Moving Pattern-based Forecasting Model of a Class of Complex
dynamical systems

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Abstract—Considering the existence of uncertainty of complex dynamical systems, in contrast to traditional modeling approach to characterizing dynamical systems in Euclidean space, the dynamics of complex industrial processes is characterized by the move of operating condition patterns in pattern moving space. First, the operating condition patterns of complex dynamical systems are partitioned into C pattern classes constructing pattern moving space, and then pattern class variable characterizing the movement of operating condition patterns in pattern moving space is defined. Each pattern class characterized (quantified) by an interval-valued number can be considered as the “calibration” in pattern moving space. For modeling the move of pattern class variable in pattern moving space, interval autoregression model (IAR) is defined and applied to modeling the movement of pattern class variable in pattern moving space. Finally, Experimental results are then presented that indicate the validity and applicability of the proposed approach.

I. INTRODUCTION

In real world, there are many complex dynamical systems in metallurgy, chemical industry and building materials etc. (e.g. blast furnace, sintering machine, cement rotary kiln and so on.) which are characterized by too many parameters, nonlinearity, time-varying, and spatial distribution[1][2][3]. Therein, there is a series of physical chemistry reactions of heat transfer and matter transfer, burning thermodynamics and chemistry reaction dynamics, moving boundary, hydrodynamics and aerodynamics. It is difficult to characterize the dynamics. In addition, the mapping relationships between inputs and outputs can’t be described by classic Newton’s laws of mechanics, but be described only by statistics. For example, even under the same experimental conditions, the obtained data usually differ from one to another. Thus, the traditional modeling approach based on input/output data characterized in Euclidean space is not appropriate for modeling the relationship between inputs and outputs of a complex industrial process.

For this class of complex production process systems, there is lack of effective dynamics description methods. We try to give a new method to describe the movement law obeying statistics by using statistical principal, pattern recognition and control theory.

For modeling of complex dynamical systems, a pattern-based process identification approach emulating human in dealing with process response was originally introduced in [4]. Generally, the existing methods for pattern-based process identification can be divided into four classes [5], namely the key-values approach [6][7], the qualitative evaluation approach [8], the elementary pattern recognition approach [9], and the curve fitting approach [10][11]. In these methods, Pattern recognition is mainly used to solve system model identification or system state estimation problems. Here pattern is not considered as a moving variable.

Pattern recognition (classification) can be considered as a mapping from patterns to class labels. The patterns belonging to the same pattern class may have many different “values” as the existing uncertainty in a real “system” that produce the patterns, but from the point of view of pattern class, they have the same “value”- the same pattern class label. So the approach of patterns characterized (quantified) by pattern class is insensitive to uncertainty in a real “system”. In this paper, in contrast to traditional modeling approach to characterizing dynamical systems in Euclidean space, the dynamics of complex dynamical systems is characterized by the moving of operating condition pattern from one pattern class to another in pattern moving space. First, the collected multi-dimensional actual operating condition data of complex dynamical systems are reduced to one dimensional data. Then, the one dimensional data are partitioned into C pattern classes constructing the pattern moving space, and pattern class variable is defined in it. Each pattern class represented by an interval-valued number can be considered as a scale value of pattern moving space. The pattern samples belonging to the same pattern class have the same scale value in pattern moving space, and are represented by the same interval-valued number. That is, we view the considered systems at the level of pattern class and capture only the dominant characteristics of the process. We are not concerned about the exact value of the operating condition data of the complex dynamical systems, but concerned about the pattern classes which the operating condition data of complex dynamical systems belong to. After pattern moving space is constructed and pattern class variable is defined in this space, the prediction model is built as follows:

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dx(k) = F(d\tilde{x}(k))
\quad = F(f(dx(k-1), dx(k-2)\cdots dx(k-m_j)))
\tag{1}

Where $dx(k)$ is pattern class variable, $F(\cdot)$ denotes classification, $\tilde{d}(k)$ is the initial prediction output of pattern class variable and $\tilde{d}(k) = f(\cdot)$ is a kind of initial prediction model.

This is a prediction process including two steps. The first step is to obtain the initial prediction output $\tilde{d}(k)$ based on the initial prediction model $\tilde{d}(k) = f(\cdot)$ modeled by interval autoregression model (IAR) proposed in this paper. The second step is to obtain the final output pattern class variable $dx(k)$ by classifying the initial prediction output $\tilde{d}(k)$.

This paper is organized as follows. After the introduction the preliminary knowledge about interval arithmetic is presented in Section 2. In Section 3, moving pattern-based forecasting model of a class of complex production process is presented in Section 2. In Section 3, moving pattern-based forecasting model of a class of complex production process. In Section 4, one real example is considered to demonstrate the validity and applicability of the proposed approach. Finally, Section 5 concludes this paper.

II. PRELIMINARY KNOWLEDGE

In this section, we briefly look at some important formulations in interval arithmetic [12].

A. Interval arithmetic

An interval-valued number can be represented by its lower and upper bounds as $A = [a^L, a^U]$, or, equivalently, by its midpoint and radius as $A = (a^C, a^R)$, where $a^C = (a^L + a^U)/2$ and $a^R = (a^U - a^L)/2$. Let $A$ and $B$ be two intervals represented by its lower and upper bounds, the operations defined on the interval-valued numbers are, respectively,

$$A + B = [a^L + b^L, a^U + b^U]$$
$$\lambda \cdot A = [\lambda \cdot a^L, \lambda \cdot a^U]$$
\tag{2}
\tag{3}

Where $\lambda$ is a constant and $\lambda > 0$.

If the interval-valued number $A$ and $B$ are represented by its midpoint and radius, the operations defined on the interval-valued numbers are respectively:

$$A + B = (a^C + b^C, a^R + b^R)$$
$$\lambda \cdot A = (\lambda \cdot a^C, |\lambda| \cdot a^R)$$
\tag{4}
\tag{5}

Where “$||$” is absolute value notation.

B. Hausdorff distance

The Hausdorff distance is often used to measure the distance between two interval data. The concept of Hausdorff distance is stated as follows. Assume that two interval-valued numbers are $X_1$ and $X_2$, $X_1 = (a_1, c_1)$, $X_2 = (a_2, c_2)$. The Hausdorff distance between $X_1$ and $X_2$ is defined as:

$$D(X_1, X_2) = |a_1 - a_2| + |c_1 - c_2|$$
\tag{6}

III. MODELING OF COMPLEX DYNAMICAL SYSTEMS BASED ON MOVING PATTERN

In this paper, the dynamics of complex dynamical systems is characterized by the move of operating condition patterns in pattern moving space. So, we need to construct pattern moving space firstly.

A. Construction of pattern moving space

Firstly, the multi-dimensional operating condition data are reduced to one dimensional data using principal component analysis. This can be considered as a process of feature extraction. Then, by fuzzy c-mean (FCM) clustering algorithm, the one-dimensional operating condition samples are partitioned into C pattern classes, that is, $P_1, P_2, \cdots, P_C$. The collection of those classes forms pattern moving space which corresponds to the considered complex production process. An explanatory example for demonstrating the constructed pattern moving space is shown in Figure 1. The samples pattern belonging to the same pattern class have the same “scale” value in pattern moving space.

![Pattern moving space composed of six pattern classes. The horizontal axis denotes the values of dimension reduced running status samples, and vertical axis denotes membership function values corresponding to dimension reduced running status samples.](image)

If the data is collected in a long enough period of time, the pattern moving space constructed based on these actual operating condition data can be considered as the running subspace of the system. Dynamics description and control problems can be discussed in this space. When the actual operating condition sample is out of the pattern moving space, the pattern moving space can be expanded by using the following method. When a new operating condition sample is classified into its belonging class, the distance between the operating condition sample and this class centre is calculated. If this distance is more than the class radius, then a new pattern class generates. Detail algorithm can refer to[13][14]. In this paper, the new operating condition sample is used as a new class centre and the membership function can be computed for the new pattern class.

B. The quantification of pattern classes

In fact, each pattern class obtained in the above section is a qualitative description about pattern. In order to characterize quantitatively the dynamics of complex industrial processes in pattern moving space, each pattern class must be quantified. In this paper, we adopt interval-valued numbers to quantify the pattern classes.

First, Finding the maximum $I_{1u}$ and minimum $I_{1l}$ among
the samples belonging to the same pattern class \( P_i \), \( i = 1, \ldots, C \), where \( C \) is the number of pattern classes constructing the pattern moving space. Then, the maximum \( I_i^+ \) and minimum \( I_i^- \) are used to construct an interval-valued number \( I_i = [I_i^-, I_i^+] \). Thus, \( I_i = [I_i^-, I_i^+] \) is the interval-valued number characterizing pattern class \( P_i \) quantitatively. Let \( I = \{I_1, I_2, \ldots, I_C\} \), it is the quantitative pattern moving space corresponding to pattern moving space \( P \) defined in above section. Here, quantified pattern classes are ordered from small to big in terms of the center values of \( I_i \) s. That is, \( a_1 < a_2 < \cdots < a_C \), where \( a_i = \frac{I_i^+ + I_i^-}{2} \), \( i = 1, \cdots, C \). An demonstrating example of quantitative pattern moving space is shown in Fig.1(b).

C. The definition of pattern class variable

In contrast to traditional modeling approach to characterizing dynamical systems in Euclidean space, in this paper, the dynamics of complex dynamical systems is characterized by the move of operating condition patterns in pattern moving space.

In moving pattern-based modeling approach, operating condition pattern \( y(k) \) characterized (quantified) in Euclidean space is re-quantified in pattern class space. That is, by computing the membership degree \( \mu_{i,k} \) of the \( k \)th operating condition pattern sample \( y(k) \) in the \( i \)th cluster based on the membership functions obtained in the process of pattern class space construction, \( i = 1, \cdots, C \), if \( \mu_{j,k} = \max_{i=1,\ldots,C} (\mu_{i,k}) \), then \( y(k) \) is classified into the \( j \)th pattern class \( P_j \), and \( I_j \) is assigned to \( y(k) \). \( I_j \) is the new “measure value” in the quantitative pattern moving space for the operating condition pattern sample \( y(k) \). Note that at any instant \( k \), \( y(k) \) is classified into one, and only one pattern class. So, \( y(k) \) have an uniquely “measure value” \( I_j \) in the quantitative pattern moving space \( I = \{I_1, I_2, \cdots, I_C\} \).

Assuming that \( \{sx(k)\} \) and \( \{mx(k)\} \) denotes measurement sample time series and sample pattern time series, respectively. Then pattern class variable must satisfy the following transformation:

\[
mx(k) = T(sx(k)) \tag{7}
\]

\[
dx(k) = F(mx(k)) \tag{8}
\]

Where \( T(\cdot) \) and \( F(\cdot) \) denote feature extraction and classification, respectively, \( dx(k) \) is the pattern class variable.

Obviously, pattern class variable has two main characteristics:

(i) It is a variable over time.
(ii) It has the class attribute.

Pattern class variable is used to describe the variation of pattern class to which operating condition pattern belong over time in pattern moving space. And it together with pattern moving space forms the basis of the proposed modeling approach. The pattern class variable’s metric form is not unique. In this paper, interval-valued number is used as its metric form. That is, given at time instant \( t \), \( mx(k) \) is classified into pattern class \( i \), then let \( dx(k) = I_i = [I_i^-, I_i^+] \).

Thus, a pattern sample time series in Euclidean space is transformed into a pattern class variable time series in pattern moving space.

For example, if the pattern moving space consists of \( C \) pattern classes, then pattern class variable \( dx(k) \) will transfer between the \( C \) pattern classes.

Thus, while quantified pattern class which can be considered as a quantitative “calibration” in pattern moving space is used to re-quantify original operating condition pattern \( y(k) \) at time instant \( k \), the corresponding pattern class variable time series \( dx(k) \) is formed. At time instant \( k \), the value of \( dx(k) \) equals to the quantified value \( I_j \) of pattern class into which the operating condition pattern \( y(k) \) is classified.

D. Modeling of Complex dynamical systems based on pattern class variable

In this paper, as interval-valued numbers is used as pattern class variable’s metric form, so, in fact pattern class variable time series is an interval-valued number time series. In our previous work, Interval T-S fuzzy model is proposed in [15][16]. For modeling the moving of pattern class variable in pattern moving space, interval autoregression model (IAR) is defined in this paper.

Definition: Given an interval-valued time series \( X(k), \quad k = 1,2,\ldots,n \), where \( X(k) \) is an interval-valued number at time instant \( k \), \( X(k) = (X_c(k), X_r(k)) \), \( X_c(k) \) and \( X_r(k) \) are the center and radius of interval-valued number \( X(k) \), respectively, \( X_c(k) \in R, \quad X_r(k) \in R, \quad X_r(k) \geq 0 \). Then Interval Autoregression Model (IAR) is defined as follows:

\[
X(k) = \theta_0 + \theta_1 \cdot X_c(k-1) + \theta_2 \cdot X_c(k-2) + \cdots + \theta_m \cdot X_c(k-m) \tag{9}
\]

Where \( m \) is the order of the IAR model, \( \theta_0, \theta_1, \theta_2, \ldots, \theta_m \) are constant interval-valued parameters, \( \theta_i = (\theta_{ic}, \theta_{ir}), i = 0,1,2,\ldots,m \).

The dynamics of complex dynamical systems is characterized by the transition of pattern class variable \( dx(k) \) in pattern moving space. The forecasting model based on pattern class variable is as follows:

\[
dx(k) = F(dx(k)) = F(f(dx(k-1), dx(k-2)\cdots dx(k-m))) \tag{10}
\]
are the mean and standard deviation of real pattern class variable \( k_{dx} \), respectively. Fig.2 shows the pattern class variable time series \( k_{dx} \) characterized by intervals in pattern moving space are shown as follows:

Furthermore, \( d\dot{x}(k) \) can be expressed as:

\[
\begin{align*}
\dot{d} x(k) &= \dot{\theta}_c + \dot{\theta}_r x(k) \\
&= (\dot{\theta}_c, \dot{\theta}_r x(k))'
\end{align*}
\]  

Where \( \theta_c = (\theta_{c0}, \theta_{c1}, \ldots, \theta_{cm})' \), \( \theta_r = (\theta_{r0}, \theta_{r1}, \ldots, \theta_{rm})' \), the notation “’” denotes the transpose of a vector or a matrix.

The parameters in (12) are obtained by minimizing the following objective function \( J \) in (13) subject to the constraints \( \theta_{jr} \geq 0 \) as the radius of an interval is greater than or equal to zero, \( j = 0, 1, 2, \ldots, m \). In fact, this is a quadratic programming problem. Where \( d\dot{x}^L(k) \), \( d\dot{x}^U(k) \) are the lower and upper bound of the initial prediction output \( d\dot{x}(k) \), respectively, \( d\dot{x}^L(k) \), \( d\dot{x}^U(k) \) are the lower and upper bound of real pattern class variable \( dx(k) \) at time instant \( k \), respectively.

\[
Min_{\theta_{c0}, \theta_{r0}} J = \sum_{k=1}^{N} |d\dot{x}^L(k) - d\dot{x}^U(k)|^2 + \sum_{k=1}^{N} (\dot{\theta}_r |x(k)|)^2
\]

subject to \( \theta_{jr} \geq 0, j = 0, 1, 2, \ldots, m \) (13)

2) After obtaining the initial prediction output \( \hat{d}(k) \), the final prediction output \( \hat{d}(k) \) can be obtained by classifying the initial prediction output \( \hat{d}(k) \) into the pattern class which is nearest to the initial prediction output \( \hat{d}(k) \) by calculating the Hausdorff distance between the initial prediction output \( \hat{d}(k) \) and each pattern class represented by interval-valued number in pattern moving space. That is, if \( j = \arg\min_{i=1, 2, \ldots, C} D(\hat{d}(k), I_i) \), then the final prediction output \( \hat{d}(k) \) is equal to \( I_j \), where \( I_j = [L^j, U^j] \) is an interval, \( I^L \) and \( I^U \) are the minimum and maximum sample value among operating condition samples that belong to pattern class \( i \), respectively, \( i = 1, 2, \ldots, C, C \) is the number of pattern class in pattern moving space. \( D(\hat{d}(k), I_i) \) denotes the Hausdorff distance between interval-valued number \( \hat{d}(k) \) and \( I_i \).
in Figure 3, \( k = 1, 2, \cdots, 100 \). From Figure 3, we can see that the transition of pattern class variable from one pattern class to another in pattern moving space can also characterize change of normalized operating condition pattern \( Z(k) \) at large.

In this example, the first 500 samples are used to train the proposed model; the others are used to validate the model. The initial prediction model based on IAR is as follows:

\[
d\tilde{x}(k) = \theta_0 + \theta_1 d\tilde{x}_c(k-1) + \theta_2 d\tilde{x}_c(k-2) + \theta_3 d\tilde{x}_c(k-3) + \theta_4 d\tilde{x}_c(k-4) \tag{18}
\]

Where \( d\tilde{x}_c(k-1) \) is the centre of \( d\tilde{x}(k-1) \). Based on formula (13), the interval parameters in IAR model are obtained and shown in Table 2.

In Figure 4, the solid line and dotted line represent the upper bound and lower bound of initial prediction output \( d\tilde{x}(k) \) obtained by the IAR, respectively, and the dash-dot line and dashed line represent the upper bound and lower bound of the real pattern class variable time series \( dx(k) \) with time from 505 to 864, respectively. From Fig.4, we can see that the upper bound of the output of IAR is always greater than equal to the lower bound of the output of IAR for the testing samples. After the initial prediction output \( d\tilde{x}(k) \) is classified, the final outputs \( \hat{dx}(k) \) of the proposed model are obtained. In Figure 5, the solid line and dotted line represent the upper bound and lower bound of the final outputs of the proposed model, the dash-dot line and dashed line represent the upper bound and lower bound of the real pattern class variable time series \( dx(k) \) with time from 505 to 864. By comparing Figure 4 with Figure 5, we can see that the small difference between initial prediction output \( d\tilde{x}(k) \) and the corresponding real pattern class variable time series \( dx(k) \) at some time instant (for example \( k=508-515, 685-693, 662-669 \) etc.) is eliminated after \( d\tilde{x}(k) \) is classified.
Table I QUANTIFICATION OF PATTERN CLASSES

<table>
<thead>
<tr>
<th>( P_i )</th>
<th>( I_i = [I_i^L, I_i^U] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>([-4.4856, -4.0391])</td>
</tr>
<tr>
<td>2</td>
<td>([-3.8872, -2.6340])</td>
</tr>
<tr>
<td>3</td>
<td>([-2.5955, -1.9612])</td>
</tr>
<tr>
<td>4</td>
<td>([-1.9172, -1.4535])</td>
</tr>
<tr>
<td>5</td>
<td>([-1.4458, -1.0406])</td>
</tr>
<tr>
<td>6</td>
<td>([-1.0377, -0.7228])</td>
</tr>
<tr>
<td>7</td>
<td>([-0.7136, -0.4651])</td>
</tr>
<tr>
<td>8</td>
<td>([-0.4576, -0.2304])</td>
</tr>
<tr>
<td>9</td>
<td>([-0.2210, -0.0196])</td>
</tr>
<tr>
<td>10</td>
<td>([-0.0168, 0.2212])</td>
</tr>
<tr>
<td>11</td>
<td>([0.2330, 0.5212])</td>
</tr>
<tr>
<td>12</td>
<td>([0.5257, 0.8140])</td>
</tr>
<tr>
<td>13</td>
<td>([0.8200, 1.0430])</td>
</tr>
<tr>
<td>14</td>
<td>([1.0558, 1.2668])</td>
</tr>
<tr>
<td>15</td>
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</tr>
<tr>
<td>16</td>
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<td>17</td>
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<td>18</td>
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<td>19</td>
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<td>20</td>
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</tr>
<tr>
<td>21</td>
<td>([2.4494, 2.7607])</td>
</tr>
<tr>
<td>22</td>
<td>([2.4972, 2.7042])</td>
</tr>
</tbody>
</table>

Table II THE IDENTIFIED INTERVAL PARAMETERS IN IAR MODEL

| \( \theta_0 \) | (0.0029, 0.1074)         |
| \( \theta_1 \) | (1.0761, 0.0343)         |
| \( \theta_2 \) | (0.1776, 0.0000)         |
| \( \theta_3 \) | (-0.0446, 0.0000)        |
| \( \theta_4 \) | (-0.2267, 0.0000)        |

V. CONCLUSION

In this paper, in contrast to traditional modeling approach to characterizing dynamical systems in Euclidean space, the dynamics of complex dynamical systems is characterized by the movement of pattern class variable in pattern moving space. That is, we view the considered complex dynamical systems at the level of pattern class and capture only the dominant characteristics of the process. Each pattern class in pattern moving space can be considered as a “scale” of operating condition patterns belonging to the same pattern class are characterized (measured) by the same “scale” value (interval-valued number). Thus, operating condition time series in Euclidean space is transformed into pattern class variable time series in pattern moving space. The proposed interval autoregression model (IAR) and interval data classifier are used to model the transition of pattern class variable in pattern moving space. Experimental results are presented that indicate the validity and applicability of the proposed moving pattern-based forecasting model.

In future work, new clustering method used to constructing pattern moving space, the metric of multi-dimensional operating condition pattern, and the model order determination of the proposed model in pattern moving space need to be researched. In addition, fault detection and interval-valued time series forecasting based on the proposed model are also our future study field.

REFERENCES