Abstract—A Rotary Left Ventricular Assist Device (RLVAD) is a mechanical pump implanted in patients with congestive heart failure to assist their left ventricle in pumping blood through the circulatory system. This blood pump is controlled by varying the rotor speed to adjust the amount of blood flow pumped into the circulatory system. If the patient is in a health care facility, the pump speed can be adjusted manually by a trained clinician to meet the patient’s blood needs depending on his or her activity level. However, an important challenge facing the increased use of the RLVAD is the desire to allow the patient to return home. The development of an appropriate feedback controller that is capable of automatically adjusting the pump speed is therefore a crucial step in meeting this challenge. In order to be able to develop such a controller an appropriate mathematical model of the combined cardiovascular system and RLVAD must first be developed. In this paper, we review progress on a state space model for this system that can be used to develop the controller. The model is 6th order, nonlinear, and time-varying and is a combination of a 5th order model of the left ventricle and circulatory system and a 1st order model of the RLVAD along with its inlet and outlet cannulae. The entire combined system is controlled by the rotational speed of the pump. Using this model we will discuss some of the challenges faced in the development of a useful feedback controller for this system. We will also present some preliminary results on a simple partial state feedback controller whose purpose is to prevent the occurrence of a dangerous phenomenon called ventricular suction.

I. INTRODUCTION

The medical community has recently placed increased emphasis on the use of mechanical heart assist devices that can substitute for, or enhance, the function of the natural heart for patients with congestive heart failure who are waiting for heart transplantation [1]. A Rotary Left Ventricular Assist Device (or RLVAD) is such a device. This device is a rotary pump that continuously draws blood out of the left ventricle and into the circulatory system. It is typically, quieter, smaller, and more efficient than the older pulsatile type devices, and consequently have received considerable acceptance in recent years. Generally speaking, the goal of the RLVAD is to assist the native heart in pumping blood so as to provide the patient with as close to a normal lifestyle as possible until a donor heart becomes available or, in some cases, until the patient’s heart recovers. In many situations, this means allowing the patient to return home and/or to the workforce.

Currently, RLVAD patients are typically kept in a critical care setting and are continuously supervised by human operators. When the patient is in a stable condition, the operator adjusts the pump speed manually so as to achieve the desired levels of blood flow and hemodynamic variables to keep the patient comfortable and insure his or her well being. However, for an active patient whose level of activity is continuously changing, this manual open-loop control becomes impractical. Its limitations will become even more apparent if the patient desires to go home so as to adopt a normal lifestyle. The inability of the manual control to respond automatically to changes in demand can dramatically impact the quality of life of these patients [2,3].

An important engineering challenge facing the increased use of the RLVAD is therefore the development of an appropriate automatic controller for the speed of the pump rotor so as to meet the body’s requirements for cardiac output (CO) and mean arterial pressure (MAP) [2, 3]. Since the rotary pump does not use valves the achievement of an appropriate rotational speed is very crucial. If the speed is too low, blood may regurgitate back from the aorta to the left ventricle through the pump resulting in what is known as “backflow”. If the speed is too high, the pump will attempt to draw more blood from the ventricle than available which may cause the ventricle to collapse. This dangerous phenomenon called “ventricular suction” must be detected quickly and the pump speed reduced before the heart muscle is damaged. While avoiding these two extremes, the pump speed must also be adjusted continuously, up and down, to meet the patient's varying levels of blood demand [4-7]. The eventual goal of a pump controller is therefore to meet all these requirements so that a RLVAD recipient patient could potentially leave the hospital and return home to a normal lifestyle.

Given that the pump is continuously interacting with the left ventricle and the circulatory system, the development of a speed controller that meets the above objectives must therefore be done using tools developed in modern control theory. This cannot be done without first having an appropriate mathematical model for this complex system. The model must be simple enough to be tractable and yet it must be comprehensive enough to capture the essential relationships between the hemodynamic variables and provide the important input and output boundary conditions...
without the ambiguity of unnecessary state variables.

In this paper, we will review current progress in the development of such a model. As a first step we present an autonomous 5-th order model of a healthy cardiovascular system which emphasizes the pressure-volume relationship of the left ventricle. By varying this relationship the model can be used to represent various degrees of congestive heart failure. We then present a 1-th order model of a typical RLVAD along with its inlet and outlet cannulae. Nonlinearities in this model are added to represent the phenomenon of ventricular suction. We then combine these two models into a 6-th order time varying nonlinear model of the left ventricle + RLVAD. The only control variable in this model is the rotational speed of the pump. Using this model, we outline some of the challenges that need to be overcome if the development of a feedback controller that can achieve the objectives mentioned above is to become a reality. We will then illustrate a simple but effective partial state feedback controller which has shown to be effective in reducing the possibility of occurrence of ventricular suction.

II. THE CARDIOVASCULAR AND RLVAD MODELS

The heart is a very complex dynamic system that is very difficult to model mathematically. Although various complete heart models including both left and right ventricles and pulmonary circulation already exist and can be supplemented with a model of the RLVAD, in this paper we are interested in a much simpler approach. We assume that the right ventricle and pulmonary circulation are healthy and normal and as a result their effect on the RLVAD, which is connected from the left ventricle to the ascending aorta, can be neglected. A 5-th order lumped parameter model which can reproduce the left ventricle hemodynamics of the heart [8, 9] can be described by the differential equation:

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt} \\
\frac{dx_3}{dt} \\
\frac{dx_4}{dt} \\
\frac{dx_5}{dt}
\end{bmatrix} = \begin{bmatrix}
-C(t) \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{C(t)} \\
-1 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 \\
-1 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
\]

where \( x_1(t) \) is the Left Ventricular Pressure (LVP(t)), \( x_2(t) \) is the Left Atrial Pressure (LAP(t)), \( x_3(t) \) is the Arterial Pressure (AP(t)), \( x_4(t) \) is the Aortic Pressure (AoP(t)) all in mmHg, and \( x_5(t) \) is the Total Flow \( Q(t) \) in ml/s. In this model, the behavior of the left ventricle is modeled by means of a time varying capacitance (or compliance) \( C(t) = \frac{1}{E(t)} \) where \( E(t) \) is the elastance of the left ventricle. The elastance \( E(t) \) describes the relationship between the ventricle’s pressure and volume [10] according to an expression of the form:

\[
E(t) = \frac{LVP(t)}{LVV(t) - V_0}
\]

where \( LVP(t) \) is the left ventricular pressure, \( LVV(t) \) is the left ventricular volume, and \( V_0 \) is a reference volume, which corresponds to the theoretical volume in the ventricle at zero pressure. Several mathematical expressions have been derived to approximate the elastance function \( E(t) \). In our work, we use the expression [11]:

\[
E(t) = (E_{max} - E_{min})E_A(t) + E_{max}
\]

where

\[
E_A(t) = 1.55 \left( \frac{t_a}{0.7} \right)^{1.9} \left( 1 + \left( \frac{t_a}{1.17} \right)^{21.9} \right)
\]

and where \( t_a = t / T_{max} \), \( T_{max} = 0.2 + 0.15t_e \) and \( t_e \) is the cardiac cycle, i.e., \( t_e = 60 / HR \), where \( HR \) is the heart-rate. The constants \( E_{max} \) and \( E_{min} \) are related to the end-systolic pressure volume relationship (ESPVR) and the end-diastolic pressure volume relationship (EDPVR) respectively. Typical values in (3) for a normal heart with a heart-rate of 60 beats per minute (bpm) are \( E_{max} = 2.0 \) and \( E_{min} = 0.06 \) mmHg/ml. In the model from which (1) was derived, preload and pulmonary circulations are represented by the capacitance \( C_R \); the aortic compliance is represented by the capacitance \( C_A \), and afterload is represented by the four-element Windkessel circuit model [12] comprising \( R_s, L_s, C_s, \) and \( R_y \). The left ventricle’s mitral and aortic valves are represented by two non-ideal switches (or diodes) consisting of a resistance \( R_M \) and ideal diode \( D_M \) for the mitral valve, and resistance \( R_A \) and ideal diode \( D_A \) for the aortic valve. The expression \( r(\xi) \) in (1) is defined by:

\[
r(\xi) = \begin{cases} 
\xi & \text{if } \xi \geq 0 \\
0 & \text{if } \xi < 0 
\end{cases}
\]

In the representation given in (1), we have kept the number of model parameters at a minimum while maintaining
enough complexity in the model so that it can reproduce the hemodynamics of the left ventricle. The various model parameters appearing in (1) and their typical associated values can be found in [13, 14].

We note that the model (1) is autonomous. Its solution is oscillatory due to the cyclic nature of the terms \( \dot{C}(t) \) and \( 1/C(t) \) in the matrices in (1). Within each cycle (called the cardiac cycle) there are three different phases of operation which occur over four different time intervals. The three phases are summarized in Table I. Clearly, every phase within the cardiac cycle is modeled by a different set of linear time-varying differential equations resulting from (1).

<table>
<thead>
<tr>
<th>Table I: Phases within each cardiac cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mitral</td>
</tr>
<tr>
<td>Closed</td>
</tr>
<tr>
<td>Open</td>
</tr>
<tr>
<td>Closed</td>
</tr>
<tr>
<td>Closed</td>
</tr>
<tr>
<td>Open</td>
</tr>
</tbody>
</table>

III. THE RLVAD MODEL

The RLVAD considered in this paper is a rotary mechanical pump connected using two cannulæ as a bridge between the left ventricle and the aorta. The pump and its two cannulæ are characterized by the following set of relationships [13]:

\[
H_j = R_j Q + L_j \frac{dQ}{dt} + \beta \omega^j 
\]

(6b)

where \( j = i, o \) represent inlet and outlet. \( H_j, \), \( H_o, \), and \( H_w \) are the pressure differences across the pump and across the inlet and outlet cannulae respectively, \( Q \) is the blood flow rate through the pump, and \( \omega \) is the pump rotational speed. The parameters \( R_j, R_o, R_w \) represent the flow resistances and \( L_j, L_o, L_w \) represent the flow inertances of the pump and cannulae respectively. \( \beta \) is a pump dependent constant. We note that \( R_j \) represents the flow resistance of the inlet cannula when no suction is present. When suction occurs, an additional resistance \( R_s \) in the form:

\[
R_s = \begin{cases} 
\alpha (LVP(t) - \bar{x}) & \text{if } LVP(t) > \bar{x} \\ 
0 & \text{if } LVP(t) \leq \bar{x} 
\end{cases} \]

(7)

is added to \( H_j \) to represent this phenomenon. Clearly, \( R_s \) is a nonlinear time-varying element whose value is zero when the pump is operating normally and is activated when \( LVP(t) \) (or \( x_i(t) \)) becomes less than a predetermined small threshold \( \bar{x} \), a condition that represents suction. The value of \( R_s \) when suction occurs increases linearly as a function of the difference between \( LVP(t) \) and \( \bar{x} \). The parameter \( \alpha \) is a cannula dependent scaling factor. Values of all parameters mentioned above for the RLVAD used in our model are given in [13, 14]. The state equation governing the behavior of the RLVAD can now be derived as:

\[
LVP(t) - AoP(t) = R'Q + L' \left( \frac{dQ}{dt} \right) + \beta \omega \]

(8)

where \( R' = R_j + R_o + R_w \) and \( L' = L_i + L_j + L_o \).

IV. THE COMBINED MODEL

The addition of the RLVAD to the left ventricle model (1) adds one state variable \( x_i(t) = Q \) which represents the blood flow through the pump and eight passive parameters \( R_j, R_o, R_w, L_i, L_j, L_o, \) and \( \beta \). The combined model is now a forced system, where the primary control variable is the pump speed \( \omega \). The state equations for this combined 6th order model can now be written in the form:

\[
\begin{bmatrix}
\dot{x}_i(t) \\
\dot{x}_j(t) \\
\dot{x}_s(t) \\
\dot{x}_o(t) \\
\dot{x}_a(t) \\
\dot{x}_u(t)
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{L_i} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{R_j C_j} & \frac{R_o C_o}{R_j C_j} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{R_s C_s} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{R_s C_s} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-R_s}{L_s} & \frac{R_s}{C_s} \\
0 & 0 & 0 & 0 & 0 & \frac{-R_o}{L_o}
\end{bmatrix}
\begin{bmatrix}
x_i(t) \\
x_j(t) \\
x_s(t) \\
x_o(t) \\
x_a(t) \\
x_u(t)
\end{bmatrix}
\]

(9)

The control variable in the above equation is \( u(t) = \alpha \dot{x}_i(t) \).

Using control terminology, this model can be expressed in the standard state-space form:

\[
\dot{x}(t) = A_k(t) x(t) + b u(t), \quad k = I, F, E \]

(10)

where the subscripts \( I, F, \) and \( K \) denote (see Table I) the Isovolumic Relaxation and Contraction phases \( (k=I) \), the Filling phase \( (k=F) \), and the Ejection phase \( (k=E) \). In the above model the matrix \( A_k \) has an interesting property in
that can be decomposed as follows:

\[ A_i = \begin{bmatrix} \hat{A}_i & A_{i2} \\ A_{i1} & A_{i2} \end{bmatrix} \]  

(11)

where the matrices \( A_{i1} \), \( A_{i2} \), and \( A_{i2} \) are 4x1, 2x5, and 2x1 respectively and are the same for \( k = I, F, \) and \( K \). These matrices are given by the expressions:

\[ A_{i1} = \begin{bmatrix} -1 \\ C(t) \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad A_{i2} = \begin{bmatrix} 0 \\ -R_x C_x \\ 0 \\ -R_x C_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ A_{i2} = \begin{bmatrix} 0 & -1 & L_x & L_x & -R_x \\ 0 & 0 & 1 & 0 & -R_x \\ 1 & 0 & 0 & -1 & 0 \end{bmatrix} \]

The 4x5 matrix \( \hat{A}_i \) is different for each phase and is given according to the following:

a) For the Isovolumic Relaxation and Contraction Phases (\( k = I \)) we have \( x_1 - x_i < 0 \) and \( x_i - x_4 < 0 \) in (9) and

\[ \hat{A}_i(t) = \begin{bmatrix} -\dot{C}(t) & 0 & 0 & 0 \\ C(t) & 0 & -1 & 1 & 0 \\ 0 & R_x C_x & R_x C_x & -1 & 0 \\ 0 & R_x C_x & R_x C_x & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \]  

(12)

b) For the Filling Phase (\( k = F \)) we have \( x_1 - x_i \geq 0 \) and \( x_i - x_4 < 0 \) in (9) and

\[ \hat{A}_i(t) = \begin{bmatrix} -(1 + R_x \dot{C}(t)) & 1 & 0 & 0 & 0 \\ R_x C(t) & -R_x C(t) & 0 & 0 & 0 \\ 0 & R_x C_x & R_x C_x & R_x C_x & 0 \\ 0 & 0 & R_x C_x & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \]  

(13)

c) For the Ejection Phase (\( k = E \)) we have \( x_1 - x_i < 0 \) and \( x_i - x_4 \geq 0 \) in (9) and

\[ \hat{A}_i(t) = \begin{bmatrix} -(R_x \dot{C}(t) + 1) & 0 & 0 & 1 & 0 \\ R_x C(t) & 0 & -1 & 1 & 0 \\ 0 & R_x C_x & R_x C_x & 0 & -1 \\ 0 & R_x C_x & R_x C_x & 0 & 1 \\ 1 & 0 & -1 & 0 & R_x C_x \end{bmatrix} \]  

(14)

V. CHALLENGES IN THE DEVELOPMENT OF A FEEDBACK CONTROLLER

As mentioned earlier, the only available mechanism to control the RLVAD is the pump rotational speed \( \omega \). The speed must be adjusted to meet the patient needs for cardiac output while at the same time insuring that suction does not occur by excessive pumping. Achieving these two objectives has been a major challenge for RLVAD developers for over 15 years and is recognized as one of the most serious limitations of this technology at the present time.

The design of a feedback controller based on the model developed in this paper is very much related to our ability to continuously measure, or estimate, in real time the patient’s hemodynamic variables (\( x_i \) through \( x_4 \) in the model). The patient’s continuously varying levels of activities are exhibited by possibly wide variations in the Systemic Vascular Resistance (\( R_v \) in (1) and (9)) which in turn affects all six state variables. Unfortunately, at present, current technology for implantable sensors that allow for continuously measuring the patient’s hemodynamic variables does not exist and will probably need many years before it can be developed. The pump flow variable \( x_6 \) appears to be the only variable that can be externally measured in real time. This may be done, for example, by using standard ultrasonic flow transducers that can be clamped on one of the pump cannulae. With this single variable, the control problem essentially reduces to designing a feedback controller based on incomplete state measurements. Given that the system is nonlinear and time varying, this remains a challenge from both the theoretical and practical considerations. An attempt to develop a very simple feedback controller based on the slope of the lower envelope of the pump flow signal has recently been made [14] but its practical usefulness in an in vivo experiment has not yet been tested. The approach of estimating the hemodynamic variables so as to implement a full state feedback controller has also been considered [15-17] but with limited success. This is largely due to the fact that this approach involves first estimating the patient’s systemic vascular parameters. Most methods developed for estimating these parameters suffer from the problem that the estimates obtained are discrete and valid only over parts of the cardiac cycle. A method for simultaneously estimating the parameters and the hemodynamic variables has been developed in [13] based on an Extended Kalman Filter approach, however, the robustness of the method with respect to parameter initialization remains questionable and its usefulness is limited due to uncertainties related to convergence of the algorithm to the correct estimates. In summary, the design of a feedback controller for the RLVAD remains a very challenging open problem.

Since the pump flow signal appears to be the only signal that is directly measurable, it is important to examine how
this signal is affected by the pump speed as it changes over its allowable safe range. Figure 1 shows a plot of this signal when our model is exited with a pump speed starting at 12,000 rpm and increasing linearly according to \( \omega = 12,000 + 100t \) and reaching a speed of 18,000 rpm after 60s. It is clear from this figure that the lower envelope of this signal seems to track the increasing pump speed up to a point when a breakdown occurs and then exhibiting a sudden drop when the speed is increased beyond the breakpoint. In Figure 1, this breakdown occurs at \( t = 35s \) which corresponds to a speed of 15,500 rpm and is indicative of the onset of suction in the model.

The same phenomenon has been observed in an in-vivo animal study\(^1\) in which the WorldHeart\(^2\) RLVAD was used. In this study the pump speed was increased linearly from 8,000 rpm according to \( \omega = 8,000 + (100/3)t \) reaching a speed of 14,000 rpm after 180s. The corresponding measured pump flow signal is shown in Figure 2. Again, in this case, a similar breakdown in the lower envelope of the signal is observed at \( t = 136s \) which corresponds to a speed 12,500 rpm and indicating the onset of suction in the animal’s left ventricle. It is interesting to note that in both the model and the in-vivo experiment, the onset of suction appears to be characterized by a sudden large drop in the lower envelope of the pump flow signal. Furthermore, when the pump is in suction for some time, a noticeable change in the characteristics of the signature of the pump flow signal is observed. This is clearly seen in Figure 3 in which the details of the in-vivo data of Figure 2 are shown over two 5s intervals of time from 60s to 64s before the occurrence of suction and from 135s to 139s during suction. The change in the frequency characteristics of the signal from a narrow-band signal to a relatively broad-band signal is very noticeable. The breakpoint in the lower envelope of the pump flow signal and the changes in its frequency characteristics before and after suction represent opportunities that can be exploited in the development of suction detection algorithms. We should note that suction detection has been a long standing problem that has been studied ever since the RLVAD was introduced. Many algorithms have been proposed based on numerous other criteria and indices [18-21]. A successful attempt to develop a suction detection algorithm based on the frequency characteristics of the pump flow signal has recently been reported in [22, 23].

A block diagram of a feedback controller based only on pump flow signal is illustrated in Fig 4. The controller consists of two basic functions. The first, labeled “Extract Minimum” will track the minimum value \( x_m \) of the pump flow signal within each cardiac cycle. Clearly the locations of these minima are synchronized within all cardiac cycles.

The second, labeled “Calculate Slope”, will estimate the slope of the envelope of minimum values. This slope is estimated by fitting, in a least-squares sense, a straight line to a moving window consisting of past minimum values of the pump flow signal. A window consisting of a large number of past values is effective in noisy pump flow data because of the noise filtering effect of the least-squares fitting method. The slope of the straight line is computed and denoted by \( dx_m/dt \), where \( x_m \) denotes the envelope of the minimum pump flow signal \( x_m \). The third function, labeled “Speed Update” provides a mechanism for adjusting the pump speed based on the calculated slope until the maximum of the minimum pump flow signal is reached.

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\(^{1}\) Authorized according to WorldHeart, Inc. IRB DO 01-06002
\(^{2}\) WorldHeart, Inc., formerly MedQuest, Inc., Salt Lake City, UT
The update rule that we use is as follows:

$$\omega(k+1) = \omega(k) + c \left( \frac{dx}{dt} \right)_{t=k}\Delta t$$  \hspace{1cm} (15)

where $k$ is the update sample, $\Delta t = 0.005$ s and $c$ is a constant gain parameter which controls the rate of speed adjustment (note that $c$ should be less than the slew rate of the pump speed). Depending on the patient, a small value of $c$ will result in a slow adjustment process, while a large value of $c$ will increase the rate of adjustment at the risk of overshoot and driving the pump into suction. We should note that the size of the window in estimating the slope and the value of $c$ in the speed update mechanism are typically chosen by a clinician and may vary depending on quality of the pump flow data and the condition of the patient. Initially, the pump speed is started below suction when $\frac{dx}{dt} > 0$ and then increased according to (15) until $\frac{dx}{dt}$ becomes close to zero, at which point it will be maintained below the suction speed. Preliminary results on the performance of this controller can be found in [14].

VII. CONCLUSION

In this paper, a $6^{th}$ order state-space model of a Rotary Left Ventricular Assist Device connected to a cardiovascular system is presented. The only control variable in the model is the rotational speed of the pump. The challenges in using this model to design a feedback controller for the RLVAD rotational speed are discussed. The characteristics of the pump flow signal - which is the only state variable that can be directly measured - when the pump is operating normally with no suction and when it is operating in suction are also described based on data obtained from the model as well as from an in-vivo animal experiment. Possible approaches for exploiting these characteristics in the development of suction detection algorithms are also discussed.

REFERENCES