Modeling and Feedback Control for Subsonic Cavity Flows: 
A Collaborative Approach


Abstract—Feedback control of aerodynamic flows is attracting the attention of researchers from a wide spectrum of specialties, because of its interdisciplinary nature and the challenges inherent to the problem. One of the main goals of the Collaborative Center of Control Science at The Ohio State University is to bring together researchers from different disciplines to advance the science and technology of flow control. This paper presents a comprehensive summary of the effort of the Center on modeling and feedback control of subsonic cavity-flow resonance. In particular, we give a detailed description of the experimental apparatus, including the wind tunnel testbed, the data measurement and acquisition system, and the real time control system. Reduced-order models of the flow dynamics based on physically-oriented linear models and Proper Orthogonal Decomposition are introduced and their effectiveness for control system design discussed. Finally, results obtained with experimental and model-based controller design are discussed.

I. INTRODUCTION

In recent years, several attempts have been made to apply closed-loop control methods to the control of flow phenomena [3], [6], [9], [10], [15], [17], [18], [19], [20], to name but a few. In particular, a benchmark problem in aerodynamic flow control is suppressing by means of active feedback control the oscillations induced by a flow propagating over a shallow cavity. This flow is characterized by strong resonance produced by a natural feedback mechanism similar to that occurring in other flows with self-sustained oscillations (e.g. impinging jet, screeching jet). In all these cases, shear layer structures impacting a discontinuity or obstacle in the flow (e.g. the cavity trailing edge) scatter acoustic waves that propagate upstream and reach the shear layer receptivity region where they tune and enhance the development and growth of shear layer structures. The resulting acoustic fluctuations can be very intense and can lead to structural damage in air vehicles. Rossiter first developed an empirical formula for predicting the cavity flow resonance frequencies, today referred to as Rossiter frequencies or modes [13], [11].

Successive studies investigated in more detail the physics of this system and have proposed ways to suppress or control the cavity-flow resonance (see Cattafesta et al. [3] for a recent review). Although flow-induced cavity resonance is a well-studied problem, the effects of the closed loop dynamic control on the flow dynamics are not well understood yet.

The Collaborative Center of Control Science (CCCS) at The Ohio State University is contributing to the advancement of the state of the art in the field of flow control with a multi-disciplinary effort to develop tools and methodologies for closed-loop flow control. The flow control team of the CCCS comprises of researchers from the Departments of Mechanical Engineering and Electrical and Computer Engineering at OSU, NASA Glenn, and the Air Force Research Laboratory - Air Vehicles Directorate at Wright-Patterson AFB. The team has synergistic capabilities in all of the required multidisciplinary areas of computational fluid dynamics, low-dimensional and reduced-order modeling, controller design, and experimental integration and implementation of the components along with actuators and sensors. The problem initially chosen as a benchmark is precisely the control of the acoustic resonance of a flow over a shallow cavity described in [16]–[18]. To this end, a small wind tunnel with a cavity recessed in the floor and equipped with state of the art diagnostics is used by the CCCS group to experimentally test and validate different control techniques. The aim of the paper is to provide a comprehensive overview of the activity of the group on modeling and feedback control design, including a presentation of the specific experimental setup and a discussion of the results obtained in experiment and simulation. The paper is organized as follows: the experimental apparatus is described in Section II. Section III discusses low dimensional modeling techniques of cavity flow dynamics from the perspective of physically-oriented and POD methods. Controller design and real time implementations are presented in Section IV, followed by concluding remarks in Section V.

II. EXPERIMENTAL APPARATUS AND ANALYSIS

In this section we outline the experimental setup described in more detail in Debiasi and Samimy [7]. The core of the experimental setup consists of an optically accessible, blow-down type wind tunnel with a test section of width $W = 400$ mm for an aspect ratio $L/D = 4$. The cavity shear-layer is gently forced by a 2-D synthetic-jet from a slot of height $h = 0.1$ mm...
embedded in the cavity leading edge, see Fig. 1. Actuation is provided by the movement of the titanium diaphragm of a Selenium D3300Ti compression driver whose voltage signal is amplified by a Crown D-150A amplifier. This assembly has a mean actuator to main flow momentum ratio $C_\mu = \frac{hw_2}{HU_\infty^2}$ in the range $10^{-6}$ to $10^{-4}$, where $u$ is the rms value of the forcing velocity at the actuator exit slot and $U_\infty$ is the velocity of the freestream in the test section above the cavity. The Pressure fluctuations are measured by Kulite dynamic pressure transducers placed in different locations in the test section. A dSPACE 1103 DSP board connected to a Dell Precision Workstation 650 computer is used to simultaneously acquire the pressure signals at 50 kHz through 16-bit channels and manipulates them to produce the desired control signal from a 14-bit output channel. Each recording is band-pass filtered between 200 and 10,000 Hz to remove spurious frequency components. In order to maximize the control board performance, its processor is used exclusively for running the control routines. For more detailed spectral analysis, simultaneous pressure recording of 262,144 samples each are band-pass filtered between 200 and 20,000 Hz and acquired at 200 kHz through a 16-bit resolution acquisition board (National Instruments PCI-6036E) operating independently in the computer. By using the Kulite sensitivity and accounting for the amplifier gain setting, the voltage values of the timetraces are converted to non-dimensional pressure referenced to the commonly used value of 20 $\mu$Pa. Thirty two narrowband power spectra, each from 8192 points, are computed using fast Fourier transform with Hanning window, converted to Sound Pressure Level (SPL) spectra, and then averaged. The resulting spectra have a spectral resolution of about 24 Hz and are accurate within $\pm 1$ dB. A stereo particle imaging velocimetry (PIV) is currently being used to obtain detailed velocity, vorticity, and turbulence data in the flow. The system is composed by a dual-head Nd-Yag laser operating at 10 Hz, with minimum time separation between the two heads of 200 ns.

Two CCD cameras (2K by 2K) with maximum acquisition frequency of 15 Hz capture the images when the laser is fired. Dedicated software is used to process the images and obtain the velocity flow field information. For this purpose the flow is seeded with sub-micron size particles using an atomizer that guarantees the uniform size and distribution of particles in the flow. These measurements provide the set of snapshots required for the derivation of the POD basis for the low dimensional model. In the initial phase of the experiments only 2 velocity components are obtained.

Debiasi and Samimy [7] observed that the experimental setup exhibits strong, single-mode resonance in the Mach number ranges 0.25-0.31 and 0.39-0.5, and multi-mode resonance in the Mach number range 0.32-0.38 as shown in Fig. 2 where the dominant peaks are represented by closed circles, whereas open circles represent other peaks appearing in multi-mode resonance. Shown are also the frequencies predicted by the semi-empirical formula of Rossiter [13], [11] and the cavity first longitudinal and transversal (vertical) modes.

### III. ANALYTICAL MODELS

From a system-theory point of view, the most outstanding difficulty in approaching cavity flow control comes from the nature of the governing Navier-Stokes equations, resulting in an intractable nonlinear infinite dimensional system. These equations cannot be solved sufficiently fast for any practical model, and they cannot be used in any internal model control scheme. Therefore, the key to the success of feedback control strategies for the considered problem lies in the development of suitable reduced-order models of the flow dynamics that can be effectively used for controller design. A physically motivated linear model was proposed and used in [15], [20]. It has been shown that a feedback controller derived from this model based on the $H^\infty$ mixed sensitivity minimization reduces the dominant resonant tone for which it is designed.
but introduces tones at other frequencies. As discussed in [21], linear models seem to be inadequate to describe cavity flow dynamics exhibiting stronger non-linearity. Recently, the use of Proper Orthogonal Decomposition (POD) along with Galerkin projection methods has become increasingly popular in many flow problems [2], [5], [14]. By means of POD and Galerkin projection, a system described by partial differential equations can be approximately reduced to a finite set of ordinary differential equations. By a boundary control separation method, it is possible to incorporate the control input explicitly in the reduced order model, which is desirable from the point of view of the control design.

In this section, we summarize the modeling results on cavity flow control from the perspectives of physics interpretation as well as POD based reduced-order methods.

A. Physics Based Linear Model

The physically motivated linear model introduced in [20] and [15] involves separate linear transfer function blocks for the shear layer $G(s)$, scattering $K_s$, acoustic feedback $A(s)$, and receptivity $K_R$, as shown in Fig. 3. The shear layer dynamics can be taken to be a second order system with a time delay:

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2} e^{-s\tau_a}$$

where the parameters have been determined from the experimental data. Defining the acoustic feedback as

$$A(s) = \frac{e^{-s\tau_a}}{1 - r(s)e^{-2s\tau_a}}, \quad r(s) = \frac{r}{1 + s/\omega_r}$$

and model the receptivity feedback and scattering as constant gains, then the plant transfer function becomes

$$P(s) = \frac{K_s G(s) A(s)}{1 - K_R K_s G(s) A(s)}.$$

The parameters of the above linear system can be optimized to match the open loop response of the cavity pressure fluctuations, at a given fixed Mach number. For the nominal Mach 0.3 case, the frequency response of the linear model and the experimental data from real time acquisition are depicted in Fig. 4. The reader is referred to [22] for details.

B. Reduced Order Modeling Based on POD

The POD method was introduced to the fluid dynamics community by Lumley [12] as a way to extract large-scale structures in a turbulent flow. The general idea is to decompose the flow field into a set of orthogonal bases that contains the most dominant characteristics of the flow. To reduce computational requirements POD modes can be obtained from highly spatially-resolved data sets like those obtained from numerical simulations or advanced laser based diagnostics using the snapshot method. A nonlinear POD based model of the cavity flow dynamics was derived in [18]. Expressing the velocity field of the flow in terms of the eigenfunction decomposition

$$u(t, x) = \sum_{i=1}^{N} a^i(t) \phi^i(x)$$

one obtains a set of ordinary differential equations of the form

$$\dot{a}^k(t) = b^k + \sum_{j=1}^{N} d^{jk} a^j(t) + \sum_{j, m=1}^{N} g^{jmk} a^j(t) a^m(t) + \langle \epsilon^k, \Gamma(t) \rangle + \sum_{j=1}^{N} \langle f^{jk}, \Gamma(t) \rangle a^j(t), \quad 1 \leq k \leq N$$

(1)

governing the time coefficients $a^i(t)$ of the POD modes. The constant scalar coefficients $b^k$, $d^{jk}$, $g^{jmk}$, and the constant vectors $\epsilon^k$ and $f^{jk}$ are obtained from Galerkin projection, while $\Gamma(t) = (\Gamma_u(t), \Gamma_v(t), \Gamma_c(t))$ is the forcing input with components given by

$$\Gamma_u(t) = V(t) \cos(\alpha), \quad \Gamma_v(t) = \begin{cases} V(t) \sin(\alpha) & V(t) > 0 \\ 0 & V(t) \leq 0 \end{cases}, \quad \Gamma_c(t) \approx 1,$$

where $\alpha = \pi/6$. The numerical value of the coefficients of the POD model have been derived from Computational Fluid Dynamics (CFD) simulations conducted in absence of external input (the baseline case) and in presence of an external
sinusoidal excitation of the form \( V(t) = A \sin(2\pi f_c t) \), with \( f_c = 500 \text{ Hz} \) and \( f_c = 900 \text{ Hz} \) respectively. The constant \( \Gamma_c \) has been normalized to unity. The reader is referred to [18] for details. A compact expression for (1) is given as

\[
\dot{a} = F + Ga + H(a) + PV + Q(a)V, \tag{2}
\]

where \( F \in \mathbb{R}^N \), \( G \in \mathbb{R}^{N \times N} \), \( H(a) \in \mathbb{R}^{N \times N} \), \( P \in \mathbb{R}^N \), and \( Q(a) \in \mathbb{R}^N \). The output equation is given by

\[
p(t) = Ma(t),
\]

where \( p(t) \) is the output pressure at the center of the cavity floor and \( M \) is the coefficient matrix relating the pressure and the time coefficients.

IV. FEEDBACK CONTROL OF CAVITY FLOW RESONANCE

A. Experimental Controllers

It has been observed in [7] that the frequency of sinusoidal forcing with the synthetic jet-like actuator has a major impact on the cavity flow resonance whereas the effect of the amplitude is relatively minor and affects the control authority only at higher Mach numbers. This prompted the development of a logic-based type of control that searches in a closed-loop fashion the forcing frequencies that reduce the cavity flow resonant peaks and then maintains the system in such conditions through an open-loop, optimal frequency forcing (OpFF for brief). The technique performed well in the experimental trials and allowed identification of optimal frequencies for the reduction of resonant peaks in the Mach number range 0.25-0.5 explored (Fig. 5).

Typical PID controllers are also tested in real time implementations to suppress the dominant Rossiter Peak of the Mach 0.3 flow. Experimental results [21] show that the PID controller based on trial-and-error tuning can reduce the main frequency of oscillation (the third Rossiter mode), while introducing strong oscillations at 1900Hz, which corresponds to the second Rossiter mode of this flow. We further introduced a Parallel-Proportional with time delay (P-P) controller in the following form

\[
CP_P(s) = K_p(1 + e^{-hs}), \quad K_p = 8, \quad h = 2.6 \times 10^{-4}.
\]

Note that the time delay \( h = 260 \mu s \) introduces 180-degree phase shift for signals operating in the neighborhood of the second Rossiter mode thus effectively placing a "zero" at the corresponding frequency. In fact, it is easily seen that

\[
1 + e^{-2.6 \times 10^{-4} \times 2\pi \times 1932j} = 0.
\]

As observed in Fig. 6, the real time implementations of the P-P controller successfully reduce the resonant peak without introducing other strong spectral peaks. More importantly, the P-P controller exhibits very good robustness against flow variations, although its parameters are tuned for the nominal flow condition at Mach 0.3.

B. \( \mathcal{H}^\infty \) Control for physics based linear model

As discussed in Section III-A, the linear physics based model with parameter tuned on the basis of experimental data gives a fair approximation of the cavity flow dynamics. However, it was observed that the system parameters are quite sensitive to flow conditions (e.g. Mach number), and a different model for each operating condition needs to be identified. Note that \( P(s) \) can be factorized as \( P(s) = N_{o1}(s)N_{o2}(s)M_n(s) \), where

\[
N_{o2}(s) = K_SG_0(s) = \frac{K_S}{1 + 2\zeta s/\omega_0 + s^2/\omega_0^2},
\]

\[
M_n(s) = e^{-h_1s},
\]

\[
N_{o1}(s) = (1 - KR_N_{o2}(s)M_n(s) - r(s)M_2(s))^{-1},
\]

Fig. 5. Effect of OpFF control at 3920 Hz, 2.5 \( V_{rms} \) on cavity flow with different Mach numbers from experimental data; thin line is the unforced flow SPL spectrum and thick line is the spectrum with OpFF control at: a) Mach 0.29; b) Mach 0.27; c) Mach 0.31.
controller has the form
\[ C(s) = C_2(s)(1 - r(s)M_2(s)) + K_R, \]
where
\[ C_2(s) = \left( \frac{\gamma}{\gamma_{\text{min}}} - \frac{\gamma_{\text{min}}}{\gamma} \right) \frac{N^{-1}_d(s)}{1 + as + bs^2} \left( \frac{1}{1 + H(s)} \right), \]
and \( H(s) = H_{\text{FIR}}(s) + H_{\text{IIR}}(s) \) are FIR and IIR filters, respectively. All the controller parameters including \( \gamma, \gamma_{\text{min}}, a, \) and \( b \) appearing in the above formula, can be explicitly computed (see [22]).

Real time implementations of the \( \mathcal{H}_\infty \) controller have shown that suppression of the main Rossiter oscillation is achieved, but at the expense of the generation of new oscillations at 1900 Hz, suggesting that linear designs may lead unsatisfactory performance, and that the linear model may be inadequate to capture the cavity flow dynamics.

**C. LQ Observer and State feedback based on POD model**

A simple analysis of the POD model discussed in Section III-B reveals the existence of an unstable equilibrium point \( a_0 \) corresponding to the mean flow. Shifting the equilibrium of (2) to the origin corresponds to singling out the effect of the mean flow from the low order model, and considering the local behavior of the system around the mean flow. Letting \( \tilde{a} = a - a_0 \), we obtain in the new set of coordinates the dynamical model
\[ \dot{\tilde{a}}(t) = \tilde{G}_a\tilde{a}(t) + \tilde{H}(\tilde{a}(t))\tilde{a}(t) + \tilde{P}V(t) + \tilde{Q}(\tilde{a}(t))V(t) \quad (3) \]
and the output equation
\[ \tilde{p} = p - p_0 = Ma - Ma_0 = M\tilde{a}. \quad (4) \]
The Jacobian linearization of (3) and (4) at the origin is readily obtained as
\[ \begin{cases} \dot{\tilde{a}} = \tilde{G}_a\tilde{a} + \tilde{P}V \\ \tilde{p} = M\tilde{a} \end{cases} \quad (5) \]
It has been verified that the system (5) is controllable and observable for all considered cases. A convenient and well-established methodology for the controller design is offered by linear-quadratic optimal control. Let the cost function \( J_c \) be
\[ J_c = \int_0^\infty (\tilde{a}^T Q_w\tilde{a} + V^TR_wV)dt \]
where \( Q_w > 0 \) and \( R_w > 0 \) are the positive definite state weighting matrix and the scalar control weight, respectively.

In our design, the weights have been chosen as \( Q_w = I_{5\times5} \) and \( R_w = 1 \). The solution of optimal state feedback controllers \( V = -K_\gamma \tilde{a} \), where the subscript \( i = 0,1,2 \) stands for the baseline case, forced case with \( f_c = 500 \) Hz and \( f_c = 900 \) Hz respectively, have been obtained from the solution of the associated Riccati equations. Similarly we design a full state observer of the form
\[ \begin{align*} \dot{\tilde{a}} &= \tilde{G}_a\tilde{a} + \tilde{P}V + L(\tilde{p} - \tilde{p}) \\ \dot{\tilde{p}} &= M\tilde{a} \end{align*} \]

and \( h_1 = \tau_s + \tau_a, M_2(s) = e^{-2\tau_a s}. \) To reduce the main Rossiter peak and obtain good robustness against model uncertainties, we have derived an \( \mathcal{H}_\infty \) controller to solve the mixed-sensitivity optimization problem:

**Mixed-sensitivity Problem:** Minimize \( \gamma \) such that the robust controller \( C(s) \) is stabilizing \( P(s) \) and
\[ \left\| \begin{bmatrix} W_1S & W_2S \\ W_1T & W_2T \end{bmatrix} \right\| \leq \gamma, \]
where \( S \) and \( T \) are the sensitivity and complementary sensitivity, respectively.

The weights \( W_1 \) and \( W_2 \) have been chosen to meet the robustness and performance criteria [22]. The resulting controller has the form
\[ C(s) = C_2(s)(1 - r(s)M_2(s)) + K_R, \]
where
\[ C_2(s) = \left( \frac{\gamma}{\gamma_{\text{min}}} - \frac{\gamma_{\text{min}}}{\gamma} \right) \frac{N^{-1}_d(s)}{1 + as + bs^2} \left( \frac{1}{1 + H(s)} \right), \]
and \( H(s) = H_{\text{FIR}}(s) + H_{\text{IIR}}(s) \) are FIR and IIR filters, respectively. All the controller parameters including \( \gamma, \gamma_{\text{min}}, a, \) and \( b \) appearing in the above formula, can be explicitly computed (see [22]).

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\[ \begin{align*} \dot{\tilde{a}} &= \tilde{G}_a\tilde{a} + \tilde{P}V + L(\tilde{p} - \tilde{p}) \\ \dot{\tilde{p}} &= M\tilde{a} \end{align*} \]
where \( \hat{a}, \hat{p} \) are respectively the estimated state and estimated output, and \( L \) is optimal output-injection gain obtained by minimizing the cost function of the same form for the dual system. The reader is referred to [23] for details.

The LQ observer and state feedback for the POD model has been validated in Simulink (Fig. 7), where the pressure trajectory is shown to decay asymptotically to the origin. However, the controller has not been implemented in real time experiments yet. This is essentially due to the fact that the POD model is developed on CFD data which has been shown to have significant difference from experimental data. We are currently using dynamic surface pressure measurements along with PIV measurements to derive the coefficient of the POD model.

![Fig. 7. Time history of the output pressure](image)

**V. CONCLUSIONS**

We provided in this paper a general overview of the research activity of the Collaborative Center of Control Science at The Ohio State University on modeling and feedback control for cavity flow dynamics. The long term goal of this research is to provide analytical linear/nonlinear models for the cavity flow dynamics and explore general feedback control strategies. On-going research regards reliable identification of POD models of the experimental cavity flow, online estimation of the time coefficients, and nonlinear POD-based control strategies.

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