Closed-loop System Identification with New Sensors

Jan Bendtsen  Klaus Trangbaek  Jakob Stoustrup

Abstract—This paper deals with system identification of new system dynamics revealed by online introduction of new sensors in existing multi-variable linear control systems. The so-called "Hansen Scheme" utilises dual Youla-Kucera parameterisation of all systems stabilised by a given linear controller to transform closed-loop system identification problems into open-loop-like problems. We show that this scheme can be reformulated to accommodate extra sensors in a nice way. The approach is illustrated on a simple simulation example.

I. INTRODUCTION

The life-time of a controller for an embedded control system might be just as long as the life-time of the embedded system itself, especially if the control system has been designed to handle aging components (e.g. by adaptive control methods, see for instance [1]) and/or faulty components (e.g. by fault tolerant control methods, see for instance [2], [3]).

In contrast, the life-time of a high level control system for a complex, industrial process is typically very short, as industrial control processes are often characterized by constant, structural modifications. The short life-time of high level process control systems is often a limiting factor for companies, when they have to decide whether to invest in advanced control design projects. Obviously, the payback time has to be shorter than the controller life-time, but this precondition might not be satisfied for complicated processes that are subject to frequent, structural changes.

Furthermore, general technological progress may make new sensor and/or actuator hardware cheaper and more attractive than at the time of the original design, and re-structuring (adding) the hardware in the loop may yield performance improvements that were deemed infeasible or too expensive at design time.

The problem here is that a vast majority of control design methodologies are monolithic in the sense that they embark from a model of an uncontrolled (open-loop) system and outputs a full, multivariable control system, which does not exploit any knowledge or functionality from previous designs. On the other hand, when new sensor and/or actuator hardware becomes available for use in a control system, it is often desirable to retain the existing control laws and apply the new control capabilities in a gradual online fashion rather than decommissioning the entire existing control system and replacing it with the new system, see for example [4], [5], [6], [7].

In order to utilise the new hardware, some sort of system identification will typically be required in order to design controllers with good stability and performance properties. Furthermore, since large-scale plants are typically not permitted to operate in open loop – the plant might for instance not operate acceptably without a controller forcing it to stay within the relevant operating range – closed-loop identification of the plant is usually necessary. However, closed-loop identification tends to be much more difficult than open-loop identification. It will therefore be convenient to adopt the system identification to the control strategy in some way, preferably even to the point of obtaining 'open-loop-like' qualities. The so-called Hansen scheme ([8], [9], [10], [11], [12]) employs the Dual Youla-Kucera parameterisation ([13], [14]) of all linear plants stabilised by a given controller to transform the closed-loop identification problem into an open-loop-like problem. See also [15] and the references therein.

In this paper, we show how the Hansen scheme can be reformulated to deal with new measurements that become available during online operation. The original plant is embedded in a larger system, in which hitherto unobservable dynamics is revealed by letting a new sensor come online. We show how the identification of the newly revealed dynamics is equivalent to the identification of a surprisingly simple dual Youla-Kucera parameter.

The outline of the rest of the paper is as follows. Section II first provides an overview of the Youla-Kucera parameterisation and the Hansen-scheme closed-loop system identification framework. Section III then presents the main contribution of this work, an extension of the Hansen scheme to accommodate new sensor measurements. Section IV then illustrates the usefulness of the scheme, and finally Section V sums up the conclusions of the work.

II. DUAL YOULA-KUCERA PARAMETERISATION

In this section we provide some preliminaries, which will be employed in the subsequent Section III. All results in this section are equally valid in continuous and discrete time. Our notation is standard, as established in e.g., [16].

A. Basic Parameterisation

Consider a LTI system $G$ mapping a set of inputs to a set of outputs:

$$ y = Gu $$

where $y \in \mathbb{R}^p$ is the measurement vector and $u \in \mathbb{R}^m$ is the input vector. If $G$ is stabilisable and detectable, it can be stabilised by some appropriate feedback controller, for instance an observer-based controller (see e.g. [16]).

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The authors are with the Department of Electronic Systems, Automation and Control, Aalborg University, Denmark {dimon, ktr, jakob}@es.aau.dk
Any proper $G$ can be written as a right, respectively left, coprime factorisation of the form:

$$ G = NM^{-1} = \tilde{M}^{-1}\tilde{N} $$

(2)

with $N, M, \tilde{M}, \tilde{N} \in \mathcal{RH}_\infty$. Correspondingly, a controller $K$ that stabilises $G$ can be factorised as

$$ K = UV^{-1} = \tilde{V}^{-1}\tilde{U} $$

(3)

where $U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$. These coprime factorisations can be chosen to satisfy the double Bezout identity

$$
\begin{bmatrix}
\dot{V} & -\tilde{U} \\
-\tilde{N} & \tilde{M}
\end{bmatrix}
\begin{bmatrix}
M & U \\
N & V
\end{bmatrix}
= 
\begin{bmatrix}
M & U \\
N & V
\end{bmatrix}
\begin{bmatrix}
\tilde{V} & -\tilde{U} \\
-\tilde{N} & \tilde{M}
\end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}
$$

(4)

For example, if $G$ has the state space realisation

$$ G = \begin{bmatrix} A & B \\ C & D \end{bmatrix} $$

(5)

with $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n}$ and $D \in \mathbb{R}^{p \times m}$ being constant matrices and $K$ an observer-based feedback controller of the form

$$ K = \begin{bmatrix} A + BF + LC + LD F & -L \\ F & 0 \end{bmatrix} $$

(6)

with $F \in \mathbb{R}^{m \times n}$ and $L \in \mathbb{R}^{n \times p}$ chosen such that the matrices $A + LC$ and $A + BF$ are stable, the double Bezout identity is satisfied by choosing the factorisation

$$
\begin{bmatrix} M & U \\ N & V \end{bmatrix} = 
\begin{bmatrix} A + BF & B & -L \\ F & I & 0 \\ C + DF & D & I \end{bmatrix}
$$

(7)

$$
\begin{bmatrix} \dot{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = 
\begin{bmatrix} A + LC & -(B + LD) & L \\ F & I & 0 \\ C & I & 0 \end{bmatrix}
$$

(8)

It is now possible to characterise all systems stabilised by a fixed controller by means of a so-called dual Youla-Kucera parameter $S \in \mathcal{RH}_\infty$. Let some system $G$, factorised as in (2), be stabilised by a feedback controller $K$. Then the set of all systems stabilised by $K$ is given by

$$ \{ G : G(S) = (N + VS)(M + US)^{-1} = (\hat{M} + S\hat{U})^{-1}(\hat{N} + S\hat{V}), \quad S \in \mathcal{RH}_\infty \}. $$

B. The Hansen Scheme

To motivate the usage of the Youla-Kucera parameterisation in system identification, we first consider normal open-loop identification of the system $G$. Some input $u$ is applied to the system, and corresponding output measurements $y$ affected by noise $n_y$ are obtained. These measurements are related through

$$ y = Gu + n_y $$

and an unbiased estimate of $G$ can be obtained if $u$ and $n_y$ are uncorrelated. Unfortunately, in a closed-loop setting $u$ is not uncorrelated with $n_y$, since the noise is fed back through the controller. To alleviate this, we employ the dual Youla-Kucera factorisation to recast the closed-loop system identification problem into an ‘open-loop-like’ problem ([11]).

Assume that a controller $K$ stabilises the plant we wish to identify, and that some nominal plant estimate $G$ is known, factorised as in (3) and (2), respectively. Then the set of all plants stabilised by $K$ can be represented as shown in Figure 1. Here, $n' = (\tilde{M} + S\hat{U})n_y$ is the measurement noise that would normally affect the measurements $y$, relocated in the block diagram to affect the output of the Youla-Kucera parameter instead, and $r_1$ and $r_2$ are external excitation signals.

![Fig. 1. Dual Youla-Kucera parameterisation used for closed-loop system identification](image)

By manipulating the block diagram and using (4), it is possible to check that $y = G(S)u + n_y$. From Figure 1 it is then possible to deduce (see e.g., [15], but please note that here we are using positive feedback control) that

$$
\begin{align*}
\zeta &= \tilde{U}r_1 + \tilde{V}r_2 \\
z &= \hat{M}y - \hat{N}u
\end{align*}
$$

(9) (10)

and, obviously, $z = Sz + n'$, $\zeta$ and $z$ are available from filtered measurements. Furthermore, if $n_y$ is independent of $r_1$ and $r_2$, then $\zeta$ is independent of $n'$ as well. Also, $S$ is known to be stable due to the dual Youla-Kucera theory (cf. the previous section). Thus, it can be seen that although $u$ and $y$ are measured in closed-loop, the identification of $S$ becomes equivalent to an open-loop identification problem.

III. New Sensor Measurement

We now turn to the problem of identifying new dynamics revealed by a new sensor plugged into an existing control system, as mentioned in the Introduction. As the sensor is plugged into the system, it reveals new information about the plant, including (possibly) extra dynamics that has been unobservable from the existing measurements. Preferably, we wish to identify only the new information revealed by the plugged-in sensor, possibly including dynamics introduced by the sensor itself.

Thus, we assume that a nominal model $(A, B, C, D)$ of the ‘old’ plant dynamics has been found, through first-principles modelling and/or system identification. This model will be denoted $G_0$ and has the state space realisation (5). $G_0$ is stabilised by a controller $K_0$ of the form (6). Now, plugging in a new sensor provides access to a new measurement $y_1 \in \mathbb{R}^p$, which is affected by the internal (unmeasurable) plant states $x$ as well as the control input $u$, as depicted in Figure 2. The new measurements are affected by noise $n_{y_1}'$, which is not necessarily uncorrelated with $n_y$.  

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Fig. 2. Plugging in a new sensor reveals the hitherto unobservable subsystem $G^*$. $x$ denotes the internal state vector of $G_0$.

The plant-controller interconnection in Figure 2 can be represented using the following state space representation. First, we embed $G_0$ in the augmented plant model

$$
G' = \begin{bmatrix} G_0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}
$$

(11)

which represents the system before introducing the new sensor, and when the new sensor is brought on-line we introduce the new plant

$$
G^* = \begin{bmatrix} G_0 \\ G_1 \end{bmatrix} = \begin{bmatrix} A & 0 & B \\ A_{21} & A_{22} & B_2 \\ C & 0 & D \end{bmatrix}
$$

(12)

where $A_{21}, A_{22}, B_2, D_{21}, D_{22}$ and $D_2$ are unknown matrices of appropriate dimensions representing $G_1$ and the couplings from $G_0$. Note that $A_{22}$ must necessarily be stable, since the closed loop as a whole is stable.

Next, we augment the controller as

$$
K^* = \begin{bmatrix} K \\ 0 \end{bmatrix} = \begin{bmatrix} A + BF + LC + LDF & -L & 0 \\ F \end{bmatrix}
$$

(13)

and it is easy to check that closing the loop with this controller and either $G'$ or $G^*$ will yield the same transfer function from $r_1$ and $r_2$ to $y_0$ as before the sensor was introduced.

The augmented coprime factorizations corresponding to (7)–(8) then become

$$
\begin{bmatrix} M' & U' \\ N' & V' \end{bmatrix} = \begin{bmatrix} A + BF & 0 & B & -L & 0 \\ 0 & I & 0 & 0 & 0 \\ -(B + LD) & -L & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ C & D & I & F & 0 \end{bmatrix}
$$

(14)

$$
\begin{bmatrix} \hat{V} & -\hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix} = \begin{bmatrix} A + LC & -(B + LD) & -L & 0 \\ 0 & I & 0 & 0 \\ -C + DF & I & 0 & 0 \\ -D & I & 0 & 0 \end{bmatrix}
$$

(15)

and hence $G(S) = G^*$. Thus, we may proceed to compute an expression for $S$ as follows:

$$
S = \hat{M}^*(G^* - G')M'
$$

(16)

for the old system and

$$
\begin{bmatrix} M^* & U^* \\ N^* & V^* \end{bmatrix} = \begin{bmatrix} A+B & 0 & B & -L & 0 \\ 0 & I & 0 & 0 \\ -(B+LD) & -L & 0 & 0 \\ I & 0 & 0 & 0 \\ C & D & I & F & 0 \end{bmatrix}
$$

for the system with the new sensor. In each of the above expressions, the dashed lines indicate how the system matrices on the right-hand side should be partitioned to correspond to the system blocks on the left-hand side.

Embedding this factorisation in the Hansen framework introduced in Section II, we can now show the following result.

**Theorem 1:** Consider the augmented plant (12) in closed loop with (13). A dual Youla-Kucera parameter system that allows open-loop-like identification of the new sensor dynamics $G_1$ is given by

$$
S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{22} & B_2 \\ C_{22} & D_2 \end{bmatrix}
$$

(18)

**Proof:** We take the starting point in (9) and first point out that the augmented system can be written as a function of the old system by means of a particular dual Youla-Kucera parameter $S = \hat{M}^*(G^* - G')M'$. To see this, insert this expression in (9) and use the factorisations $G^* = (\hat{M}^*)^{-1}N^*$ and $G_0 = N'(M')^{-1}$ to obtain

$$
G(S) = (N' + V'\hat{M}^*(G^* - G')M') \times (M' + U'\hat{M}^*(G^* - G')M')^{-1}
$$

$$
= (N' + V'\hat{N}^*(M' - \hat{M}N')) \times (M' + U'\hat{N}^*(M' - \hat{M}N'))^{-1}
$$

Here we use the Bezout identities $N'\hat{V}' - V'\hat{N}' = 0, V'\hat{M}' - N'\hat{U}' = I$ and $\hat{V}'M' - \hat{U}'N' = I$ to see that

$$
N' + V'(\hat{N}^*M' - \hat{M}^*N') = \hat{N}^*
$$

Similar, from the Bezout identities $N^*\hat{U}' - U'\hat{M}' = 0, M^*\hat{V}' - U'\hat{N}' = I$ and $\hat{V}'M' - \hat{U}'N' = I$, we see that

$$
M' + U'(\hat{N}^*M' - \hat{M}^*N') = \hat{M}^*
$$

and hence $G(S) = G^*$. Thus, we may proceed to compute an expression for $S$ as follows:

$$
S = \hat{M}^*(G^* - G')M'
$$

(16)
Now, by looking at $\tilde{M}^*$ in (17), it is recognised that $\tilde{M}^*$ receives no input via its first input channel, and its last input channel is simply an identity matrix. Thus we have

$$S = \tilde{M}^* \begin{bmatrix} A & 0 & B \\ A_{21} & A_{22} & B_2 \\ 0 & 0 & 0 \end{bmatrix} M' \begin{bmatrix} A + BF & B \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A + BF & B \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{21} & A_{22} & B_2 \\ C_{21} & C_{22} & D_2 \end{bmatrix}$$

By a simple state transformation, $S_1$ can be reduced to

$$S_1 = \begin{bmatrix} A + BF & 0 & B \\ \frac{A_{21} + B_2 F}{C_{21} + D_2 F} & A_{22} & B_2 \end{bmatrix} \begin{bmatrix} A + BF & B \\ 0 & 0 \end{bmatrix}$$

Here, we introduce the filter

$$\Phi = \begin{bmatrix} A + BF & B \\ F & I \\ 0 \end{bmatrix}$$

This filter takes $\zeta$ defined in (9) as input and yields the output

$$\begin{bmatrix} F\xi + \zeta \\ \xi \end{bmatrix} = \Phi \zeta$$

where $\xi$ is the state vector of $\Phi$. This allows us to write $S$ as the factorisation

$$S = \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} \Phi,$$

where

$$\Gamma = \begin{bmatrix} A_{22} & B_2 & A_{21} \\ C_{22} & D_2 & C_{21} \end{bmatrix}. \quad (19)$$

We thus arrive at our main contribution, the setup for the modified Hansen scheme depicted in Figure 3.

The procedure is straightforward: first generate a data sequence by adding excitation signals through $r_1$ and $r_2$, then compute the signals

$$\begin{bmatrix} F\xi + \zeta \\ \xi \end{bmatrix} = \Phi(\hat{U}'r_1 + \hat{V}'r_2)$$

and $z = \hat{M}'y - \hat{N}'u$ by filtering. $\Gamma$ can now be obtained by a standard open loop identification method. Once $\Gamma$ has been found, the extension parameters in (12) are given directly by (19). Alternatively, the plant transfer function can be computed by inserting $\Gamma$ in the loop in Figure 3.

It is worth noting that this setup carries over the nice non-correlation qualities of the original Hansen scheme, whereas identifying the transfer function from $u$ to $y_1$ directly from closed loop data can cause bias problems, especially if the noise is correlated with the noise affecting the control system.

Remark 1 The signals generated by $\Phi$ can have a strong correlation between the elements. This makes it difficult to identify the matrices in $\Gamma$ independently, although the resulting transfer function from $u$ to $y_1$ will usually be correct. This issue is inherent to the identification problem itself, and the only solution seems to be to acquire more data. $\triangleright$

Remark 2 It may be slightly surprising that the setup depicted in Figure 3 still shows the 'old' factorisation, i.e., the factors $M', N', U', V'$ rather than $M^*, N^*, U^*, V^*$. However, this is due to the connection between the old and the augmented system, i.e., $G^* = G'(S)$, where, essentially, all the new dynamics is isolated in the $S_1$-parameter. $\triangleright$

IV. SIMULATION EXAMPLE

We now illustrate the feasibility of the identification scheme by a numerical example. We consider the discrete-time system

$$x_{k+1} = Ax_k + Bu_k,$$

$$y_k = Cx_k + n_y$$

where

$$A = \begin{bmatrix} 0.7 & 0.7 & 0.4 \\ 0.65 & 0.3 & 0.09 \\ -0.8 & 0.27 & 0.94 \end{bmatrix}, B = \begin{bmatrix} 0.07 \\ 0.03 \\ 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

and $n_y$ is a Gaussian white noise signal with variance 0.0050. The system is open-loop unstable (with poles in $z = -0.1084, z = 1.0242 \pm 0.4047j$), so it is not possible to disconnect the controller to obtain good data for identification. This system has been chosen because it poses a relatively difficult identification task when we are only allowed to impose mild excitation signals.

A stabilizing observer-based controller for the system has been found using standard optimal control design methods. The feedback and observer gains were found to

$$F = [-1.5 -2.6 -2.7]$$

and $L = [-0.36 -0.27 0.23]$ respectively.
The system is then augmented with a new sensor, which has its own dynamics, feedthrough from the control input, etc. It is described by the following augmented system matrices specified in (12):

\[
A_{21} = \begin{bmatrix} 0 & 0.1 & 0.3 \end{bmatrix}, \quad A_{22} = 0.9, \quad B_2 = 0.094 \\
C_{21} = \begin{bmatrix} 0 & 0 & -0.4 \end{bmatrix}, \quad C_{22} = 1.2, \quad D_2 = 0.2
\]

Furthermore, it is affected by Gaussian noise \(n_y\), which is correlated with \(ny\) as given by the covariance matrix

\[
E\left( \begin{bmatrix} n_y \\ n_y^* \end{bmatrix} \begin{bmatrix} n_y \\ n_y^* \end{bmatrix}^T \right) = \begin{bmatrix} 0.0025 & 0.0025 \\ 0.0025 & 0.005 \end{bmatrix}
\]

All the information about the new sensor is considered unknown at the point where the sensor is brought on-line.

The true parameters inserted). As can be seen, there is very good agreement between the two, especially for frequencies up to 1 rad/sec.

For comparison purposes, we also perform a ‘direct’ system identification, i.e. identifying a transfer function from \(u\) to \(y_1\) using these signals directly. We then compute the transfer function from \(u\) to \(y_1\) for the true system, using the S-parameter found above, and the new ‘direct’ identified model. Bode plots of the results are shown in Figure 6, from which it is very apparent that the Hansen-scheme-based model is much closer to the real system than the ‘direct’ identified model. This is most likely because \(u\) and \(y_1\) are correlated through the old plant-controller loop, and due to the fact that the noise signals are correlated.

We now apply pseudo-random excitation signals \(r_1\) and \(r_2\) to the control loop as indicated in Figure 3. Figure 4 shows plots of the excitation signals, the control input, as well as the ‘old’ output \(y_0\) and the new sensor output \(y_1\). As can be seen from the figure, the excitation signals are of small amplitude compared to \(u\) and \(y\) and are mainly in the low end of the frequency spectrum, i.e., they do not interfere aggressively with the closed-loop operation of the plant.

Next, we filter \(r_1\), \(r_2\), \(u\) and \(y_1\) as given in the previous section and use the filtered signals \(\zeta\) and \(z\) for system identification of the S parameter system. Figure 5 shows a Bode plot of the identified system together with the corresponding Bode plot of the true S (computed using equation (1), with

Fig. 4. Data sequences for identification. From the top: excitation sequences \(r_1\) and \(r_2\), control signal \(u\), old output \(y_0\) and new output \(y_1\).

Fig. 5. Bode plot of \(S\). Solid: True. Dashed: Identified by Hansen scheme.

Fig. 6. Bode plot of transfer function from \(u\) to \(y_1\). Solid: Real. Dashed: Identified directly using these signals directly. We then compute the transfer function from \(u\) to \(y_1\) for the true system, using the S-parameter found above, and the new ‘direct’ identified model. Bode plots of the results are shown in Figure 6, from which it is very apparent that the Hansen-scheme-based model is much closer to the real system than the ‘direct’ identified model. This is most likely because \(u\) and \(y_1\) are correlated through the old plant-controller loop, and due to the fact that the noise signals are correlated.
Finally we take a look at the identified $\Gamma$ in Figure 7, comparing it to the real $\Gamma$ given by (19). As seen, the fit is very poor, which is not surprising, see Remark 1. Additional tests indicate that using a much larger data sequence will make the estimated $\Gamma$ converge to the correct one, but as seen above, this is not necessary if we are only interested in getting the correct transfer function to the new output.

In conclusion, we have demonstrated that the proposed scheme can identify new dynamics for relatively difficult unstable systems with only a small amount of excitation.

V. DISCUSSION

Closed-loop system identification is much more difficult than open-loop system identification, due to the fact that inputs and noise cannot be considered uncorrelated because of the controller feedback. The so-called Hansen scheme is a factorisation-based approach to alleviate some of these difficulties by taking the starting point in a 'nominal' system model and identifying the unknown dynamics by means of a dual Youla-Kucera parameter in an essentially open-loop setting.

This paper showed how the Hansen scheme can be extended to deal with new measurements that become available during online operation. The original plant is embedded in a larger system, in which hitherto unobservable dynamics is revealed by letting a new sensor come online. It was then shown how the identification of the newly revealed dynamics is equivalent to the identification of a surprisingly simple dual Youla-Kucera parameter.

The novel scheme was shown to be superior to simple, 'direct' system identification of the new dynamics in a simple example. One might argue that similar results can be obtained by generating signals through a simulation of the closed loop with only excitation signals as inputs. However, formulating the problem in the Youla-Kucera-Hansen framework paves the way for controller redesign and transfer along the lines demonstrated in [17].

REFERENCES