Abstract—We propose a new position control scheme for vertical take-off and landing (VTOL) unmanned airborne vehicles (UAVs). Traditionally, the control schemes for this type of vehicle assume that the system attitude is accurately known (or measured). Unfortunately, there does not exist any sensor that directly measures the orientation of a rigid body. Instead, to obtain the orientation of the aircraft, separately designed attitude-estimation schemes relying, for instance, on an inertial measurement unit (IMU) must be employed. Consequently, one drawback of this common practice, on top of the possible inaccuracies in recovering the systems attitude, is mainly related to the difficulty of proving the stability of the overall closed-loop system (observer-controller). Motivated by this problem, we propose a new position control scheme that does not require the recovery of the system’s attitude. Instead, we rely on a direct use of the vector measurements provided by the IMU. Our approach can efficiently handle large linear accelerations, which is not the case in traditional controllers relying on IMU-based attitude observers that assume that the accelerometer provides a measurement of the gravity vector in the body-attached frame.

I. INTRODUCTION

Vertical take-off and landing (VTOL) unmanned airborne vehicles (UAVs) are more commonly being sought to perform a number of tasks, including surveillance, structure inspection, and a variety of other applications where human presence is either difficult or dangerous to achieve. One of the limitations associated with the operation of these aircraft is attributed to the requirement of skilled and trained operators to pilot the system. This limits the usefulness of these systems to persons without the required training. Fortunately, this problem has motivated several groups in the research community to develop flight-control systems for these types of aircraft which allow the aircraft to operate more autonomously, thereby reducing the required skill of the pilot or operator. By relieving the operator of some of the pilot-related duties, more attention can be placed on viewing a camera or other sensors to address the primary mission objective, instead of primarily concentrating on flying the aircraft. As a result of these efforts the research community has seen substantial and interesting advancements in the design of position controllers, for example see [1], [2], [3], [4], [5], [6], [7], [8].

The existing controllers are designed to provide the necessary control input (in terms of the system angular velocity or control-torque that is applied to the rotational dynamics of the system) based upon measurements obtained using a common-sensor set. These sensors usually include a global positioning system (GPS) (to obtain the system position and velocity), and a sensor to provide the attitude of the aircraft (with respect to a fixed inertial frame). However, there does not exist any inertial sensor (to our knowledge) that directly measures the orientation of a rigid body with respect to an inertial frame. In fact, the commercially available orientation sensors employ Kalman filtering or some other attitude observer in order to provide the orientation of the aircraft. Therefore, there can be un-modeled dynamics or other errors associated with the filter (or attitude observer) that may not be accounted for by the proposed control scheme.

Due to the unavailability of such a sensor which directly measures orientation, the research community has also been motivated to develop a number of attitude-observers. Typically, the existing attitude-observers utilize a set of vectors (usually assumed to be known in the inertial frame) which are measured in the body-fixed frame. Some examples of attitude observers which use vector measurements can be found in [9], [10], [11], [12], [13] and [14]. However, the main limitation of these types of observers is due to the limited number of sensors that can measure the body-referenced coordinates of a vector which is known in the inertial frame. Typically, the two sensors used most commonly in this capacity are the accelerometer and magnetometer, which due to developments in technologies such as Integrated Micro-Electrical-Mechanical systems (IMEMS), are small, inexpensive, and widely available. The magnetometer is used to measure the known inertially-referenced ambient magnetic field in the body-fixed frame, where the accelerometer is typically used to measure the body-referenced gravity vector. Herein lies a problem since the accelerometer also measures forces due to the acceleration of the device which are likely significant due to aircraft motion, especially in the context of position control.

To address the problem associated with the accelerometers dependance on linear accelerations, a different type of attitude observer has been previously proposed which uses a GPS in addition to the vector measurements from the accelerometer and magnetometer. Examples of this type of observers can be found in [15], [16], and [17]). These so-called velocity-aided attitude observers, alternatively use the accelerometer to measure the apparent acceleration of the system, rather than assuming only gravity is measured, resulting in significant improvements in performance when the system experiences relatively large linear accelerations.

A second challenge associated with the position control
The rotation matrix \( R(\eta, q) \) corresponding to the unit quaternion \( Q = (\eta, q) \) can be determined using the so-called Rodrigues formula

\[
R(\eta, q) = I_{3\times3} + 2S(q)^2 - 2\eta S(q),
\]

where \( S(\cdot) \) is the skew-symmetric matrix

\[
S(u) = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix},
\]

where \( u = [u_1, u_2, u_3]^T \). The set \( \mathbb{Q} \) requires the use of some unique mathematical operations which we will define now. Given \( Q, P \in \mathbb{Q} \) where \( P = (p_0, p) \) the quaternion product is defined by

\[
Q \odot P = (p_0\eta - q^T p, \quad \eta p + p_0q + S(q)p).
\]

Note that the quaternion product of two unit-quaternions preserves the properties of the unit quaternion, i.e \( Q \odot P \in \mathbb{Q} \). The unit-quaternion inverse is given by \( Q^{-1} = (\eta, -q) \) which has the property \( Q \odot Q^{-1} = Q^{-1} \odot Q = (1, 0) \) and \( Q \odot (1, 0) = (1, 0) \odot Q = Q \), where \((1, 0)\) is known as the identity quaternion.

### B. Attitude Dynamics

Let \( \omega \) denote the body-referenced angular velocity of the frame \( B \) wrt \( I \) (expressed in \( B \)). Using the body-referenced angular velocity an expression for the time-derivative of the quaternion \( Q \) is given by

\[
\dot{Q} = \frac{1}{2}Q \odot \begin{bmatrix} 0 \\ \omega \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \eta I_{3\times3} + S(q) \end{bmatrix} \omega.
\]

Similarly, an expression for the time-derivative of the rotation matrix \( \dot{R}(\eta, q) \) is given by \( \dot{R}(\eta, q) = -S(\omega)R(\eta, q) \).

### C. Bounded Functions

Consider a bounded, differentiable function, denoted as \( h(\cdot) : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \), where we also denote \( \phi(u) := \frac{\partial}{\partial u} h(u) \), which satisfies the following properties:

\[
\begin{align*}
& u^T h(u) > 0 \quad \forall u \in \mathbb{R}^3, \|u\| \in (0, \infty) \\
& 0 \leq \|h(u)\| < 1 \quad \forall u \in \mathbb{R}^3, \|u\| \in [0, \infty). \\
& 0 < \|\phi(u)\| \leq 1
\end{align*}
\]

Throughout the paper we make use of one example of this type of function which is given by \( h(u) = (1 + u^T u)^{-1/2} u \) from which one can derive the expression for \( \phi(u) \) to be

\[
\phi(u) = (1 + u^T u)^{-3/2} \left( I_{3\times3} - S(u)^2 \right).
\]
II-B we consider the following well-known model for a VTOL UAV ([5], [7], [8])

\[ \dot{p} = v, \quad (9) \]
\[ \dot{v} = \mu + \delta(v), \quad \mu = \ldots, \quad (21) \]

The extracted attitude \( Q_d \) has the time derivative

\[ \dot{Q} = \frac{1}{2} \left[ -q^T \eta_d I_{3 \times 3} + S(q) \right] \omega, \quad (11) \]

where \( u_t = T/m_b, \) \( T \) is the system thrust, \( m_b \) is the system mass, \( e_3 = \text{col}[0,0,1], \) \( g \) is the gravitational acceleration, and \( \delta(v) \) is a disturbance which is dependant on aerodynamic drag forces. The control input of the system is defined as \( u = [u_t, \omega]^T. \) The system output is defined as \( y = [p,v,b_1,b_2]^T \) where \( b_2 \) is the signal obtained using an accelerometer, \( b_1 = Rr_1 \) is a signal obtained using a magnetometer, and \( r_1 \) is the magnetic field of the surrounding environment (assumed constant). Note that the system attitude \( Q, R \) is not assumed to be a known output of the system. The accelerometer model is given by

\[ b_2 = R(\dot{v} - ge_3) = -u_t e_3 + R \delta = R r_2, \quad (12) \]

where \( r_2 \) is known as the system \emph{apparent acceleration} such that \( \dot{v} = ge_3 + r_2, \) or equivalently

\[ r_2 = -u_t R^T e_3 + \delta. \quad (13) \]

III. PROBLEM FORMULATION

Let \( p_r \) denote a desired reference position, which is assumed to be constant (or slowly-varying), and let \( e_p = p - p_r. \) Our main objective is to develop a control law for the system inputs \( u_t \) and \( \omega, \) using the available system outputs \( y = [p,v,b_1,b_2], \) such that the system states \( (e_p, v) \) are bounded and \( \lim_{t \to \infty} (e_p, v) = 0. \) To develop the control laws we first make a number of simplifying assumptions.

\textbf{Assumption 1 (Aerodynamic Forces)}: In light of the fact that the disturbance force \( \delta \) is due to aerodynamic forces exerted on the vehicle we make the following simplifying assumptions:

\begin{itemize}
  \item[(a)] The aerodynamic disturbance \( \delta \) is dissipative with respect to the system translational kinetic energy and satisfies \( \delta^T \delta \leq 0. \)
  \item[(b)] The aerodynamic disturbance force \( \delta \) is only dependant on the system translational velocity, and there exists a positive constant \( c_1 \) such that \( \delta \) is bounded by \( \|\delta\| \leq c_1 \|v\|^2. \)
  \item[(c)] There exists positive constants \( c_2 \) and \( c_3 \) such that the time-derivative of the aerodynamic disturbance force is bounded by \( \|\dot{\delta}\| < c_2 + c_3 \|v\|^3. \)
\end{itemize}

Assumption 1(a) and 1(b) can be realized when the system is operating in an environment where the exogenous airflow is negligible (no wind). Assumption 1(c) can be satisfied when the system geometry is sufficiently symmetrical such that the system aerodynamic forces do not significantly depend on the system orientation. Although this assumption may be reasonable for certain VTOL type aircraft, for example the ducted-fan, this assumption may not be the case with certain systems, for example fixed wing aircraft, where the system aerodynamics depend largely on the orientation of the vehicle.

\textbf{Assumption 2}: Given two positive constants \( \gamma_1, \gamma_2, \) there exists a positive constant \( c_w(\gamma_1, \gamma_2) \) such that the matrix

\[ W = -\gamma_1 S(r_1)^2 - \gamma_2 S(r_2)^2, \quad (14) \]

is bounded by \( c_w \leq \lambda_{\min}(W), \) where \( \lambda_{\min}(W) \) denotes the minimum eigenvalue of \( W. \)

Assumption 2 is satisfied provided that the value for \( r_2 \) is non-vanishing and is non-collinear to the magnetic field vector \( r_1. \) Note that in the case where \( r_2 = 0, \dot{v} = ge_3 \) which is unlikely under normal operating conditions.

Now that we have established the required assumptions, let us consider the model for the system acceleration from (10): due to the underactuated nature of this system, the translational acceleration is driven by the system thrust and attitude \( \mu(u_t, R). \) That is, if \( \mu \) were a control input, setting \( \mu = -ge_p - k_v v \) would satisfy the objectives (since \( v^T \delta(v) \leq 0. \) However, since \( \mu \) is a function of the system state, we define the \( \mu_d \in \mathbb{R}^3 \) as the desired acceleration, and introduce the new error signal

\[ e_d = \mu - \mu_d. \]

\[ (15) \]

Subsequently, a new objective is to force \( e_d \to 0 \) in order to obtain the desired translational dynamics. Since the signal \( \mu \) is dependant on the system thrust and attitude, based upon the value of the desired acceleration \( \mu_d \) we wish to obtain a suitable desired attitude, denoted as \( Q_d = (\eta_d, q_d) \in \mathbb{Q}, \) and system thrust \( u_t, \) such that the following equation is satisfied

\[ \mu_d = ge_3 - u_t R_d^T e_3, \quad (16) \]

where \( R_d = R(\eta_d, q_d), \) is the rotation matrix corresponding to the unit-quaternion \( Q_d, \) as defined by (3). An extraction method which satisfies these requirements has been previously given in [8]. For the sake of clarity, this attitude and thrust extraction algorithm is restated in the following section.

\textbf{A. Desired Attitude and Thrust Extraction}

Given an arbitrary vector \( \mu_d \in \mathbb{R}^3 \) we wish to find the value of attitude \( R_d \in SO(3) \) and system thrust \( u_t \in \mathbb{R} \) that satisfies

\[ ge_3 - u_t R_d^T e_3 = \mu_d, \quad (17) \]

A solution to this problem, which has been proposed in [8], is provided as follows: Given \( \mu_d \) where \( \mu_d \notin L, \)

\[ L = \{ \mu_d \in \mathbb{R}^3 : \mu_d = \text{col}[0,0,\mu_{d3}]; \mu_{d3} \in [g, \infty) \}, \quad (18) \]

then a value of the thrust \( u_t \) and attitude \( Q_d = (\eta_d, q_d) \) which satisfies (17) is given by

\[ u_t = \|\mu_d - ge_3\|, \quad (19) \]
\[ \eta_d = \left( \frac{1}{2} \left( 1 + \frac{g - e_3^T \mu_d}{\|\mu_d - ge_3\|} \right) \right)^{1/2}, \quad (20) \]
\[ q_d = \frac{1}{2\|\mu_d - ge_3\|\eta_d S(\mu_d)e_3}. \quad (21) \]

The extracted attitude \( Q_d \) has the time derivative

\[ \dot{Q}_d = \frac{1}{2} \left[ \eta_d I_{3 \times 3} + S(q_d) \right] \omega, \quad (22) \]
where the desired angular velocity \( \omega_d \) is given by
\[
\omega_d = M(\mu_d)_\mu_d,
\]
(23)
\[
M(\mu_d) = \frac{1}{4\eta^2} du^4_t (-4S(\mu_d)e_3e^T_3 + 4\eta^2 u_t S(e_3) + 2S(\mu_d)
-2e^2_3 u_t S(e_3)) S(\mu_d - ge_3)^2.
\]
(24)

It is worth noting that there are an infinite number of solutions for the desired attitude which achieve the desired system acceleration. The above attitude extraction offers a unique solution since it effectively includes an extra constraint by fixing the desired yaw angle.

**B. Attitude Error**

Let \( Q_e = (\eta_e, q_e) \in \mathbb{Q} \) and \( R_e = R(\eta_e, q_e) \in SO(3) \) denote the attitude error which are defined by \( Q_e = Q \circ Q_d^{-1} \), and \( R_e = R_d^TR \), where \( Q_d \) is the unit quaternion obtained using (20) and (21). In light of the derivatives \( Q \) and \( Q_d \), as defined by (11) and (22), respectively, the time derivative of the attitude error is found to be
\[
\dot{Q}_e = \frac{1}{2} Q_e \circ \begin{bmatrix} 0 & \omega_e \end{bmatrix}, \quad \dot{R}_e = -S(\omega_e)R_e, \quad \omega_e = R_d^T(\omega - \omega_d),
\]
(25)
where \( \omega_d \) is the desired angular velocity as defined by (23).

### IV. POSITION CONTROLLER

As previously discussed, the control scheme is based upon a value of the desired attitude, in terms of \( R_d \) or \( Q_d \), which is obtained using the value of the desired translational acceleration \( \mu_d \). To avoid the singularity in the attitude extraction algorithm, defined by (18), we must ensure that the expression for \( \mu_d \) is bounded a priori. This can be accomplished using the bounded function \( h(\cdot) \) defined in section (II-C). We choose the following expression for the desired translational acceleration:
\[
\mu_d = -k_p h(e_p) - k_v h(v),
\]
(26)
from which we obtain the required system thrust \( u_t \) and desired attitude \( Q_d = (\eta_d, q_d) \) from (19)-(21), and the rotation matrix \( R_d \) using (3). To avoid the singularity in the attitude extraction (to ensure a solution for \( Q_d \) and \( R_d \) exists) we choose positive gains \( k_p \) and \( k_v \) which satisfy the following requirement
\[
k_p + k_v < g,
\]
(27)
which ensures the singularity can never be reached. Due to this choice of gains, from (19) one can deduce that the thrust is non-vanishing and bounded. That is there exists positive constants \( c_t \) and \( \tilde{c}_t \) such that
\[
0 < c_t \leq u_t \leq \tilde{c}_t, \quad \tilde{c}_t = g - k_p - k_v, \quad c_t = g + k_p + k_v.
\]
(28)
Furthermore, as a consequence of the lower bound for the thrust, in [8] the authors find an upper bound for the matrix \( M(\mu_d) \), defined by (24), which is given by
\[
\| M(\mu_d) \| \leq \sqrt{2/c_t}.
\]
(29)

In light of the choice for \( \mu_d \), the derivatives of the position error and velocity are now given by
\[
\dot{e}_p = v, \quad \dot{v} = -k_p h(e_p) - k_v h(v) + e_{\mu} + \delta.
\]
(30)
(31)
To obtain the desired angular velocity \( \omega_d \), using (8), (23), (30) and (31) we first calculate the derivative of \( \mu_d \) to be
\[
\dot{\mu}_d = f_{\mu_d} - k_v \phi(v) e_\mu - k_v \phi(v) e_\delta, \quad f_{\mu_d} = -k_p \phi(e_p) + k_v \phi(v) (k_p h(e_p) + k_v h(v)).
\]
(32)
(33)
To define the control law, we are required to use vector measurements (instead of the attitude \( Q \) or \( R \)), however the inertial referenced vector \( r_2 \) which corresponds to the accelerometer measurement is not available. To obviate the requirement of \( r_2 \), we define the error signal \( \tilde{v} = v - \hat{v} \) where \( \hat{v} \in \mathbb{R}^3 \) is an adaptive state which is defined such that \( \tilde{v} \) is related to \( r_2 \) in some manner. To this end, we now propose the following control law
\[
\omega = M(\mu_d) (f_{\mu_d} - k_v \phi(v) R_d b_2 + u_t e_3) + \beta, \quad \beta = \gamma_1 S(R_d r_1) b_1 + \gamma_2 k_1 S(R_d (v - \hat{v})) b_2,
\]
(34)
\[
\dot{\beta} = g e_3 + R_d^T b_2 + k_1 (v - \dot{v}) + R_d^T S(b_2) \beta/k_1, \quad \beta > 0, \quad M(\mu_d) \text{ is the function defined by (24),}
\]
(35)
(36)
and \( \phi(\cdot) \) is the bounded function defined by (8). The role of \( \beta \) in the feedback structure is to use the vector measurements in order to force the system attitude to the desired value (correction term which forces \( Q \rightarrow Q_d \)). The remaining term in the expression for \( \omega \) is related to the desired angular velocity \( \omega_d \) (tracking term).

In light of these control laws we propose the following theorem:

**Theorem 1:** Consider the system given by (9)-(11), where we apply the control laws \( u_t \) and \( \omega \) as defined by (19) and (34) where the gain restriction (27) is satisfied. Let Assumptions 1 and 2 be satisfied. Then the system thrust \( u_t \) is bounded and non-vanishing, and for all initial conditions \( \eta_e(t_0) \neq 0 \) there exists gains \( \gamma_1, \gamma_2, k_1 > 0 \) such that for \( \gamma_1 > \gamma_2, \gamma_2 > \gamma_1, \gamma_2 > \gamma_1 > \gamma_2 \) the system states \((e_p, v)\) are bounded and \lim_{t \rightarrow -\infty} (e_p, v) = 0.

**Proof:** First, let us define the following error function
\[
\psi = k_1 \tilde{v} - (I - R_e) r_2,
\]
where \( \tilde{v} = v - \hat{v} \) and \( I = I_{3x3} \) is the identity matrix. Using the error function for \( \psi \), in addition to the fact that \( S(R_d u) = R_d S(u) R_d^T \) and \( R_d^T R = R_e \), the expression for the function \( \beta \), defined by (35), can now be written as
\[
\beta = \gamma_1 R_d S(r_1) R_e r_1 + \gamma_2 R_d S(r_2) R_e r_2 + \gamma_2 R_d S(\psi) R_e r_2. \quad (37)
\]
At this point in the proof, we wish to study the dynamics of the attitude error (in terms of the quaternion scalar \( \eta_e \)) as a result of the proposed control law, which requires we first calculate the expression for the desired angular velocity \( \omega_d \). Due to the control law \( \mu_d \) from (26), the derivative \( \mu_d \) from (32), and using (23) we find the expression for the desired angular velocity is given by \( \omega_d = M(\mu_d) (f_{\mu_d} - k_v \phi(v) e_\mu - k_v \phi(v) e_\delta) \), which we use, along with (25), (37), the control law \( \omega \) from...
Given these bounds, we now apply Young’s inequality to a number of the undesired terms in the expression for $\dot{V}$:

$$
\gamma v^T f_1(u) q_e \leq \frac{\gamma c_1}{2} v^T h(v) + \frac{2\sqrt{\gamma c_2^2}}{\epsilon_1} \sqrt{\gamma + \sigma(t)} q_e^T q_e,
$$

$$
\gamma k_v k_v v^T R_d^T S(b_2) M(\mu_d) \phi(v) (f_1(u) + f_2(\delta)) q_e
\leq \frac{\gamma c_3 k_v c_4}{2} \gamma v^T \psi \psi + \frac{4k_v k_v}{\epsilon_2} \gamma^2 \gamma_{L_2}(\gamma_{\epsilon_1} + c_1 \sigma(t)^2)^2 q_e^T q_e,
$$

where the norm of the matrix $M(\mu_d)$ is given by (29). To determine the bound of the term involving the derivative of $r_2$, we differentiate the expression for $r_2$ given by (13), which we have omitted due to space constraints. However, based upon the result for the derivative, in addition to the bound of $b_2$ and $\delta$ from (46) and (44), respectively, we find that there exists positive constants $d_{1,2,3,4,5}$, such that $r_2 \leq d_1 + d_2 ||v|| + d_3 ||v||^2 + d_4 ||v||^3 + d_5 ||v||^4$.

For the sake of simplicity, from this result we further conclude that there exists positive constants $c_3$ and $c_4$, such that $r_2 \leq c_3 + c_4 \sigma(t)^4$. Note that the constants $d_4$ and therefore $c_3$ and $c_4$, depend on the gains $k_p, k_v, \gamma_1$ and $\gamma_2$, but they do not depend on the gain $k_1$, which will be useful later in the proof.

As a result of this analysis, we again use Young’s inequality to establish the following bounds:

$$
\gamma k_v k_v v^T f_2(r_2) q_e \leq \frac{\gamma k_v c_1}{2} \gamma v^T \psi \psi + \frac{2k_v c_4}{\epsilon_3} (c_3 + c_4 \sigma(t)^4)^2 q_e^T q_e,
$$

$$
\gamma k_v k_v v^T q_e^T \psi + \frac{\gamma c_3 c_4 \sigma(t)}{2} \eta_0 \eta_e q_e^T q_e.
$$

Finally, we also find the bound of the following function

$$
\gamma k_v k_v v^T q_e^T R_d^T M(\mu_d) \phi(v) (f_1(u) + f_2(\delta)) q_e
\leq \frac{2\sqrt{\gamma c_2^2}}{\epsilon_2} \eta_0 v^T \psi + \frac{\gamma c_3 c_4 \sigma(t)}{2} \eta_0 q_e^T q_e.
$$

Recall from Assumption 2 the norm of the matrix $W$ has a lower bound which is denoted as $c_w$. Therefore, in light of these results we find the expression $\dot{V}$ is bounded by

$$
\dot{V}(t) \leq -k_v v^T h(v) (v - c_1/2)\eta_0^2 q_e^T q_e \left(\frac{\alpha_1(t)}{\epsilon_1} + \frac{\alpha_2(t)}{\epsilon_2} + \frac{\alpha_3(t)}{\epsilon_3}\right) - \frac{\alpha_4(t)}{\epsilon_4 |\eta_e|}
-\frac{2\sqrt{\gamma c_2^2}}{\epsilon_2} \eta_0 v^T \psi + \frac{\gamma c_3 c_4 \sigma(t)}{2} \eta_0 q_e^T q_e.
$$

Note that when $\eta_e(t) = 0$ we cannot guarantee stability using (52) since in this case $\dot{V}$ could potentially be positive. Consequently, we must show that $\eta_e(t) \neq 0$ for all $t > t_0$. Due to this requirement we are forced to exclude the initial condition $\eta_e(t_0) = 0$. To show that $\eta_e(t)$ is never zero, we first introduce the positive constant $\rho$ which is the desired minimum bound for $|\eta_e(t)|$. Therefore, $\rho$ must be chosen to satisfy...
0 < ρ < |η_e(t_0)|. Subsequently, based upon the definition of the Lyapunov function candidate (41), we choose the gain γ as follows:

\[ γ = \frac{k_p \left( \sqrt{1 + \|e_p(t_0)\|^2} - 1 \right)}{\sqrt{1 + \|v(t_0)\|^2} + \frac{k_v}{2} \|\psi(t_0)\|^2 + \xi}, \]

where ξ > 0, and \( \dot{\gamma} \) is chosen to satisfy

\[ 0 < \dot{\gamma} < \gamma \left( η_e(t_0)^2 - ρ^2 \right). \]

The remaining gains are chosen as follows: Choose \( k_p, k_v > 0 \) such that (27) is satisfied, and choose \( 0 < \epsilon_1 < 2k_v \). Recall from (14) that the gain \( c_w > 0 \) can be increased using the gains \( γ_1 \) and \( γ_2 \). Therefore, there exists gains \( γ_1, γ_2, \) and \( ε_i, i = 2, 3, 4 \), such that for all \( γ_1 > γ_1, γ_2 > γ_2, \) and \( ε_i > ε_i \) the following inequality is satisfied

\[ 2c_w > \frac{1}{ρ^2} \left( \frac{α_1(t_0)}{ε_1} + \frac{α_2(t_0)}{ε_2} + \frac{α_3(t_0)}{ε_3} + \frac{α_4(t_0)}{ε_4} \right) + \frac{2 \sqrt{2} k_v (γ_1 c_1 + c_3 σ(t_0)^2)}{ρ^2} \]

where we note that the functions \( σ(t) \) and \( α_i(t) \) through \( α_4(t) \) are non-increasing if \( V \leq 0 \). Finally, we choose the gain \( k_1 \) to satisfy \( k_1 > k_1 := k_1 > \epsilon_2 k_v + ε_3 / 2 + ε_4 γ_2 σ_0 (2γ k_v) \). Due to these choices for the gains we conclude that \( V(t_0) \leq 0 \), and a sufficient condition for \( V(t) \leq 0 \) is \( |η_e(t)| ≥ ρ \). We will now show that indeed \( ρ ≤ |η_e(t)| \) for all \( t > t_0 \). Suppose that there exists a time \( t_1 \) such that for all \( t_0 ≤ t < t_1, |η_e(t)| ≥ ρ \) and \( |η_e(t_1)| < ρ \) when \( t = t_1 \). At the time \( t_1 \) from (41), it is clear that \( V(t_1) ≥ γ \left( 1 - η_e(t_1)^2 \right) > γ \left( 1 - ρ^2 \right) \). However, due to the choice of the gain \( γ \) and \( \dot{γ} \), given by (54) and (55), respectively, the value of the Lyapunov function candidate at the initial time \( t_0 \) must satisfy \( V(t_0) < γ + γ \left( 1 - η_e(t_0)^2 \right) < γ \left( 1 - ρ^2 \right) \) and therefore \( V(t_1) > V(t_0) \). This is a contradiction since \( V(t) ≤ 0 \) for all \( t_0 ≤ t < t_1 \). Therefore, we conclude that \( |η_e(t)| ≥ ρ \) and \( V(t) ≤ 0 \) for all \( t > t_0 \), and the states \( (v, ψ) \) are bounded. Therefore the expressions for \( ̇v, ̇ψ, \) \( ̇η_e, \) and \( ̇V \) are bounded. Barbalat’s Lemma therefore implies that \( \lim_{t→∞} (v, ψ, q_e) = 0 \). Since \( ψ \to 0 \) and \( q_e \to 0 \), this further implies that \( ̇v \to 0 \). Furthermore, since \( \lim_{t→∞} ̇v = 0 \), and \( \lim_{t→∞} δ = 0 \), it follows from the expression of the velocity dynamics \( ̇v = -k_p h(e_p) - k_v h(ψ) - δ = 0 \), then \( \lim_{t→∞} e_p = 0 \) which satisfies the control objectives.

V. CONCLUSION

We proposed a new position controller for VTOL-UAVs which uses low cost sensors, namely an accelerometer and magnetometer, in addition to a GPS. The main advantage of this controller is that it does not require direct measurement of the system attitude, nor does it require the use of an attitude observer. Alternatively, in this control scheme we use the accelerometer and magnetometer signals directly in the position controller (and not for attitude estimation). Furthermore, the accelerometer is now used to obtain the system apparent acceleration, instead of measuring the gravity vector in the body-fixed frame (which would normally require the system to be non-accelerating), which offers superior performance when the system is subjected to linear accelerations. We have shown that, through an appropriate choice of the control gains, the system position is guaranteed to converge to the desired position for almost all initial conditions.

REFERENCES