = 0\) over the interval \((0, t)\) implies \(y^{(k)}(t) = 0\) \(\forall k \in \mathbb{N}\) over the same interval. By the definition of the observability matrix \(O\):

\[
O x(t) = \begin{bmatrix} y(t) & y'(t) & \cdots & y^{(n-p)} \end{bmatrix} = 0
\]

where \(n = \text{rank}(A)\) and \(p = \text{rank}(C)\). Since, by hypothesis, \(\text{rank}(O) = n\) we have \(x(t) \equiv 0\) on the given interval. Using the same line of reasoning on the controllability matrix \(C\) we have \(Cx = 0 \Rightarrow u(t) \equiv 0\) on \((0, t)\).