Comparison of Two Nonlinear Model Predictive Control Methods and Implementation on a Laboratory Three Tank System

A. Bamimore, O. Taiwo, and R. King

Abstract—Almost all industrial processes exhibit nonlinear dynamics, however most model predictive control (MPC) applications are based on linear models. Linear models do not always give a sufficiently adequate representation of the system and therefore Nonlinear Model Predictive Control (NMPC) techniques have to be used. In this article, two techniques of NMPC, namely successive linearization nonlinear model predictive control (SLNMPC) and wiener nonlinear model predictive control (WNMPC) are applied to nonlinear process systems. The major advantage of the two methods being that the NMPC problem is reduced to a linear model predictive control (LMPC) problem at each time step which thereafter allows the optimization problem to be solved using quadratic programming (QP) techniques. Another advantage of these methods is the reduced computational time in calculating the control effort which makes them suitable for online implementation.

Both simulation and experimental results show the superiority of the SLNMPC over WNMPC in handling process nonlinearity. The work also shows the favourable performance of the NMPC over LMPC, as expected.

I. INTRODUCTION

MODEL Predictive Control (MPC) refers to a class of computer control algorithms in which a dynamic model of process is used to predict and optimize its performance. At each sample time a predictive controller takes measurement of the system output (or state if available), uses the internal model to predict the behaviour of the system over a prediction horizon and then computes a finite horizon control sequence that optimizes some open-loop performance objective while making sure that no constraints are violated. This control sequence is implemented until the next measurement becomes available. Then the optimisation problem is solved again.

Linear model predictive control (LMPC) is well established industry standard for controlling constrained multivariable processes Garcia et al. [1989]. In LMPC, the plant behaviour is described by linear dynamic models however most chemical processes are highly nonlinear hence the inadequacy of using linear models to represent them. This limitation of LMPC has brought about the development of NMPC in which a more accurate nonlinear model is used for prediction and optimization. This however does not come without its own problems. While in LMPC a convex optimisation problem is solved at each sample time, in NMPC the problem becomes non-convex in which an optimal solution cannot be guaranteed. In addition to this, NMPC optimisation problem tends to become too large to be solved online. To reduce the computational complexity, it has been proposed that the problem can be reduced to either linear program or quadratic problem.

Inspite of the above, there are systems where nonlinear effects are significant enough to justify the use of nonlinear model predictive control (NMPC). These include at least two broad categories of applications:

1. Regulatory control problems, where the process is highly nonlinear and subject to large frequent disturbances.
2. Servo control problems, where the operating points change frequently and span a sufficiently wide range of nonlinear process dynamics.

To address this problem, quite a number of methods have been proposed by researchers for representing the internal model used in NMPC for predictive controller design, some mechanistic and some empirical. (Henson[1998], Kouvaritakis and Cannon [2001], Rossiter [2003], and Allgower et al. [2009]). Among these are Hammerstein, Wiener, Volterra, artificial neural network and successively linearized models. Camacho and Bordons [2004] give a review of these methods. In this work, NMPC by successive linearization and by the use of Wiener model are explored for controlling nonlinear process systems. Both techniques have been known to be computational less expensive because of the problem of having to solve a quadratic program at each sampling period.

In SLNMPC, the model used for output prediction is obtained by relinearizing the nonlinear model at every sampling instant and at the current operating point while the original nonlinear model can be used to compute the effect of past input moves. This reduces optimization problem to be solved at every sampling instant to a quadratic program. In WNMPC, since wiener model allows a nonlinear plant to be represented by a linear dynamic part and nonlinear static part, the problem is transformed to a linear one by employing the linear dynamic model for predictive controller design and using the inverse of the nonlinear part to compensate for the nonlinear effect.
II. SUCCESSIVE LINEARIZATION NONLINEAR MODEL PREDICTIVE CONTROL (SLNMPC)

Model for a nonlinear process
It is a common practice to model a nonlinear system using (Richer and Lee [1995]):
\[ \dot{x} = f(x, u, d) + w^x \]
\[ d = w^d \]
\[ y = g(x, u, d) + v \]
Where \( x \) is the state vector, \( y \) is the output vector, \( u \) is the vector of manipulated variables, and \( w^x, w^d, \) and \( v \) are zero-mean, white noise with specified covariances. The \( w^x \) vector represents short-term disturbances having zero mean; \( d \) represents sustained disturbances – integrated white noise.

Estimation of state
By linearization and discretization of equations (1) – (3) for a sampling period of \( t_k \) time units, an optimal estimate predictions of state and output can be made using the Kalman Filter. Accordingly, let \( x_{k-1|k-1}, y_{k-1|k-1}, \) and \( d_{k-1|k-1} \) represents the estimates of the state, the output and the disturbance at time \( k - 1 \) given information up to \( k - 1 \), then
\[ \begin{bmatrix} x_k^* \\ d_k^* \\ j \geq 1 \end{bmatrix} = A x_{k-1|k-1} + B u_{k-1|k-1} + L_k (y_k - \hat{y}_k^{k|k-1}) \]
where \( y_k \) is the measured output, \( \hat{y}_k^{k|k-1} \) is estimated output, and \( L_k \) is Kalman Filter gain matrix. It is assumed to be linear time-invariant in this work as this does not introduce serious error in state estimation.

Linearization for prediction/control
With the availability of the estimates of \( x_k^* \) and \( d_k^* \) at time \( k \) from equation (4) and the current inputs being \( u_{k-1} \), the problem is to compute \( u_k^* \), which will be sent to the plant (to be implemented from \( t_k \) to \( t_{k+1} \)). Also, with the expectations that \( w^x \) will be zero and that \( d \) will be constant:
\[ d_{k+j|k} = d_k^* \]
(5)
Equations (1) and (2) can be linearized with respect to \( x \) and \( u \) to obtain:
\[ \dot{x} = A_k (x - x_k^*) + B_k (u - u_k^*) \]
\[ y = g_k (x - x_k^*) + d_k (u - u_k^*) \]
where
\[ A_k = \frac{\partial f}{\partial x} |_{x_k^*,u_k^*,d_k^*} \]
\[ B_k = \frac{\partial f}{\partial u} |_{x_k^*,u_k^*,d_k^*} \]
\[ C_k = \frac{\partial g}{\partial x} |_{x_k^*,u_k^*,d_k^*} \]
are matrices of appropriate sizes.

The next step is to discretize the linearized model. However, a complication arises in that the reference point for linearization is the current state, i.e. an unsteady-state. A convenient way of dealing with this is to include the initial condition
\[ f_{ok} = f(x_k^*, u_{k-1}, d_k^*) \]
as a column of the \( B \) matrix corresponding to a constant input. We can then write equation (6) as
\[ \dot{x} = A_k (x - x_k^*) + [B_k \ f_{ok}] [u - u_k^*] \]
(9a)
or \( \dot{x}^* = A_k x^* + [B_k \ f_{ok}] u^* \)
(9b)
where \( x^* = x - x_k^* \), \( u^* = u - u_k^* \) are deviation variables
We then discretize equations (9b) and (7) to give:
\[ x_{k+j+1}^* = \Phi_k x_k^* + \Gamma_k u_{k+j}^* + \Gamma'_k \]
(10)
\[ y_{k+j}^* = C_k x_{k+j}^* \]
(11)
where for \( j \geq 0 \):
\[ x_{k+j}^* = x_k^* + x_{k+j|k} - x_{k|k} \]
\[ y_{k+j}^* = y_k^* - g(x_{k|k}, u_{k-1}, d_{k|k}) \]
\[ u_{k+j}^* = u_k^* + u_{k+j|k} - u_{k-1} \]
Where \( \Phi_k \) and \( \Gamma_k \) are state matrices obtained from \( A_k, B_k \) and the sampling time, \( t_k \).

Linear prediction of future outputs
The above equations are used to develop a linear prediction of future outputs for onward computation of control actions as is found in the usual MPC formulations (Richer and Lee [1995]).
For a ʻprediction horizonʻ of \( P \) sampling periods \((P \geq 1)\) we obtain:
\[ Y_{k+1|k} = Y^o_{k+1|k} + S^u \Delta U_k \]
(12)
where
\[ Y_{k+1|k} = \begin{bmatrix} y_{k+1|k} \\ y_{k+1|k}^- \end{bmatrix}, \Delta U_k = \begin{bmatrix} \Delta u_k = u_k - u_{k-1} \\ \Delta u_{k+1|k} = u_{k+1|k} - u_k^* \end{bmatrix} \]
(13)
\[ Y^o_{k+1|k} = \begin{bmatrix} C_k (\Phi_k + 1) \Gamma'_k + g(x_{k|k}, u_{k-1}, d_{k|k}) \\ C_k \sum_{i=1}^P (\Phi_k^p - \Gamma'_k + g(x_{k|k}, u_{k-1}, d_{k|k}) \end{bmatrix} \]
(14)
\[ S^u_k = \begin{bmatrix} \Phi_k^p - \Gamma'_k \]
(15)
\[ S^u_k \] is of dimension \( n_u \times n_i \), where \( n_u \) is the number of outputs and \( n_i \) is the number of inputs

Calculation of control action
The control signal is calculated by solving the quadratic program problem:
\[ \min_{\Delta U} \left(\|R_k x_{k+1|k} - Y^o_{k+1|k}\|^2_{\Lambda^y} + \|\Delta U\|^2_{\Lambda^u}\right) \]
(16)
Subject to
\[ u_{k+j}^l \leq u_{k+j} \leq u_{k+j}^{hi} \quad j = 0, m - 1 \]
\[ -\Delta u_{k+j} \leq \Delta u_{k+j} \leq \Delta u_{k+j}^{hi} \quad j = 0, m - 1 \]
\[ y_{k+j|k} \leq y_{k+j|k} \leq y_{k+j|k}^{hi} \quad j = 1, P \]
(19)
Where \( R_k x_{k+1|k} \) is the vector of future set-points (corresponding to the predicted output \( Y_{k+1|k} \), and \( \Lambda^y \) and \( \Lambda^u \) are tuning parameters.

III. WIENER MODEL BASED NONLINEAR MODEL PREDICTIVE CONTROL (WNMPC)

A. Wiener Model Structure
For the purposes of control system design, any nonlinear system as represented using equations (1) – (3), can be approximated using Wiener model. A Wiener model consists
of a dynamic linear element (LDE) in cascade with a static nonlinear part (NL), as shown in Fig. 1.

\[ u(k) \xrightarrow{\text{LDE}} z(k) \xrightarrow{\text{NL}} y(k) \]

**Fig.1: Block representation of Wiener model**

For the linear element, a state space description is used as follows:

\[ x(k+1) = Ax(k) + Bu(k) \]
\[ z(k) = Cx(k) + Du(k) \]  \hspace{1cm} (20)

For the static nonlinear element, the polynomial function can be used

\[ y(k) = h(z(k)) = \sum_{i=0}^{N} a_i z^i(k) \]  \hspace{1cm} (21)

It will be assumed here that the function \( h \) has an inverse and thus can be approximated too using a polynomial function.

**B. Wiener based nonlinear model predictive control Algorithm**

In LMPC, the general optimization problem to be solved at every sampling instant is posed as equations (16) – (19). If the state vector at the present time and the future behaviour of the variables are assumed to be known, they can be written in a matrix form:

\[ x(k) = [z^T(k+1) \ldots z^T(k+P)]^T \]
\[ \Delta U(k) = [\Delta u^T(k+1) \ldots \Delta u^T(k+M)]^T \]
\[ y(k) = [y^T(k+1) \ldots y^T(k+P)]^T \]
\[ r(k) = [r^T(k+1) \ldots r^T(k+P)]^T \]

Then, the predicted output for the linear model is

\[ \hat{z}(k) = \beta \Delta U + \xi x(k) + d(k) \]  \hspace{1cm} (22)

Where

\[ \beta = \begin{bmatrix} C^T A^2 B & C^T AB & C^T B & D & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C^T A^{P-1} B & C^T A^{P-2} B & C^T A^{P-3} B & \ldots & C^T A^{P-M} B & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \]

\[ \xi = \begin{bmatrix} C^T A \xi \[21] \end{bmatrix} \]

And \( d(k) = [d(k+1|k) \ldots d(k+P|k)] \)

Then, the predicted output for the complete model is

\[ \hat{y}(k) = \begin{bmatrix} \hat{h}(\hat{z}(k+1)) \\ \hat{h}(\hat{z}(k+2)) \\ \vdots \\ \hat{h}(\hat{z}(k+P)) \end{bmatrix} = h(\hat{z}(k)) \]  \hspace{1cm} (23)

We now define some points related to the MPC structure:

(1) Since the polynomial function \( h \) was assumed to be invertible, it is possible to write the desired signal referred to the output of linear model as transformation of the set point \( r^*(k) \) as,

\[ r^*(k) = h^{-1}(r(k)) \]  \hspace{1cm} (24)

(2) If \( \hat{y}_{k+1|k}^{\text{high}} \) and \( \hat{y}_{k+1|k}^{\text{low}} \) are the upper and lower bounds for the output variables \( \hat{y}_{k+1|k} \), then these magnitudes can be translated to the linear model as

\[ \hat{z}_{k+1|k}^{\text{high}} = h^{-1}(\hat{y}_{k+1|k}^{\text{high}}) \]
\[ \hat{z}_{k+1|k}^{\text{low}} = h^{-1}(\hat{y}_{k+1|k}^{\text{low}}) \]  \hspace{1cm} (25)

(3) Disturbances are typically handled by assuming that a step signal has entered at the output and that it will remain constant for all future time \( (d(k) = d(k+j), j = 1, \ldots, P) \). In this case the step disturbance is computed:

\[ d(k) = h^{-1}(y^m(k)) - \hat{z}(k) \]  \hspace{1cm} (26)

where \( \hat{z}(k) \) is the current predicted output for the linear model and \( y^m(k) \) is the current measured output for the process. It is straightforward that introducing this bias in the error, as a perturbation, allows to remove any model errors offset in steady-state.

Finally, the WNMPC can be posed as quadratic optimization problem (QP),

\[ \min_{\Delta U} \left\{ \| \hat{z}(k) - r^*(k) \|_2^2 + \| \Delta U \|_2^2 \right\} \]

subject to equation (22) and constraints (17), (18) and (28)

\[ \hat{z}_{k+1|k}^{\text{low}} \leq \hat{z}_{k+1|k} \leq \hat{z}_{k+1|k}^{\text{high}} \]

**IV. EXAMPLES**

**A. STIRRED-TANK REACTOR (CSTR)**

**Process Description and modeling**

The process under consideration here is the constant volume continuous stirred-tank reactor in which an irreversible, exothermic reaction \( A \rightarrow B \) occurs that is cooled by single coolant stream (see fig.2). Interested reader should consult Cervantes et al. (2003), and Prakash and Srinivasan (2009) for more information about this process.

**Fig.2: Continuous stirred tank reactor**

The process is modeled by the following equations:

\[ \frac{dc_A(t)}{dt} = \frac{q(t)}{V} (c_A(t) - c_A(t)) \]
\[ \frac{dc_B(t)}{dt} = -\frac{q(t)}{V} (c_B(t) - c_B(t)) \]
\[ \frac{dT(t)}{dt} = \frac{Q(t)}{V} (T(t) - T(t)) - \frac{\Delta H}{RT(t)} \]

The measured concentration has a time delay \( t_d = 0.5 \text{ min} \) modelled by

\[ c_{\text{meas}}(t) = c_A(t - t_d) \]

The objective of the control system is to control the measured concentration of \( A, c_A \) by manipulating the coolant flow rate \( q_c \). Process nominal operating data is displayed in Table 1.
Identification of the process
The results of the identification experiment performed gives the linear dynamic element (LDE) of the wiener model as:

\[ \frac{dC_A}{dt} = \begin{bmatrix} -9.9839 \\ 1796.8 \end{bmatrix} \left[ \begin{array}{cc} C_A \\ T \end{array} \right] + \begin{bmatrix} 0 \\ -0.8772 \end{bmatrix} q_c \] (32)

\[ z = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} \] (33)

The nonlinear static element and its inverse are plotted in fig.3

Nonlinear predictive control system design
The two techniques (SLN MPC and WNMPC) for designing nonlinear model predictive control system discussed in sections 2 and 3 respectively are employed to design predictive controllers for the CSTR. For the purpose of making comparison, linear model predictive controller is also designed for the same system. The designed controllers are implemented on a simulation model of the CSTR developed in MATLAB/SIMULINK. The MPC tuning parameters are shown in table 2.

The simulation results as depicted in fig.4 show that SLN MPC has the best performance with fastest settling time both for the positive and negative set-point changes. WNMPC is the next in performance by exhibiting faster response than LMPC for a negative set-point change. However, LMPC response for a positive set-point change is better than that of a WNMPC. This is due to the fact that the system is being driven to higher gain region.

The performance indices computed for each of the techniques as displayed in table 3 also indicates the superiority of SLN MPC over WNMPC and LMPC, having the least error.

B. THREE-TANK-SYSTEM

Process description and modeling
The principal structure of the three tank plant considered here for study is shown in fig. 5 below (See Amira (2002) for detailed description). The controlled variables are the levels, \( h_1 \) and \( h_2 \) inside tanks 1 and 2. The level \( h_3 \), in tank 3, though not being controlled is to be maintained not to overflow or run dry. The maximum level of each tank is 62 cm(+/-1 cm). Inflow, \( q_1 \), of tank 1 and inflow \( q_2 \) of tank 2 are considered as the manipulated variables.

\[ S \frac{dh_1}{dt} = q_1 - q_{13} \] (34)
\[
\begin{align*}
S_{\text{dt}}^{\text{dh}_3} &= q_{13} - q_{32} \\
S_{\text{dt}}^{\text{dh}_2} &= q_2 + q_{32} - q_{20}
\end{align*}
\]  
where \( q_{ij} = \mu_i, S_p, \text{sign}(h_i - h_j) \cdot \sqrt{2g|h_i - h_j|} \) \hspace{1cm} (37)

\[ q_{20} = \mu_2, S_p, \sqrt{2gh_2} \] \hspace{1cm} (38)

The state space model of the three tank system around the

\[ \dot{x} = Ax + Bu, \quad y = Cx \] \hspace{1cm} (39)

where

\[
A = \begin{bmatrix}
a_{11} & 0 & a_{13} \\
0 & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
B = \begin{bmatrix}
1/S_1 \\
0 \\
0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}.
\]

\[
a_{11} = \frac{(-1/S_1)\mu_1 S_p \sqrt{2g \text{sign}(h_{10} - h_{30})}}{2\sqrt{|h_{10} - h_{30}|}},
\]

\[
a_{13} = \frac{(1/S_3)\mu_1 S_p \sqrt{2g \text{sign}(h_{10} - h_{30})}}{2\sqrt{|h_{10} - h_{30}|}},
\]

\[
a_{22} = \frac{(-1/S_3)\mu_3 S_p \sqrt{2g \text{sign}(h_{30} - h_{20})}}{2\sqrt{|h_{30} - h_{20}|}} - \frac{(1/S_1)\mu_2 S_p \sqrt{2g \text{sign}(h_{20})}}{2\sqrt{|h_{20}|}}
\]

\[
a_{23} = \frac{(1/S_3)\mu_3 S_p \sqrt{2g \text{sign}(h_{30} - h_{20})}}{2\sqrt{|h_{30} - h_{20}|}}
\]

\[
a_{31} = \frac{(1/S_1)\mu_1 S_p \sqrt{2g \text{sign}(h_{10} - h_{30})}}{2\sqrt{|h_{10} - h_{30}|}},
\]

\[
a_{32} = \frac{(1/S_3)\mu_3 S_p \sqrt{2g \text{sign}(h_{30} - h_{20})}}{2\sqrt{|h_{30} - h_{20}|}}
\]

\[
a_{33} = \frac{(-1/S_1)\mu_1 S_p \sqrt{2g \text{sign}(h_{10} - h_{30})}}{2\sqrt{|h_{10} - h_{30}|}} - \frac{(1/S_3)\mu_3 S_p \sqrt{2g \text{sign}(h_{30} - h_{20})}}{2\sqrt{|h_{30} - h_{20}|}}
\]

\[
\begin{bmatrix}
[\text{Output, H}_1, \text{cm}] \\
[\text{Output, H}_2, \text{cm}]
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
[\text{Input, } \text{H}_1] \\
[\text{Input, } \text{H}_2] \\
[\text{Input, } \text{H}_3]
\end{bmatrix}
\] \hspace{1cm} (40)

The nonlinear element and its inverse are plotted in fig. 6.

**Simulation and Experimental Results**

Again, predictive controllers were designed for the three-tank plant by the three techniques, SLNMP, WNMPC and LMPC, and implemented both on a simulated model of the three-tank and on an experimental set-up of the tank housed in our Process System Engineering Laboratory. The tuning parameters are displayed in Table 5. Figs. 7-10 show the response of the process when the second output set-points have changed, while the first output is set to the nominal value 41.5cm. SLNMP and WNMPC show similar and better performance than LMPC with shorter settling time. In addition to this, H_1 reaction to set-point change in H_2 is least for SLNMP followed by WNMPC. Performance indices are summarized in Tables 6 and 7.

**Table 4: Steady state operation table**

<table>
<thead>
<tr>
<th>h_{10}, h_{30}, h_{20} in cm</th>
<th>41.5, 26.5, 11.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outflow coefficients, ( \mu_1, \mu_2, \mu_3 )</td>
<td>(0 – 1)</td>
</tr>
<tr>
<td>Area of tank (S_1 to S_3) in cm^2</td>
<td>149</td>
</tr>
<tr>
<td>Area of connecting pipes in cm^2, S_p</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Identification of the process**

The results of the identification experiment performed gives the linear dynamic element (LDE) of the wiener model as:

\[
\begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} = \begin{bmatrix}
-0.0112 & 0 & 0.0112 \\
0 & -0.0404 & 0.0112 \\
0.0112 & 0.0112 & -0.0224
\end{bmatrix} \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} + \begin{bmatrix}
0.0067 & 0 \\
0 & 0.0067 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix}
\]

\[
\begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} = \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} + \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} + \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix} + \begin{bmatrix}
[\text{LDE}] \\
[\text{LDE}]
\end{bmatrix}
\]

**Table 5: MPC tuning parameters [three-tank-system]**

<table>
<thead>
<tr>
<th>( \Lambda^y - \Lambda^u )</th>
<th>101 – 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P - m )</td>
<td>8 – 1</td>
</tr>
<tr>
<td>( y^k_{\text{low}} - y^k_{\text{high}} )</td>
<td>0 – 63</td>
</tr>
<tr>
<td>( u^k_{\text{low}} - u^k_{\text{high}} )</td>
<td>0 – 100</td>
</tr>
</tbody>
</table>

**Table 6: Performance indices, three-tank, for the simulation results**

<table>
<thead>
<tr>
<th>SLNMP</th>
<th>WNMPC</th>
<th>LMPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>188.6</td>
<td>218.1</td>
</tr>
<tr>
<td>ITAE</td>
<td>5.8e4</td>
<td>7.1e4</td>
</tr>
<tr>
<td>ITSE</td>
<td>2.4e5</td>
<td>2.8e5</td>
</tr>
<tr>
<td>ISE</td>
<td>772.4</td>
<td>898.7</td>
</tr>
</tbody>
</table>

**Table 7: Performance indices, three-tank, for the experimental results**

<table>
<thead>
<tr>
<th>SLNMP</th>
<th>WNMPC</th>
<th>LMPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>213.3</td>
<td>213.3</td>
</tr>
<tr>
<td>ITAE</td>
<td>6.9e4</td>
<td>6.9e4</td>
</tr>
<tr>
<td>ITSE</td>
<td>2.5e5</td>
<td>2.5e5</td>
</tr>
<tr>
<td>ISE</td>
<td>760.4</td>
<td>760.4</td>
</tr>
</tbody>
</table>
In this paper, two techniques of NMPC have been applied to control nonlinear processes and compared with that of LMPC. These two methods enjoy the advantage of having to solve a quadratic programming problem at each sampling instant compared to the original formulation of NMPC which results to non-convex nonlinear programming. Both the simulation results and results of real-time implementation carried out on our experimental three-tank system show that there could be improvement in the performance of a process plant by appropriately applying NMPC. In all of the cases studied, NMPC outperforms the LMPC without excessive control actions.

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