Decentralized Receding Horizon Control Using Communication Bandwidth Allocation

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Abstract—The decentralized receding horizon control (DRHC) of a team of cooperative vehicles with limited communication bandwidth is considered. It is well known that the more communication the better stability and performance properties of the cooperative vehicles; however, in reality the available communication is often limited. This motivates our research to develop a new algorithm for efficient usage of available communication capacity so that the teaming behavior is optimized. The proposed algorithm uses a bandwidth allocation method; the key idea is to reduce the overall mismatch between predicted and actual plans of each neighbor by efficient communication bandwidth allocation.  

I. INTRODUCTION

The recent advances in distributed computation allow using optimization-based control methods such as receding horizon control (RHC) [1, 2], and decentralizing the control problems [3-5]. Such fact has motivated researchers to develop RHC based decentralized control architectures [1, 3-5] for cooperative multi-agent systems. However, proving stability and feasibility of DRHC-based cooperative control systems is still a challenge [3, 6-8] and hence remains an ongoing research area.

The stability and performance of DRHC-based cooperative control systems may be enhanced by modifying the cost function and constraints [3-8]. However, the communication based methods offer another potential method for improving the performance of DRHC in the context of cooperative control of multi-agents. This paper aims at improving the performance of DRHC by proposing a DRHC technique that includes allocating the available, although limited, communication resources to the different agents in the team. In this paper, bidirectional communication is assumed available to the neighboring vehicles; however, the bandwidth in the communication channels is limited. The latter is a practical consideration that may lead to delayed information exchanges among agents and, if not handled properly, may cause instability. The key idea in this paper is to enable the DRHC to allocate a greater portion of the available bandwidth to agents in need, with objective of maintaining fleet cohesion.

II. PROBLEM FORMULATION

A. Interaction and Information Exchange Graphs

The interaction between cooperative vehicles is usually represented by an “interaction graph” including nodes and arcs. The nodes represent the vehicles and an arc between two nodes denotes a coupling term in the objectives and/or in the constraints associated to the nodes. Also, it is usually assumed that the exchange of information has a particular structure; in this paper it is assumed that the information exchange graph is fixed and that each vehicle can communicate information with only a subset of the other vehicles in the team. Furthermore, we assume the interaction graph and the information exchange graph coincide; that is, only the vehicles that have interaction with each other, such as in collision avoidance algorithms, will exchange information.
Considering a set of $N_v$ vehicles cooperating to perform a common mission, the $i^{th}$ vehicle corresponds to the $i^{th}$ node of the graph. If an arc $(i,j)$ connecting the $i^{th}$ and $j^{th}$ node is present, it means that the $i^{th}$ and $j^{th}$ vehicles have a coupling term in their cost function and/or in their constraints (interaction), and communicate with each other. This relationship is termed as a neighborhood for the $i^{th}$ and $j^{th}$ vehicles. This leads to the interaction graph:

$$G(t) = \{V,E\}$$

(1)

where $V$ is the set of nodes (vehicles) and $E \subseteq V \times V$ the set of arcs $(i,j)$, with $i, j \in V$. The interaction graph is indirect i.e. $(i,j) \in E$ implies $(j,i) \in E$ even though it does not appear in $E$. This graph topology enables us to represent all configurations of the subgroups in terms of interaction and information exchange graphs. For the remainder of the paper, let $N^*_n$ denote the number of neighbors of vehicle $i$. Also, the terms “agent”, “vehicle” and “team member” bear the same meaning.

### B. DRHC formulation

With Receding Horizon Control (RHC) –also known as model predictive control- a cost function is optimized over a finite time called prediction horizon $T$, or in short horizon. The first portion of the computed optimal input is applied to the plant during a period of time called the execution horizon $\delta$, or sampling period. The reader is referred to [11] for a comprehensive review of RHC.

Let us assume that the execution horizon $\delta$ is equal to the communication period. We can thus suppose there is synchronization between the communication rate and the sampling rate of RHC. Then, let the following represent the concatenated state and input vectors of the neighbors of $i^{th}$ vehicle at time $t_k$, where $t_{k+1} = t_k + \delta$ is the discrete time and $t_0 = 0$:

$$\tilde{x}^i(t_k) = [\ldots, x^i(t_{k}), \ldots]^T \quad : j \in V, \; (i,j) \in E$$

$$\tilde{u}^i(t_k) = [\ldots, u^i(t_{k}), \ldots]^T \quad : j \in V, \; (i,j) \in E$$

(2)

where $x^i(t)$ and $u^i(t)$ are the state and the input vectors of the $i^{th}$ vehicle, respectively, at time $t$. Also, let the following include the state and the input vectors of $i^{th}$ vehicle and the concatenated vectors $\tilde{x}^i(t_k)$ and $\tilde{u}^i(t_k)$, respectively:

$$\tilde{x}^i(t_k) = [x^i(t_k), \tilde{x}^i(t_k)]^T$$

$$\tilde{u}^i(t_k) = [u^i(t_k), \tilde{u}^i(t_k)]^T$$

(3)

vectors $\tilde{x}^i(t_k)$ and $\tilde{u}^i(t_k)$ represent the updated information available to the $i^{th}$ vehicle at time $t_k$. The following represents the decentralized cost function for the $i^{th}$ vehicle in the team at time $t_k$:

$$J^{i}_{t_k}(\tilde{x}^i(t_k),\tilde{u}^i(t_k)) = J^{i}_{T}(x^i(t_{k}),u^i(t_{k}))+ \sum_{(i,j) \in E} [L^{ij}_{T}(x^i(t_{k}),u^i(t_{k}))+L^{ji}_{T}(x^i(t_{k}),x^j(t_{k}))]$$

(4)

where, the term $L^{ij}_{T}(x^i(t_{k}),u^i(t_{k}))$ in (4) is associated to the cost of the individual vehicle $i$ over the prediction horizon $T$. Also, the term $L^{ij}_{T}(x^i(t_{k}),x^j(t_{k}))$ represents the coupling cost between $i^{th}$ and $j^{th}$ vehicles over $[t_{k-1},t_k+T]$. As seen from (4) the decentralized cost function includes the cost associated to each vehicle and that of its neighboring vehicles, and not all the vehicles in the team.

Suppose that the following represents the nonlinear dynamics of the $i^{th}$ vehicle:

$$\dot{x}^i(t) = f(x^i(t),u^i(t)), \quad f(0,0) = 0$$

(5)

Assume $x^i_{t_k}(t)$ denotes the state vector of the $i^{th}$ vehicle at time $t$, calculated by solving the optimization problem $p^i(t_k)$ at time $t_k$ and also $x^i(t)$ denotes the actual state of $i^{th}$ vehicle at time $t$. The DRHC problem $p^i(t_k)$ is then defined for the $i^{th}$ vehicle at time $t_k$ as follows:

$$DRHC Problem \ p^i(t_k) :$$

$$\tilde{u}^i_{t_k}(\cdot) = \arg\min J^i_{T}(\tilde{x}^i(t_k),\tilde{u}^i(t_k))$$

(6)

Subject to:

$$\dot{x}^i_{t_k}(t) = f(x^i_{t_k}(t),u^i_{t_k}(t));$$

$$x^i_{t_k}(t_k) = x^i(t_k); \quad t \in [t_k, t_k+T]$$

(7a)

$$x^i_{t_k}(t) \in X^i; \quad u^i_{t_k}(t) \in U^i; \quad t \in [t_k, t_k+T]$$

(7b)

$$\dot{x}^i_{t_k}(t) = f(x^i_{t_k}(t),u^i_{t_k}(t));$$

$$x^i_{t_k}(t_k) = x^i(t_k); \quad t \in [t_k, t_k+T]; \quad (i,j) \in E$$

(7c)

$$x^i_{t_k}(t) \in X^i; \quad u^i_{t_k}(t) \in U^i; \quad t \in [t_k, t_k+T]; \quad (i,j) \in E$$

(7d)

$$x^i_{t_k}(t_k) + T \in X^i_f; \quad (i,j) \in E$$

(7e)

In Eq. (7), $J^i_{T}(\cdot)$ comes from Eq. (4). Vectors $X^i$ and $X^i_f$ denote the set of admissible states, inputs and final states, respectively, for $i^{th}$ vehicle. Signal $\tilde{u}^i_{t_k}(\cdot)$ denotes the trajectory of optimal inputs for all vehicles over $[t_{k-1}, t_k+T]$.

The control action obtained by solving the optimization problem $p^i(t_k)$ is implemented during the execution time $\delta$ until the next update. Repeating this procedure online yields the closed-loop solution of RHC.

### C. DRHC Algorithm

Each vehicle $i$ at any sampling time sends its states to its neighboring vehicles. Furthermore, every vehicle $i$ receives
the information on states of its neighbors. Based on such 
information, each vehicle \( i \) solves the optimal 
problem \( P^i(t_k) \) using Algorithm 1:

**Algorithm 1:** At any time instant \( t_k \), each vehicle \( i \):
1. Let \( k=0 \)
2. Send \( x^i(t_k) \) to the neighboring vehicles and receive the 
   most updated information from neighboring vehicles 
   \( (\tilde{\mathbf{x}}^j(t_k)) \).
3. Solve \( P^i(t_k) \) and generate the control action \( u^i(t_k) \) 
   for \([t_k,t_k+T]\).
4. Execute the control action for individual vehicle \( i \) over the 
   time interval \([t_k,t_{k+1}]\).
5. \( k=k+1 \). Goto step 2.

This algorithm is repeated until the assigned target (here 
origin) is reached. The targets are assumed to be known and 
assigned to each agent a priori.

D. Formation Cost

For the particular case of formation control, consider a 
group of mobile robots or flying vehicles that are required 1) 
to keep certain relative positions, and 2) to visit a set of 
targets. The decentralized individual and coupling cost 
functions for each vehicle are then defined as follows:

\[
L^i_T(x^i(t_k), u^i(t_k)) = \int_{t_k}^{t_k+T} \left( \|x^i_k(\tau)\|^2_Q + \|u^i_k(\tau)\|^2_R + \|x^i_{t_k}(t_k + T)\|^2_P \right) d\tau
\]

\[
l^i_T(x^i(t_k), x^i(t_k)) = \int_{t_k}^{t_k+T} \left( \|x^i_k(\tau) - x^j_k(\tau)\|^2_S \right) d\tau
\]

where \( P, Q, R \) and \( S \) are positive definite symmetric matrix. 
As there is no non-convex coupling constraint, this cost 
function allows applying a relative position constraint 
among the vehicles. Such approach is used extensively in 
the literature \([3, 4]\).

III. STABILITY ANALYSIS

Each vehicle relies on models of its neighbors to predict 
their. Note that \( x^{i,j}_{t_k}(t) \) is the state vector of \( i^{th} \) vehicle at 
time \( t \), computed by \( j^{th} \) vehicle at time step \( t_k \). Also, it is 
assumed that \( x^{i,j}_{t_k}(t) = x^{i,j}_{t_k}(t) \).

**Theorem 1 (Stability):** Assume the matrix penalties \( P, Q, R \) and \( S \) are 
symmetric and positive definite in (8). Then, a 
sufficient condition for the asymptotic stability of DRHC 
problem \( P^i(t_k) \) at the origin, with cost functions (8) is:

\[
e^i(t_k) \leq \kappa^i(t_k)
\]

where \( e^i(t_k) \) is the prediction mismatch of the optimization 
problem \( P^i(t_k) \), over \([t_k+1,t_k+T] \), given as follows:

\[
e^i(t_k) = \sum_{j(k,j) \in E} \int_{t_k}^{t_k+T} \left( \|x^{i,j}_{t_k}(t) - x^{j,j}_{t_k}(t)\|^2_Q + \|u^{i,j}_{t_k}(t) - u^{j,j}_{t_k}(t)\|^2_R \right) dt
\]

and the bound \( \kappa^i(t_k) \) is given as:

\[
\kappa^i(t_k) = \int_{t_k}^{t_k+T} \left( \|x^{i,j}_{t_k}(t)\|^2_Q + \|u^{i,j}_{t_k}(t)\|^2_R \right) dt
\]

**Proof:** The proof is removed due to page restriction. 
Stability condition (9) implies a reduced prediction 
mismatch decreases the left-hand side of inequality (9) and 
eases satisfying the stability condition as opposed to a larger 
mismatch.

IV. PERFORMANCE ANALYSIS

For formation control, it is desired that vehicles keep 
certain relative positions. Hence, the decentralized 
performance metric is formulated as follows:

\[
I^i(\tilde{x}^i(t_k)) = \sum_{j(k,i) \in E} \int_{t_k}^{t_k+T} \left( \|x^i(t) - x^j(t) - \hat{r}^j(t)\|^2_S \right) dt
\]

where \( \hat{r}^j(t) \) is the vector of desired relative positions 
between the \( i^{th} \) and the \( j^{th} \) vehicles. It is desired 
that \( I^i(\tilde{x}^i(t_k)) = 0 \). Any deviation from that will be studied 
using perturbation analysis as follows:

\[
x^i(t) \rightarrow x^i(t) + \Delta x^i(t)
\]

\[
x^i(t) \rightarrow x^i(t) + \Delta x^i(t)
\]

\[
I^i(\tilde{x}^i(t_k)) \rightarrow I^i(\tilde{x}^i(t_k)) + \Delta I^i(\tilde{x}^i(t_k))
\]

substituting (13) in (12) and using triangular inequality of 
norms yield:

\[
\Delta I^i(\tilde{x}^i(t_k)) \leq \sum_{j(k,i) \in E} \int_{t_k}^{t_k+T} \left( \|x^i(t) - x^j(t)\|^2_S \right) dt
\]

\[
\sum_{j(k,i) \in E} \int_{t_k}^{t_k+T} \left( \|x^{i,j}_{t_k}(t) - x^{j,j}_{t_k}(t)\|^2_S \right) dt \leq
\]
for the sake of stability it is required that \( \varepsilon_0 \leq \kappa^i(t_k) \) according to (9); and this must be taken into account. This algorithm is a modified version of Algorithm 1 while at every iteration, steps 1, 2 and 3 will be executed first to allocate the communication resources efficiently. The next section will demonstrate, via simulations, the 3rd step of the algorithm.

VI. SIMULATIONS

A formation of a fleet of miniature rotorcrafts with the 3DOF nonlinear dynamics is considered [12]. The main rotor and tail rotor thrusts are saturated at: 0 ≤ \( T_{mr} \leq 1000 \) and 0 ≤ \( T_{tr} \leq 20 \) where \( T_{mr} \) and \( T_{tr} \) are the main rotor and tail rotor thrust forces respectively. Also: \( V_{max} = 10 \text{ m/sec} \) (Velocity constraint). The SNOPT optimization package [13] is used to solve the RHC problem. The actual trajectories for 6 vehicles in a triangular formation are shown in Figure 1. The corresponding distance history is depicted in Figure 2 for some typical scenario. In this formation, it is desired that moving vehicles keep a relative distance of 3m while flying in a triangular formation. As seen from Figure 2 the vehicles reach the desired distances after some time.

Algorithm 2: At any time instant \( t_k \), each vehicle \( i \):
1. Communicate \( \varepsilon^i(t_k) \) to neighboring vehicles.
2. Find \( q^i \) member in the set of neighbors so that:
   \[ e^q = \max \{ e^i(t_{k-1}) - \tau; (i, j) \in E \}. \]
A. Error (Performance) & Communication Rate & Communication Delay vs. Mismatch:

Figure 3 shows the maximum error in desired relative distance (12) versus mismatch (10) for different simulations. As seen from Figure 3 the error will increase with the mismatch. Such numerical result corroborates Equation (14).

Four different simulations are run with different sampling time (communication rate). As seen from Figure 4, a faster communication rate (smaller sampling time) results in a decrease in the mismatch. As another case study, the effect of communication delay on mismatch is investigated. Figure 5 shows the mismatch time history for 7 different simulations. The simulations differ only in communication delays. As seen from Figure 4 and Figure 5, the overall mismatch will increase with the communication delay and a higher sampling time. Consequently, from the stability and performance analysis results we can conclude that communication delay and slower communication rate can have an adverse effect on the stability and performance, which is intuitively expected.

B. Limited bandwidth Communication

In this subsection the proposed algorithm 2 is applied to the case where the communication bandwidth is limited. Consider a network of vehicles, where the communication channel of each vehicle is used to communicate with neighboring vehicles. Hence, in such situation, the following communication constraint must be satisfied by each vehicle $i$ when communicating with neighbors:

$$\sum_{j: (i,j) \in E} \frac{K_{ij}}{\tau_{ij}} \leq B_i$$  \hspace{1cm} (15)

where, $\tau_{ij}$ is the delay for transmitting the information form $i$ to $j$. $K_{ij}$ is the size of $i^{th}$ message (bits/sample) and $B_i$ (bits/sec) is the bandwidth available with the communication channel of vehicle $i$. In normal condition when none of the neighbors of $i^{th}$ vehicle are in critical situation, equal bandwidth portions will be allocated to all neighbors. However, in the emergency case where an agent $q$ in the neighboring set of $i^{th}$ vehicle, is in a critical situation, the neighboring vehicles of $q$ assign more bandwidth for communicating with $q$. The communication delay in this case is calculated as follows:

$$\tau_{iq} = \frac{N_i}{\eta B} \quad ; \quad q \mid (i,q) \in E \quad ; \quad 1 < \eta$$   \hspace{1cm} (16)

where, $\eta$ is a design parameter and defines the portion of bandwidth allocated to the $q^{th}$ vehicle. Hence, considering (15) for the rest of neighboring vehicles of $i$, the delay is assigned as follows:

$$\tau_{ij} \geq \frac{K(N_i - 1)}{B(1 - \frac{\eta}{N_i})} \quad ; \quad j \mid (i,j) \in E \quad , j \neq q$$ \hspace{1cm} (17)

This pattern will be used for the same scenario as in previous subsections. In the following simulations we assume $\frac{K}{B} = 2$, and choose $\eta = 1.25$ and $\varepsilon_s = 100$. $K$ is the size of messages and $B$ comes from the capacity of communication channel. $\eta$ and $\varepsilon_s$ are design parameters; $\eta$ determines the portion of bandwidth allocated for communication with the agent in worse (or critical) situation and $\varepsilon_s$ must be chosen so that $\varepsilon_s \leq \kappa_{\min}^i(t)$ for the sake of stability according to (9).

**Remark 1:** As the agents move closer to their target, $\kappa^i(t)$ approaches zero, this can be seen from (11) and from the simulations. This means $\kappa^i_{\min}(t) \rightarrow 0$ which makes it difficult to choose an appropriate $\varepsilon_s$. To circumvent this problem, we neglected the steady-state response and chose $\kappa^i_{\min}(t)$ based on the transient response. A comprehensive investigation for choosing $\varepsilon_s$ and $\eta$ is required.
The simulation results for two different cases are depicted in Figure 6 and Figure 7 for the formation flight of three vehicles. In Figure 6, the error parameter (12), corresponding to the second vehicle (the error profile for other vehicles follows the same pattern), is plotted versus time for two different cases. First, an equal bandwidth allocation strategy is utilized using Algorithm 1; the average error for this case is 225. Second, proposed Algorithm 2 allocates bandwidth so that the average error is reduced to 70. In this simulation any of the agents may be in the critical situation at any time but most of the time the second vehicle is in critical situation.

The bandwidth allocation leads to varying communication delays for each vehicle as seen in Figure 7, where the time history of the delay allocation (due to bandwidth allocation) is plotted for the communication channel of the third vehicle. The communication delay is denoted by \( \tau \) and \((d-1)\delta \leq \tau \leq d\delta \) where \( d \in \mathbb{N} \). As seen from Figure 7 whenever there is no critical situation both the neighboring vehicles 1 and 2 are assigned the same communication delay, namely \( d=4 \). However, in the case of one agent in critical situation where the communication delay of one vehicle is reduced to \( d=3 \), the penalty is that the communication delay corresponding to the other neighbor will increase to \( d=5 \) to satisfy the communication constraint (15).

VI. CONCLUSION

The results of analyzing the feasibility, stability and performance of DRHC imply that the mismatch between predicted and actual plans of each agent plays an important role in the stability and performance of the entire fleet. Based on this key result, a new improved algorithm for DRHC is proposed which leads to superior stability and performance of the team by communication bandwidth allocation.

REFERENCES