Coordinated Decentralized MRAC of Delayed Plants with Actuator Failures

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Abstract—This paper develops adaptive state feedback coordinated decentralized control scheme for a class of dynamic systems with state delay in subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem. The main contributions of this paper are the development of a new controller parametrization which attempt to anticipate the future states and failures, the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms, and a stability proof.

I. INTRODUCTION

Control design for systems with actuator failures is an area of research that has been studied a lot in recent years. Many important results have been obtained. The designs have been based mainly on the following approaches: actuator fault detection and isolation methods [1], [2], [3], the robust fault accommodation approach [4], multiple model switching and tuning methods [5], [6], and the sliding mode method [7].

One of the main research directions is adaptive control, see e.g. [8], [9], [10], [11], [12], and references therein.

In the recent series of papers, see, e.g. [10] and the books [11], [12] the model reference adaptive control (MRAC) technique was successful applied for the numerous problems with actuator failures in the centralized framework and in the delay-free case.

Yet, relatively few results using adaptive control for the important class of delayed systems with actuator failures are available in the literature. Time-delay is a natural component of dynamic processes in many engineering fields and its presence in the plants considerably complicates the design problem, see e.g. the recent papers [13] and [14] for centralized control cases. In [13] a fault detection and accommodation procedure is considered for stable nonlinear state delay plants with, based on an the iterative design of an observer which monitors the variations of the system dynamics. Within the framework of Linear Matrix Inequalities techniques, a robust state feedback linear controller $u = K_x(t)$ is designed for the stabilization of the linear plant with input delay, and actuator failures of stuck-type [14]. In [15] we proposed two adaptive state feedback control schemes for a class of linear systems with state delay in the presence of unknown actuator failures for the centralized control problem. To the best of the authors’ knowledge, the decentralized MRAC design problem for plants with actuator failures and time delay has not been solved yet.

In this paper, using a Lyapunov approach and our adaptive decentralized control scheme with model coordination, see, e.g. [16], [17], we present a decentralized model reference adaptive controller (DMRAC) for a class of uncertain dynamic systems with state delays in subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem.

A special Lyapunov-Krasovskii functional is used to design the update mechanism for the controller parameters. For the updating of the controller parameters, we use a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes [15].

The main contributions of the paper are:

1) the enlargement of the class of systems with actuator failures that can be handled using model reference adaptive control;
2) a direct coordinated decentralized adaptive control law parametrization which attempt to anticipate the future states and actuator failures in subsystems;
3) the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms and to prove stability.

This paper is organized as follows: In section 2 the problem statement and preliminaries are presented. In section 3 the decentralized controller parametrization with reference model coordination is presented. The section includes the adaptive control scheme and the proof of stability. The simulation results are presented in Section 4.

II. PLANT MODEL AND PROBLEM FORMULATION

We consider a class of uncertain systems, which are composed of $M$ multi-input multi-output subsystems with state delays in subsystems and in interconnections whose control components may fail at the time of operation described by...
equations, suitably initialized, of the form
\[
\dot{x}_i(t) = A_{rx_i}(t) + A_{r}(t) + B_{r}u_{pi}(t)
\]
where, for the \( i \)-th subsystem \( x_i \in \mathbb{R}^{n_i} \) is the state vector, \( u_{pi}(t) = [u_{p1}(t), \ldots, u_{pig}(t), \ldots, u_{pim}(t)]^T \) is applied to the plant real control input vector, \( b_{ri} \) (\( q = 1, 2, \ldots, m_i \)) is the \( q \)-th column of \( B_i \), \( \bar{u}_i = [\bar{u}_{i1}, \ldots, \bar{u}_{iq}, \ldots, \bar{u}_{im}]^T \) is some constant vector and \( u_i(t) = [u_{i1}(t), \ldots, u_{iq}(t), \ldots, u_{im}(t)]^T \) is the control vector to be designed. The constant matrices \( A_{ri}, A_{ti} \in \mathbb{R}^{n_i \times n_i}, B_i \in \mathbb{R}^{n_i \times m_i}, A_{il} \in \mathbb{R}^{n_i \times m_i} \) have unknown elements. \( \tau_i \in \mathbb{R}^{+}, i, l, 1, \ldots, M \) are known time delays and \( \sum_{i=1}^{m} n_i = n \).

The indicator matrix \( Y_i = \text{diag}[1, \ldots, 0, \ldots, 0] \in \mathbb{R}^{m_i \times m_i} \) describes the working condition of the actuators. \( u_{iq} = 1 \) denotes that the \( q \)-th actuator of the \( i \)-th system is in normal mode, and the actually applied \( q \)-th control input to the subsystem is \( u_{iq} \). \( u_{iq} = 0 \) denotes a failed actuator, e.g. stuck at zero position, and the actual applied \( q \)-th control input to the subsystem equals \( \bar{u}_{iq} \). If all actuators of the subsystem are in normal mode, the matrix \( Y_i \) is an identity matrix \( Y_i = I \). The constant value \( \bar{u}_{iq} \) and the failure time instant \( t_{iq} \) are unknown, i.e. the type of actuator failures considered here is the same as in [9]:

\[
u_{pi}(t) = \bar{u}_{iq}, \quad t \geq t_{iq}, \quad q = 1, 2, \ldots, m_i.
\]

Note that it is postulated that a failed actuator never returns to normal operation. For the decentralized control problem considered in this paper, the first assumption is that \( (A1) \) the system (1) without actuator failures is decentralized state feedback stabilizable.

The problem is to design an adaptive feedback control, and tune, online, the controller parameters in order to achieve desired closed loop specifications when there are up to \( m - 1 \) unknown actuator failures. The desired specification in this paper is that with a failure model (2), all signals of the closed loop system remain bounded, and that the each subsystem state \( x_i(t) \) asymptotically exactly follows the state \( x_i(t) \) of a stable reference model without delays

\[
\dot{x}_i(t) = A_{ri}x_i(t) + B_{ri}r_i(t)
\]
where \( A_{ri} \in \mathbb{R}^{n_i \times n_i}, B_{ri} \in \mathbb{R}^{n_i} \) are known constant matrices, and \( r_i(t) \in \mathbb{R}^{n_i} \) is a bounded reference input signal. I.e. we demand that \( \lim_{t \to \infty} \|e_i(t)\| = \|x_i(t) - x_i(t)\| = 0, \quad i = 1, \ldots, M, \) i.e. also in the presence of up to \( m - 1 \) actuator failures.

As in [10] for the centralized case, we assume that \( (A2) \) if the plant parameters and the actuator failures (up to \( m - 1 \) failures) are known, the remaining subsystem actuators can still achieve the desired control objective.

### III. PROPOSED ERROR EQUATION PARAMETRIZATION

Motivated by our previous works, see, e.g. [18], [16], [17], we will use the decentralized adaptive control scheme with reference model coordination to achieve the control objective. The control law for the \( i \)-th local subsystem \( u_{pi}(t) \) is chosen to be of the form

\[
u_{pi}(t) = u_{pi}(t) + u_{pci}(t)
\]

where the part of the control law \( u_{pi}(t) \) is based only on the local signals of the \( i \)-th subsystem, and the component \( u_{pci}(t) \) is the coordinated component which is based on the reference signals of the all other subsystems. Exchange of the reference signals between subsystems can be easily implemented in real-life control systems. Let us assume that all the parameters of (1) and the actuator failures are known, and let us define \( u_{pi}(t) \) as

\[
u_{pi}(t) = u_{pi}(t) = \Theta_{ri}^T e_i(t) + \Theta_{ti}^T x_i(t) + \Theta_{qi}^T x_{qi}(t - \tau_i)
\]

\[
+ \Theta_{ri}^T u_{ri}(t) + \Theta_{ti}^T u_{ti}(t)
\]

where the constant matrices \( \Theta_{ri}^q = [\theta_{ri}, \theta_{ri+1}, \ldots, \theta_{rim}] \in \mathbb{R}^{n_i \times m_i}, \Theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{irm}] \in \mathbb{R}^{n_i \times m}, \Theta_{ti}^q = [\theta_{ti}, \theta_{ti+1}, \ldots, \theta_{trim}] \in \mathbb{R}^{m_i \times m_i}, \Theta_{qi} = [\theta_{qi}, \theta_{q1+1}, \ldots, \theta_{qim}] \in \mathbb{R}^{m_i \times m_i} \) to be defined for perfect model-following, and the vector \( \Theta_i^q = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{irm}]^T \in \mathbb{R}^{m_i} \) we introduce, as in [10], [12], for cancel of the failed actuators effect. The coordinated component \( u_{ci}(t) \) is used to cancel the effect of the interconnections, as defined next.

The \( k \)-th component of \( u_{pi} \), \( q = 1, 2, \ldots, m_i \), can be write

\[
u_{pi}(t) = u_{pi}(t) + u_{ci}(t)
\]

Suppose there are \( g \) failed actuators, that is

\[
u_{pi}(t) = \bar{u}_{ij}, \quad \text{if} \quad j = 1, 2, \ldots, g, \quad 1 \leq g \leq m_i - 1
\]

than with definition of \( u_{pi}(t) \) in (6) there exist [10], [15] constant parameters \( \theta_{oxiq}^q, \theta_{oxiq}^q, \theta_{oriq}^q \) and \( \theta_{oxiq}^q \) such that \( A_{ri} - A_{ri} + b_{ri} \Theta_{oxiq} = 0, A_{ti} + b_{ri} \Theta_{oxiq} = 0, b_{ri} \Theta_{oxiq} - b_{ri} = 0 \) and

\[
A_{ri} - A_{ri} + \sum_{q \neq 1, \ldots, g} b_{ri} \Theta_{oxiq} = 0
\]

Introducing the parameter errors \( \tilde{\theta}_{oiq}(t), \tilde{\theta}_{oiq}(t), \tilde{\theta}_{oiq}(t), \tilde{\theta}_{oiq}(t) \)

\[
\tilde{\theta}_{oiq}(t) = \Theta_{oiq}(t) - \tilde{\theta}_{oiq}, \quad \tilde{\theta}_{oiq}(t) = \Theta_{oiq}(t) - \tilde{\theta}_{oiq}
\]

\[
\tilde{\theta}_{oiq}(t) = \Theta_{oiq}(t) - \tilde{\theta}_{oiq}, \quad \tilde{\theta}_{oiq}(t) = \Theta_{oiq}(t) - \tilde{\theta}_{oiq}
\]

(8)
where $\theta_{eq}(t)$, $\theta_{slq}(t)$, $\theta_{sly}(t)$, $\theta_{sly}(t)$ and $\theta_{slq}(t)$ are the adaptation gain matrices and applying (6) to the actual plant (1), then using (7) and (3) we obtain

$$
\dot{e}_i(t) = A_{rl}e_i(t) + \sum_{l=1}^{M} m_i b_{ilq} a_{ilq}^T x_l(t - \tau_{li}) \\
- \sum_{q \neq j_1, \ldots, j_g} m_i b_{iq} \theta_{eq}^\tau e_i(t - \tau_i) + \sum_{q \neq j_1, \ldots, j_g} m_i b_{iq} \eta_{iq}(t) \\
+ \sum_{q \neq j_1, \ldots, j_g} m_i b_{iq} [\theta_{eq}^\tau e_i(t) + \theta_{slq}^\tau x_i(t) + \theta_{sly}^\tau x_i(t - \tau_i)] \\
+ \theta_{slq}(t) + \eta_{iq}(t)
$$

Let us define the coordinated component of control $u_{c_{ilq}}(t)$ as

$$
u_{c_{ilq}}(t) = \sum_{l=1}^{M} \theta_{c_{ilq}}^\tau x_l(t - \tau_{li})
$$

Then with (10), using the value $b_{iq} = b_i \theta_{orig}^{-1}$ from (7) and the error $e_i(t - \tau_{li}) = x_i(t - \tau_{li}) - x_i(t)$ we obtain

$$
\dot{e}_i(t) = A_{rl}e_i(t) + \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} a_{ilq}^T x_i(t - \tau_{li}) \\
+ \sum_{q \neq j_1, \ldots, j_g} m_i b_{ilq} \theta_{orig}^{-1} [\theta_{eq}^\tau e_i(t) + \theta_{slq}^\tau x_i(t)] \\
+ \theta_{slq}(t) + \nu_{c_{ilq}}(t) \\
+ \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} \theta_{c_{ilq}}^\tau x_i(t - \tau_{li})
$$

where $\tau_{li} = \tau_i$ and

$$a_{ilq} = \begin{cases} a_{ilq}, & l \neq i; \\
\bar{a}_{ilq}, & l = i. \end{cases}
$$

Because we can write

$$
\sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} a_{ilq}^T x_i(t - \tau_{li}) \\
= \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} \theta_{c_{ilq}}^\tau x_i(t - \tau_{li})
$$

where $\theta_{c_{ilq}}^*$ is a some constant matrix we have

$$
\dot{e}_i(t) = A_{rl}e_i(t) + \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} a_{ilq}^T e_i(t - \tau_{li}) \\
+ \sum_{q \neq j_1, \ldots, j_g} m_i b_{ilq} \theta_{orig}^{-1} \theta_{c_{ilq}}^\tau x_i(t - \tau_{li}) \\
+ \theta_{slq}(t) + \nu_{c_{ilq}}(t) \\
+ \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} \theta_{c_{ilq}}^\tau x_i(t - \tau_{li})
$$

with $\theta_{c_{ilq}}(t) + \theta_{c_{ilq}}(t) = \theta_{c_{ilq}}^*$.

For the convenience of the analysis we introduce the following auxiliary variables and signals

$$
\dot{\theta}_{f_{ilq}} = \begin{bmatrix} \theta_{eq}^\tau(t), \theta_{slq}^\tau(t), \theta_{c_{ilq}}^\tau(t), \theta_{slq}(t), \theta_{ilq}(t) \end{bmatrix}
$$

$$
\omega_{f_{ilq}}(t) = [e_i^T(t), x_i^T(t), x_i^T(t - \tau_i), r_i(t), 1]
$$

and write the $q$th component of $u_i$ from (6) as

$$
u_{ilq}(t) = \theta_{f_{ilq}}^T(t) \omega_{f_{ilq}}(t) + \sum_{l=1}^{M} m_i \theta_{c_{ilq}}^\tau(t) x_i(t - \tau_{li})
$$

then we obtain the basic tracking error equation for stability analysis and adaptation algorithms design

$$
\dot{e}_i(t) = A_{rl}e_i(t) + \sum_{l=1}^{M} m_i b_{ilq} \theta_{orig}^{-1} a_{ilq}^T e_i(t - \tau_{li}) \\
+ \sum_{q \neq j_1, \ldots, j_g} m_i b_{ilq} \theta_{orig}^{-1} \theta_{c_{ilq}}^\tau(t) x_i(t - \tau_{li})
$$

IV. ADAPTATION ALGORITHMS AND STABILITY ANALYSIS

To design the update laws for the control parameter matrices $\theta_{eq}(t)$ and $\theta_{c_{ilq}}(t)$ in the adaptive control (15) we use the following Lyapunov-Krasovskii type functional

$$
V(*) = \sum_{i=1}^{M} V_i, \quad V_i = V_{ei} + V_{ni} + \sum_{l=1}^{M} \left( V_{ni\dot{d}} + V_{eli} \right)
$$

$$
V_{ei} = e_i^T(t) P_i e_i(t), \quad V_{eli} = \frac{1}{M} \int_{t-\tau_i}^{t} e_i^T(s) Q e_i(s) ds
$$

$$
V_{ni} = \sum_{q \neq j_1, \ldots, j_g} \left| \theta_{orig}^{-1} \right| \left( \eta_{ilq}^T \Gamma_{ilq}^{-1} \eta_{ilq} + \int_{t-h_i}^{t} \eta_{ilq}^T(s) \Gamma_{ilq}^{-1} \eta_{ilq}(s) ds \right)
$$

where

$$
\eta_{ilq} = \dot{\theta}_{f_{ilq}}(t) + \eta_{ilq} + \eta_{lq}(t) + \eta_{lq}(t - h_i) \\
\eta_{ilq} = \dot{\theta}_{c_{ilq}}(t) + \eta_{ilq} + \eta_{lq}(t) + \eta_{lq}(t - h_i)
$$

and $Q_e = Q_e^T > 0$, $\Gamma_{ilq} = \Gamma_{ilq}^T > 0$ and $\Gamma_{ilq}^T = \Gamma_{ilq} > 0$ are matrices of corresponding dimensions. The matrix $P_i = P_i^T > 0$ is from the Lyapunov equation

$$
A_{rl}^T P_i + P_i A_{rl} + Q_i + Q_e \leq 0, \quad Q_i^T = Q_i > 0
$$

and

$$
\eta_{lq} = \frac{1}{2} r_0 \theta_{orig}^{-1} [b_i^T P_i 0 0 0 0 0] \quad r_0 > 0
$$

where $r_0 > 0$ is some scalar constant. This constant $r_0$ and the time-varying vectors $\eta_{ilq}(t)$ and $\eta_{ilq}(t)$ are “artificial” whose values will be defined later. These parameters are used
only in the process of the stability proof. The parameters \( h_i \) and \( h_{il} \) are design parameters in the delayed components of adaptation algorithms, as will be seen below.

We now choose the adaptation algorithms as

\[
\theta_{fiq}(t) = -\eta_{iq}(0) - \eta_{iq}(t) - \eta_{iq}(t - h_i) - \int_0^t \eta_{iq}(s) ds,
\]

\[
\eta_{fiq}(t) = sign[\theta_{orig}^*] \Gamma_{i1q} \omega_i(t) e_i^T(t) P_i b_{ri}
\tag{21}
\]

\[
\theta_{diq}(t) = -\eta_{diq}(0) - \eta_{diq}(t) - \eta_{diq}(t - h_d) - \int_0^t \eta_{diq}(s) ds,
\]

\[
\eta_{diq}(t) = sign[\theta_{orig}^*] \Gamma_{i2q} x_{rl}(t - \tau_d) e_i^T(t) P_i b_{ri}
\tag{22}
\]

or in differential form

\[
\dot{\theta}_{fiq}(t) = -\dot{\eta}_{iq}(t) - \dot{\eta}_{iq}(t - h_i),
\]

\[
\dot{\eta}_{fiq}(t) = sign[\theta_{orig}^*] \Gamma_{i1q} \omega_i(t) e_i^T(t) P_i b_{ri}
\tag{23}
\]

\[
\dot{\theta}_{diq}(t) = -\dot{\eta}_{diq}(t) - \dot{\eta}_{diq}(t - h_d),
\]

\[
\dot{\eta}_{diq}(t) = sign[\theta_{orig}^*] \Gamma_{i2q} x_{rl}(t - \tau_d) e_i^T(t) P_i b_{ri}
\tag{24}
\]

where \( h_i \) and \( h_{il} \) are some design parameters.

Remark 1: Although only the integral component \( \eta_{iq}(t) \) (\( \eta_{iq}(0) = 0 \) and \( \eta_{iq}(t - h_i) = 0 \) in (23)) of the adaptation algorithm is needed for stability and exact asymptotic tracking, the use of the proportional and the proportional delayed terms in the adaptation algorithm (23) makes it possible to achieve better adaptation performance than the traditional integral (I) and proportional integral (PI) schemes see, e.g. [15] in which a proportional integral time delay (PITD) adaptation algorithm is used for the decentralized adaptive control. This adaptation algorithm includes the traditional I and PI schemes [19] as a special case. The design parameters \( h_i \) and \( h_{il} \) are chosen in the same way as the traditional gains \( \Gamma_{i1q} \) and \( \Gamma_{i2q} \) in (21) - (23).

Using (19), the time derivatives of the components of (17) along (16) can be written as

\[
\dot{V}_{ei|16}|_{(16)} = e_i^T(t) [A_i^T P_i + P_i A_i] e_i(t)
\]

\[+ 2 e_i^T(t) P_i \sum_{i=1}^m \sum_{q=1}^{m_i} b_{ri} \theta_{orig}^* a_{ilq}^{-1} T_{fiq}(t - \tau_d)]
\]

\[+ 2 e_i^T(t) P_i \sum_{q \neq j = 1}^{m_j} b_{ri} \theta_{orig}^* T_{fiq}(t) \omega_i(t)
\]

\[+ 2 e_i^T(t) P_i \sum_{q \neq j = 1}^{m_j} \sum_{l=1}^M b_{rl} \theta_{orig}^* \Gamma_{i1q} x_{rl}(t - \tau_d)]
\tag{25}

\[
\dot{V}_{eii|16}|_{(16)} = \frac{1}{M} \left( e_i^T(t) Q_e e_i(t) - e_i^T(t - \tau_d) Q_e e_i(t - \tau_d) \right)
\tag{26}
\]

\[
V_{\eta|16} = 2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \dot{\theta}_{fiq}(t) \Gamma_{i1q}^{-1} \dot{\eta}_{iq}
\]

\[+ 2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \eta_{iq}^T \Gamma_{i1q}^{-1} \dot{\eta}_{iq}
\]

\[+ 2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \left( \eta_{iq}(t) + \eta_{iq}(t - h_i) \right)^T \Gamma_{i1q}^{-1} \dot{\eta}_{iq}
\]

\[+ \eta_{iq}^T(t) \Gamma_{i1q}^{-1} \eta_{iq}(t) - \eta_{iq}(t - h_i) \Gamma_{i1q}^{-1} \eta_{iq}(t - h_i)
\tag{27}
\]

Combining (23) and (27) it follow that

\[
V_{\eta|16} = -2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \eta_{iq}^T \Gamma_{i1q}^{-1} \eta_{iq}
\]

\[+ 2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \left( \eta_{iq}(t) + \eta_{iq}(t - h_i) \right)^T \Gamma_{i1q}^{-1} \eta_{iq}
\]

\[+ \eta_{iq}^T(t) \Gamma_{i1q}^{-1} \eta_{iq}(t) - \eta_{iq}(t - h_i) \Gamma_{i1q}^{-1} \eta_{iq}(t - h_i)
\tag{28}
\]

and view of (20) we obtain

\[
V_{\eta|16} = - r_0 \sum_{q \neq j_1, \ldots, j_k}^{m_i} e_i(t)^T P_i b_{ri} b_{ri}^T P_i e_i(t)
\]

\[+ 2 \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \left( \eta_{iq}(t) + \eta_{iq}(t - h_i) \right)^T \Gamma_{i1q}^{-1} \eta_{iq}
\]

\[+ \eta_{iq}^T(t) \Gamma_{i1q}^{-1} \eta_{iq}(t) + \eta_{iq}(t - h_i) \eta_{iq}(t - h_i)
\tag{29}
\]

Similarly we have

\[
V_{\eta|16} = - \sum_{q \neq j_1, \ldots, j_k}^{m_i} \left| \theta_{orig}^* \right|^{-1} \left( \eta_{diq}(t) + \eta_{diq}(t - h_d) \right)^T \Gamma_{i2q}^{-1} \eta_{diq}
\]

\[+ \eta_{diq}^T(t) \Gamma_{i2q}^{-1} \eta_{diq}(t) + \eta_{diq}(t - h_d) \eta_{diq}(t - h_d)
\tag{30}
\]

Then using (19), (29) and (30) we get

\[
V_{i|16} \leq - \frac{1}{M} \sum_{l=1}^{M} e_i^T(t - \tau_d) Q_e e_i(t - \tau_d)
\]

\[+ \frac{M}{M} \sum_{l=1}^{M} \sum_{q=1}^{m_i} e_i^T(t) P_i b_{ri} \theta_{orig}^* a_{ilq}^{-1} T_{fiq}(t - \tau_d)]
\]

\[+ \frac{M}{M} \sum_{l=1}^{M} \sum_{q=1}^{m_i} e_i^T(t) P_i \sum_{q \neq j = 1}^{m_j} b_{ri} \theta_{orig}^* \Gamma_{i1q} x_{rl}(t - \tau_d)]
\]

\[+ \frac{M}{M} \sum_{l=1}^{M} \sum_{q \neq j = 1}^{m_j} \sum_{l=1}^M b_{rl} \theta_{orig}^* \Gamma_{i1q} x_{rl}(t - \tau_d)]
\tag{25}

\[
V_{eii|16} = \frac{1}{M} \left( e_i^T(t) Q_e e_i(t) - e_i^T(t - \tau_d) Q_e e_i(t - \tau_d) \right)
\tag{26}
\]

\[
- \frac{M}{M} \sum_{l=1}^{M} e_i^T(t - \tau_d) Q_e e_i(t - \tau_d)
\]

\[+ \frac{M}{M} \sum_{l=1}^{M} \sum_{q=1}^{m_i} e_i^T(t) P_i b_{ri} b_{ri}^T P_i e_i(t), \quad \beta > 0
\tag{31}
\]
After completing the squares we have

\[
\dot{V}_i|_{(16)} \leq -e_i^T(t)Q_ie_i(t) - \frac{1}{\mu} \sum_{j=1}^{M} e_j^T(t - \tau_{ij})Q_{iej}(t - \tau_{ij})
\]

\[
+ \frac{1}{\mu} \sum_{j=1}^{M} e_j^T(t - \tau_{ij}) \Psi_{dij} e_j(t - \tau_{ij})
\]

(32)

where \( \Psi_{dij} = \sum_{q=1}^{m_{ij}} \frac{1}{\beta_{ijq}} \alpha_{ijq} \alpha_{ijq}^T \)

Let us \( Q_i = Q_{ei1} + Q_{ei2} \) with \( Q_{ei1} = Q_{ei1}^T \) and \( Q_{ei2} = Q_{ei2}^T \) are some matrices.

If we select the value \( r_0 \) from the inequality \( r_0 > \frac{\lambda_{max}(\Psi_{dij})}{\lambda_{min}(Q_{ei1})} \)

from (32) where \( \lambda_{max}(\cdot) \) and \( \lambda_{min}(\cdot) \) are the maximum and minimum eigenvalues of (\( \cdot \)), we obtain from (32)

\[
\dot{V}_i|_{(16)} \leq - \sum_{i=1}^{M} \left( e_i^T(t)Q_ie_i(t) \right)
\]

\[
+ \frac{1}{\mu} \sum_{j=1}^{M} \left[ e_j^T(t - \tau_{ij})Q_{ej}e_j(t - \tau_{ij}) \right] \leq 0
\]

(33)

for \( t \in (T_i, T_{i+1}) \), i.e. \( \dot{V}_i|_{(16)} \) is negative semi-definite. Thus [20], we have proved that the adaptive control (10) and the update laws (21) and (23) guarantee that \( V(t) \) and, therefore, \( e(t), \ \theta_{1ig} \in \mathbb{L}_{\infty} \). The remainder of the stability analysis follows directly using the steps in [21].

We have thus obtained the following result

**Theorem 1:** Consider system (1) and the reference model (3). Then the adaptive control (10) with update laws (21), (23) and actuator failures model (2) assures that the closed loop signals are bounded and that the tracking error \( e(t) \) converges to zero asymptotically.

**Remark 2:** We note that the coefficient matrices \( Q_i, Q_e \) and the scalar \( r_0 \) are used only for analysis and do not influence the control law. Decentralized controller gains adjust automatically to counter the non-desirable effects of delayed interconnections, actuator failures and parameter uncertainties.

**V. SIMULATION**

To illustrate the application of the proposed adaptive scheme, let us consider a plant with two subsystems described as

\[
\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} u_1(t)
\]

\[
\dot{x}_2(t) = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix} u_2(t)
\]

\[
\begin{align*}
x_1(0) = x_2(0) &= \begin{bmatrix} 0 & 0 \end{bmatrix}^T.
\end{align*}
\]

(34)

We choose the reference model as

\[
\begin{align*}
\dot{x}_{r1}(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x_{r1}(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} r_1(t) \\
\dot{x}_{r2}(t) &= \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix} x_{r2}(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} r_2(t)
\end{align*}
\]

\[
x_{r1}(0) = x_{r2}(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}
\]

(35)

All parameters except the time delays (\( \tau_1 = 5 \), \( \tau_2 = 5 \), \( \tau_3 = 4 \), \( \tau_21 = 4 \)) are unknown to the controller.

The adaptation algorithms (21) in our simulation are

\[
\theta_{ig}(t) = -PITD(\Theta_i(t) e_i^T(t) P_i b_i)
\]

where \( PITD(\cdot) \) is the operator form for \( PITD(\mathbf{Z}_i(t)) = k_f \int_0^t \mathbf{Z}_i(s) ds + k_p \mathbf{Z}_i(t) + k_D \mathbf{Z}_i(t - h_i) \), where the parameter values were chosen as \( h_1 = h_2 = 1, \ k_f = 1, \ k_p = 0.5, \ k_D = 0.05 \), with \( \Theta_i = [e_i(t) x_{r1}(t) x_{r2}(t - \tau_1) r_1(t) 1 x_{r2}(t - \tau_2)]^T \), and \( P_i \) from (19)

\[
P_1 = \begin{bmatrix} 9.1667 & 2.5000 & 1.6667 \\ 2.5000 & 7.5000 & 2.5000 \\ 1.6667 & 2.5000 & 5.0000 \end{bmatrix}.
\]

The input signals of the reference models \( r_1 \) and \( r_2 \) are \( r_1 = r_2 = 2 \sin(t) \).

In the simulation study, we suppose that the second control input of the subsystem 1 fails at \( t = 30 \) seconds and the second control input of the subsystem 2 fails at \( t = 50 \) seconds.

Simulation results are found in Figures 1–3.
We have developed a coordinated adaptive decentralized control scheme for state delayed systems with unknown actuator failures. This scheme ensures asymptotic exact tracking in the presence of unknown plant parameters and unknown actuator failure parameters. Simulation results verified the desired performance of the developed coordinated adaptive decentralized controller. For the updating of the decentralized controller parameters, we develop a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes. A special Lyapunov-Krasovskii functional is introduced to design the update mechanism for the controller parameters and prove stability.

VI. CONCLUSION

We have developed a coordinated adaptive decentralized control scheme for state delayed systems with unknown actuator failures. This scheme ensures asymptotic exact tracking in the presence of unknown plant parameters and unknown actuator failure parameters. Simulation results verified the desired performance of the developed coordinated adaptive decentralized controller. For the updating of the decentralized controller parameters, we develop a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes. A special Lyapunov-Krasovskii functional is introduced to design the update mechanism for the controller parameters and prove stability.

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