Adaptive Observer Design for the Bottomhole Pressure of a Managed Pressure Drilling System

Øyvind Nistad Stamnes, Jing Zhou, Glenn-Ole Kaasa and Ole Morten Aamo

Abstract—In this paper a reduced order observer that adapts to unknown friction and density, and estimates the bottomhole pressure in a well during drilling, is presented. The design is based on a newly developed third order nonlinear model with a nonlinear output equation containing a product between an unknown parameter and unmeasured state. Based on a Lyapunov approach the pressure estimate is shown to converge to the true pressure under reasonable conditions. Application of the observer to real data from a North Sea oil well demonstrates promising behaviour.

Index Terms—Drilling, nonlinear observer, adaptive observer, pressure estimation.

I. INTRODUCTION

As an introduction to drilling consider the drill rig set-up illustrated in Fig. 1. The figure illustrates a jacket platform performing offshore drilling. At the top of the derrick the drill string is attached to the topdrive which is a motor that turns the drill string. The drill string can move up and down inside the derrick as the topdrive is attached to a hook that can be lowered or raised. As the drilling progresses the top of the drill string sinks towards the drill floor. After approximately 27m a new stand of drill pipe is connected to the top and drilling resumes. This procedure is referred to as a pipe connection. For a typical rate of penetration of 15 m/hr a pipe connection is performed roughly every two hours.

During drilling, down hole cuttings need to be transported out of the bore hole. This is done by using a mud circulation system. On board the rig, tanks filled with drilling mud feed the main mud pump which pumps the drilling fluid through the topdrive and into the drill string. The mud then flows down through the bit and up through the annulus carrying the cuttings along before the flow exits through a choke. After exiting, the fluid is recycled and returned to the mud tanks.

The example illustrated in Fig. 1 has a rotating control device which seals off the annulus from the outside while a choke controls the flow of mud out from the annulus.

The main reason for pressure control is to maintain the annulus pressure profile within its margins, i.e., above the pore pressure of the reservoir or the collapse pressure of the bore hole, and below the fracturing pressure of the bore hole. Another important reason for pressure control is to prevent uncontrolled reservoir influx which in the worst case scenario can lead to a surface blowout with large financial losses, environmental damage and possible loss of lives.

The pressure in the annulus is mainly affected by the hydrostatic weight and the pressure due to friction losses [1]. In addition, if the annulus is closed off, the pressure at the top of the annulus induced by choking will significantly affect the pressure in the well.

There are several operational procedures that affect the pressure in the annulus. Pipe connection affects the pressure as the main pump must be disconnected to attach a new section of drill pipe, this leads to zero flow and loss of pressure due to friction. Moving the drill string all the way in/out of the well (tripping) changes the volume in the annulus. Tripping out pipe causes reduced pressure in the annulus, and tripping in pipe creates a surge in the pressure. Similar effects can be experienced due to wave-induced motion (heave) when drilling from a floater.

Fig. 1. Offshore drilling from a jacket platform. Drill mud flows from the main pump through the drill string, drill bit and out through the choke. The mud transports cuttings out of the wellbore and helps to maintain the desired pressure in the borehole.

A. Pressure Control

As described in the previous section there is a demand for accurate control of the annulus pressure. As a response to these demands a fairly new (for offshore drilling) technology for pressure control has emerged [2]. It is named Managed Pressure Drilling (MPD) and is defined by the IADC Underbalanced Operations Committee as: "Managed
Pressure Drilling is an adaptive drilling process used to precisely control the annular pressure profile throughout the well bore. The objectives are to ascertain the down hole pressure environment limits and to manage the annular hydraulic pressure profile accordingly.” [3].

In many cases the bottomhole pressure (the pressure at the bit) is used as the variable to control [4], [5], [6], [7]. The bottomhole pressure is measured, but the signal is usually transmitted by mud-pulse telemetry which is powered by a mud flow turbine. It is therefore hampered with slow sampling and no signal when the circulation is low, e.g., during pipe connection procedures. Since the measurement is unreliable the pressure needs to be estimated, which is non-trivial due to uncertainties in friction and density.

B. Pressure Estimation

Some existing pressure estimation schemes are found in the literature. The multiphase flow dynamics of a well can be described fairly accurately by a set of partial differential equations derived from mass balance equations and a simplified momentum balance known as the drift-flux formulation [9], [10]. The PDEs can be discretized and implemented for simulation as as a large set of ordinary differential equations that can be used to predict the pressure in the well provided all parameters are known and inputs (such as pump flows and choke flows) are measured and fed into the simulator. Existing schemes like these do not use the estimation error to adjust the future estimate and hence they are non-robust to modeling errors. In [11] a new MPD concept which uses a modified version of OLGA 2000 to provide an estimate of the pressure profile in the annulus is presented. OLGA 2000 is a powerful multiphase flow simulator developed for the petroleum industry [12]. The robustness of complex estimation schemes like these in conjunction with a control system is hard to analyze in a rigid manner. For these simulations, verification by extensive Monte Carlo simulations or trials is the only method to guarantee proper functionality.

The complexity of such schemes is increased by the fact that many of the parameters in such models are uncertain/unknown and possibly slowly changing, which implies that they would need to be tuned as operating conditions change. This tuning can be done by an experienced operator or by using automatic tuning methods such as parameter estimation algorithms. In [13] an unscented Kalman filter is used to update the friction estimate in both the drill string and the annulus. The scheme uses a measurement of the bottomhole pressure to update the parameters every 30 seconds. Although no formal proofs are shown the estimation scheme shows promising behavior with better estimates of the bottomhole pressure than without the unscented Kalman filter, and fairly accurate estimation of the friction factors.

Previous attempts at using low order models for control and estimation of the bottomhole pressure can be found in [14] and [5]. In [14] nonlinear model predictive control (NMPC) was used together with an unscented Kalman filter to control the bottomhole pressure. A third order nonlinear model was used as the basis for the control and estimation

\[
\frac{V_d}{\beta_d} \dot{p}_p = q_{pump} - q_{bit} \\
\frac{V_a}{\beta_a} \dot{p}_c = q_{bit} + q_{back} - q_{choke} + q_{res} - V_a,
\]

where \( p_p \) is the pump pressure and \( p_c \) is the choke pressure. \( V_d \) is the volume in the drill string, \( V_a \) is the volume in the annulus. \( \beta_d \) and \( \beta_a \) are the bulk moduli of the fluid in the drill string and the annulus respectively. \( q_{pump} \) is the volume flow through the mud pump. \( q_{bit} \) is the volume flow through
the bit. $q_{\text{back}}$ is the flow through the back pressure pump, $q_{\text{choke}}$ is the flow through the choke and $q_{\text{res}}$ is the influx from the reservoir.

The volume flow dynamics is derived from a momentum balance and is governed by

$$M \dot{q}_{\text{bit}} = p_p - p_c - F_d |q_{\text{bit}}|q_{\text{bit}}$$
$$- F_a |q_{\text{bit}} + q_{\text{res}}| (q_{\text{bit}} + q_{\text{res}}) + (\rho_d - \rho_a) g h_{\text{bit}}.$$  (3)

Here $F_d$ and $F_a$ are the friction factors in the drill string and the annulus, respectively, $\rho_d$ and $\rho_a$ are the average densities in the drill string and the annulus, $g = 9.81 \text{ m/s}^2$ and $h_{\text{bit}}$ is the vertical depth of the bit, see Fig 2. Furthermore $M = M_a + M_d$ with

$$M_a = \rho_a \int_0^{l_w} \frac{1}{A_a(x)} dx, \quad M_d = \rho_d \int_0^{L_{DN}} \frac{1}{A_d(x)} dx.$$  (4)

where $l_w$ is the length of the annulus, $L_{DN}$ is the total length of the drill string, and $A_a(x)$ and $A_d(x)$ are the cross sectional areas of the annulus and the drill string, respectively.

The pressure at the bit depends on the choke pressure, pressure due to rate of change in $q_{\text{bit}}$, friction pressure and hydrostatic pressure, and is given as

$$p_{\text{bit}} = p_c + M_a q_{\text{bit}} + F_a |q_{\text{bit}} + q_{\text{res}}| (q_{\text{bit}} + q_{\text{res}}) + \rho_a g h_{\text{bit}}.$$  (5)

Substituting (3) into (5) gives

$$\dot{p}_{\text{bit}} = \frac{M}{M} p_p - \frac{M}{M} p_c + \frac{M}{M} \rho_a + \frac{M}{M} (\rho_d - \rho_a) g h_{\text{bit}}$$
$$+ \left( \frac{M_d}{M} F_a - \frac{M_d}{M} F_d \right) |q_{\text{bit}} + q_{\text{res}}| (q_{\text{bit}} + q_{\text{res}}).$$  (6)

Using the notation $a_1 = \frac{\partial q_{\text{bit}}}{\partial p}, \quad b_1 = \frac{\partial q_{\text{bit}}}{\partial q_{\text{bit}}}, \quad a_2 = \frac{F_a}{M}, \quad a_5 = \beta_\gamma, \quad u_p = \frac{q_{\text{pump}}}{\gamma_{\text{choke}}}, \quad v_3(t) = \frac{V_a(t)}{v_3}, \quad v_5(t) = h_{\text{bit}}(t)$ equations (1) – (3) can be written more compactly as

$$\dot{p}_p = -a_1 q_{\text{bit}} + b_1 u_p$$  (7)
$$\dot{q}_{\text{bit}} = a_2 (p_p - p_c) - \frac{F_a}{M} |q_{\text{bit}}|q_{\text{bit}}$$
$$- \frac{F_a}{M} |q_{\text{bit}} + q_{\text{res}}| (q_{\text{bit}} + q_{\text{res}}) + \frac{(\rho_d - \rho_a) g}{M} v_3$$  (8)
$$\dot{p}_c = \frac{a_5}{v_3} (q_{\text{bit}} + q_{\text{res}} + u + v_2).$$  (9)

III. OBSERVER

As the friction factor in the annulus depends on several uncertain parameters such as viscosity of the fluid, pipe roughness and flow regime, it is assumed unknown. The density in the annulus is also encumbered with uncertainty as the amount of cuttings in the drill mud affects it, and is also assumed unknown. Let the unknown parameters be denoted as

$$\theta_1 = \frac{F_d + F_a}{M} > 0 \Rightarrow F_a = M \theta_1 - F_d$$  (10)
$$\theta_2 = \frac{(\rho_d - \rho_a) g}{M} \Rightarrow \rho_a = \rho_d - \frac{M}{g} \theta_2.$$  (11)

In the choice of the unknown parameter $\theta_2$ certain assumptions have been made. The reason for choosing $\theta_2$ as an unknown is because $\rho_a$ is encumbered with uncertainty. From (4) one can see that $M_a$ is linearly dependent on $\rho_a$, which implies that $M = M_d + M_a$ will also depend on $\rho_a$. Neglecting this dependency can be justified by two observations. Firstly, $M_a$ affects only transients in the flow dynamics, which are fast compared to the dominating pressure dynamics. Secondly, the sensitivity of $M$ w.r.t. changes in $\rho_a$ is small as $M_d$ is greater than $M_a$. Treating $M$ as a known constant considerably reduces observer complexity. We will furthermore assume zero reservoir influx, hence $q_{\text{res}} = 0$. In view of these assumptions, (8) can be simplified to

$$\dot{q}_{\text{bit}} = a_2 (p_p - p_c) - \theta_1 |q_{\text{bit}}|q_{\text{bit}} + \theta_2 v_3,$$  (12)

and (6) can be written as

$$p_{\text{bit}} = p_c + M_a (a_2 (p_p - p_c) - \theta_1 |q_{\text{bit}}|q_{\text{bit}} + \theta_2 v_3)$$
$$+ (M \theta_1 - F_d) |q_{\text{bit}}|q_{\text{bit}} + (\rho_d g - \theta_2) v_3.$$  (13)

The goal for this section is to design an observer that estimates $p_{\text{bit}}$ and adapts to the unknown parameters $\theta_1$ and $\theta_2$. The estimated states and estimated parameters will be denoted with a hat. Before continuing the following assumptions regarding boundedness and knowledge of signals will be made:

**Assumption 1:** All signals in (7) – (8) are bounded $\Leftrightarrow p_p, p_c, q_{\text{bit}}, v_3 \in L_{\infty}$ and $v_3 \in L_{\infty}$.

Considering that the system is stable and $v_3$ is the vertical depth of the well this assumption is mild.

**Assumption 2:** The following signals are assumed known: $p_p, p_c, u_p, v_3, v_3$.

Standard top side measurements include $p_p$ and $p_c$. The pump flow $u_p$ can be estimated accurately by using the known pump speeds ($\omega_p$), the number of pistons ($N_p$) and volume per stroke per piston ($V_p$) according to $u_p = N_p V_p 2\pi \omega_p$. The depth of the bit ($v_3$) and its rate of change ($v_3$), are given indirectly by the known geometry of the well path and the topside measurement of the block (top drive) position.

**Assumption 3:** $\dot{\theta}_1 = \dot{\theta}_2 = 0$.

Both parameters vary slowly therefore the assumption is valid.

A. Error Dynamics

Motivated by [15], define the following change of coordinates

$$\xi = q_{\text{bit}} + l_1 p_p,$$  (14)

where $l_1$ is a feedback gain. From (7) and (12), the dynamics of $\xi$ is

$$\dot{\xi} = -l_1 a_1 q_{\text{bit}} - \theta_1 |q_{\text{bit}}|q_{\text{bit}} + \theta_2 v_3 + a_2 (p_p - p_c) + l_1 b_1 u_p.$$  (15)
An observer for \( q_{\text{bit}} \) is
\[
\dot{\hat{\xi}} = -l_1a_1\hat{q}_{\text{bit}} - \hat{\theta}_1|\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}} + \hat{\theta}_2v_3 \nonumber + a_2(p_p - p_c) + l_1b_1u_p, \tag{16}
\]
\[
\hat{q}_{\text{bit}} = \hat{\xi} - l_1p_p. \tag{17}
\]
Noticing that
\[
\theta_1|q_{\text{bit}}|q_{\text{bit}} - \hat{\theta}_1|\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}} = \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}}) \nonumber + \hat{\theta}_1|\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}}, \tag{18}
\]
and from (14) and (17) that \( \hat{\xi} = \xi - \hat{\xi} = \hat{q}_{\text{bit}} \), the dynamics of the state estimation error becomes
\[
\dot{\hat{\xi}} = -l_1a_1\hat{q}_{\text{bit}} - \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}}) \nonumber - \hat{\theta}_1|\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}} + \hat{\theta}_2v_3. \tag{19}
\]
Let the parameter errors and the regressor be denoted as
\[
\hat{\theta} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix}, \quad \phi(\hat{q}_{\text{bit}}, v_3) = \left[ -|\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}} \right]. \tag{20}
\]
Using (20) and \( \hat{\xi} = \hat{q}_{\text{bit}} \), (19) can be rewritten as
\[
\dot{\hat{\xi}} = -l_1a_1\hat{\xi} - \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}}) + \hat{\theta}^T \phi. \tag{21}
\]

B. Lyapunov Analysis
For the error system \((\xi, \hat{\theta})\) with \( \hat{\xi} \) dynamics described by (21) and \( \hat{\theta} \) dynamics to be found, consider the candidate Lyapunov function
\[
U(\xi, \hat{\theta}) = \frac{1}{2} \xi^T + \frac{1}{2} \hat{\theta}^T \Gamma^{-1} \hat{\theta}, \tag{22}
\]
where \( \Gamma = \Gamma^T > 0 \) is the adaptation gain matrix. Using (21), the time derivative of \( U \) is
\[
\dot{U} = -l_1a_1\xi^2 - \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}})\dot{\xi} + \hat{\theta}^T (\phi \xi + \Gamma^{-1} \dot{\hat{\theta}}). \tag{23}
\]
Choosing the \( \hat{\theta} \) dynamics to be
\[
\dot{\hat{\theta}} = -\Gamma \phi \xi, \tag{24}
\]
gives
\[
\dot{U} = -l_1a_1\xi^2 - \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}})\dot{\xi}. \tag{25}
\]
Since \( \theta_1 > 0 \) and \( \xi = \hat{q}_{\text{bit}} \), it follows that \( \theta_1(|q_{\text{bit}}|q_{\text{bit}} - |\hat{q}_{\text{bit}}|\hat{q}_{\text{bit}}) \geq 0 \), so
\[
\dot{U} \leq -l_1a_1\xi^2. \tag{26}
\]
Since \( a_1 > 0 \), choosing \( l_1 > 0 \) gives \( \dot{U}(\xi, \hat{\theta}) \leq 0 \). Noticing that \( \xi = \hat{\theta} = 0 \) is an equilibrium point for the system defined by (21) and (24), and that the system is locally Lipschitz in \((\xi, \hat{\theta})\), uniformly in \( t \) under Assumption 1, the LaSalle-Yoshizawa Theorem [16] can be invoked to conclude that all solutions to (21) and (24) are uniformly bounded and that
\[
\lim_{t \to \infty} -l_1a_1\xi^2 = 0. \tag{27}
\]
This implies that \( \hat{q}_{\text{bit}} \) and \( \hat{\theta} \) are bounded, and that \( \hat{q}_{\text{bit}} \to q_{\text{bit}} \) as \( t \to \infty \). There is no guarantee that the parameter estimates converge to their true values. The results derived hold for all \((\xi, \hat{\theta}) \in \mathbb{R}^3\).

C. Adaptive Law
In (24) \( \dot{\xi} \) is unknown which implies that the adaptive law \( \dot{\hat{\theta}} = -\hat{\theta} \) cannot be implemented in this form. This problem will be dealt with now. Define
\[
\sigma = \theta + \eta(\hat{q}_{\text{bit}}, v_3), \tag{28}
\]
where \( \eta \) is a function of known/measured signals that is to be designed to assign \( \sigma \) the desired dynamics. Differentiating \( \sigma \) with respect to time (remembering that \( \dot{\hat{\theta}} = 0 \)) gives
\[
\dot{\sigma} = \frac{\partial \eta}{\partial q_{\text{bit}}} \dot{q}_{\text{bit}} + \frac{\partial \eta}{\partial v_3} \dot{v}_3. \tag{29}
\]
Substituting for \( \dot{q}_{\text{bit}} \) by differentiating (17) w.r.t. time and using (7) gives
\[
\dot{\sigma} = \frac{\partial \eta}{\partial q_{\text{bit}}} (\dot{\xi} - l_1p_p) + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \tag{30}
\]
\[
= -l_1\frac{\partial \eta}{\partial q_{\text{bit}}} (-a_1\hat{q}_{\text{bit}} + b_1u_p) + \frac{\partial \eta}{\partial q_{\text{bit}}} \dot{\xi} + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \tag{31}
\]
where \( \dot{\xi} \) is known from (16). From Assumption 2, only \( q_{\text{bit}} \) in (31) is unknown. To deal with this an estimate \( \hat{\sigma} \) is used
\[
\dot{\hat{\sigma}} = -l_1\frac{\partial \eta}{\partial q_{\text{bit}}} (-a_1\hat{q}_{\text{bit}} + b_1u_p) + \frac{\partial \eta}{\partial q_{\text{bit}}} \dot{\hat{\xi}} + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \tag{32}
\]
\[
\hat{\sigma} = \hat{\sigma} - \eta(\hat{q}_{\text{bit}}, v_3). \tag{33}
\]
Since \( \hat{\sigma} \) the dynamics of the estimation error is obtained from (31)–(32) as
\[
\dot{\hat{\sigma}} = l_1a_1 \frac{\partial \eta}{\partial \hat{q}_{\text{bit}}} \hat{\xi}, \tag{34}
\]
where the fact \( \hat{\xi} = \hat{q}_{\text{bit}} \) has been used. Comparing (34) to (24) suggests that \( \eta \) should be chosen such that
\[
l_1a_1 \frac{\partial \eta}{\partial \hat{q}_{\text{bit}}} = \Gamma \phi. \tag{35}
\]
Using (20) and integrating (35) w.r.t. \( \hat{q}_{\text{bit}} \) gives
\[
\eta(\hat{q}_{\text{bit}}, v_3) = \left[ \frac{\hat{q}_{\text{bit}}^3}{3l_1a_1} \right] \tag{36}
\]
The partial derivatives of \( \eta(\hat{q}_{\text{bit}}, v_3) \) needed in (32) are
\[
\frac{\partial \eta}{\partial \hat{q}_{\text{bit}}} = \left[ \frac{\hat{q}_{\text{bit}}^2}{l_1a_1} \right], \quad \frac{\partial \eta}{\partial v_3} = \left[ \frac{0}{l_1a_1} \right]. \tag{37}
\]

D. Initial Conditions
There are two initial conditions that need to be set. One is \( \xi(0) \) in (16) and the other is \( \hat{\sigma}(0) \) in (32). The initial conditions should be constructed by using the relationships
\[
\hat{\xi}(0) = \hat{q}_{\text{bit}}(0) + l_1p_p(0), \tag{38}
\]
\[
\hat{\sigma}(0) = \hat{\sigma}(0) + \eta(\hat{q}_{\text{bit}}(0), v_3(0)), \tag{39}
\]
where \( p_p(0) \) and \( v_3(0) \) are known since they are measured. The user can now come up with initial estimates of \( \hat{q}_{\text{bit}}(0) \) and \( \hat{\sigma}(0) \) and then use relations (38) and (39) to compute the corresponding \( \xi(0) \) and \( \hat{\sigma}(0) \).
E. Convergence of \( \hat{p}_{\text{bit}} \)

The goal of Section III is to design an observer so that the estimated pressure at the bit \( \hat{p}_{\text{bit}} \) tracks \( p_{\text{bit}} \). In Sections III-A – III-D an observer for the unmeasured state \( q_{\text{bit}} \) has been designed. In this section convergence properties of \( \hat{p}_{\text{bit}} = p_{\text{bit}} - \hat{p}_{\text{bit}} \) will be proved. Motivated by (13) an estimate of \( \hat{p}_{\text{bit}} \) is

\[
\hat{p}_{\text{bit}} = p_c + M_a (a_2(p_p - p_c) - \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} + \hat{\theta}_2 v_3) + (M \hat{\theta}_1 - F_d) |q_{\text{bit}}| q_{\text{bit}} + (\rho_d g - M \hat{\theta}_2) v_3. \tag{40}
\]

Using (13) the error in the estimate can be expressed as

\[
\hat{p}_{\text{bit}} = M_a (-\hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} - \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}}) + \hat{\theta}_2 v_3 - F_d (|q_{\text{bit}}| q_{\text{bit}} - |q_{\text{bit}}| q_{\text{bit}}) + M (\hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} - \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}}) - M \hat{\theta}_2 v_3. \tag{41}
\]

Using \( M = M_a + M_d \) and (18), (42) can be rewritten as

\[
\hat{p}_{\text{bit}} = M_a (\hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} - \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}}) - M_d \hat{\theta}_2 v_3 - F_d (|q_{\text{bit}}| q_{\text{bit}} - |q_{\text{bit}}| q_{\text{bit}}) = M_a (\hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} - |q_{\text{bit}}| q_{\text{bit}}) + \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} - \hat{\theta}_2 v_3) = - F_d (|q_{\text{bit}}| q_{\text{bit}} - |q_{\text{bit}}| q_{\text{bit}}) = - F_d (|q_{\text{bit}}| q_{\text{bit}} - |q_{\text{bit}}| q_{\text{bit}}). \tag{42}
\]

From the error equation (43) and remembering that \( (q_{\text{bit}} - \hat{q}_{\text{bit}}) \to 0 \) from the previous Lyapunov analysis it can be seen that if \( \dot{\theta}_T \phi \to 0 \) then \( \hat{p}_{\text{bit}} \to 0 \). In view of (21), \( \hat{q}_{\text{bit}} \to q_{\text{bit}} \) and \( \xi \to 0 \), the convergence \( \dot{\theta}_T \phi \to 0 \) follows directly from the extended Barbalat’s Lemma [17, Lemma 1], provided \( \dot{\theta}_T \phi \) is uniformly continuous. This is the case if \( \hat{\theta}, \phi, \hat{\theta} \) and \( \phi \) are bounded. The previous analysis has established that \( \hat{\theta}, \phi, \hat{\theta} \in L_\infty \), so it remains to show that \( \phi \in L_\infty \). Using (17) we obtain

\[
\phi = \left[ -2 |q_{\text{bit}}| \hat{q}_{\text{bit}} \right] v_3 = \left[ -2 |q_{\text{bit}}| \left( \frac{\xi}{v_3} - l_1 \hat{p}_p \right) \right]. \tag{44}
\]

From (16) we can conclude that \( \xi \in L_\infty \) as \( \hat{q}_{\text{bit}}, \hat{\theta}, \hat{\theta}, \hat{p}_p, p_c, u_p \in L_\infty \) from the previous Lyapunov analysis and Assumption 1. Similarly from (7) and Assumption 1 we can conclude that \( \hat{p}_p \in L_\infty \). Finally \( v_3 \in L_\infty \) by Assumption 1.

F. Summary Adaptive Observer

The adaptive observer is summarized in Table I, and has the following properties:

- All solutions to (21), (24) are uniformly bounded.
- \( \lim_{t \to \infty} \hat{q}_{\text{bit}} = 0 \)
- \( \lim_{t \to \infty} \hat{\theta} = 0 \)
- \( \lim_{t \to \infty} \dot{\theta}_T \phi = 0 \)
- \( \lim_{t \to \infty} \hat{p}_{\text{bit}} = 0 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_d )</td>
<td>42 ( m^3 )</td>
<td>Volume drill string (m³)</td>
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<tr>
<td>( \beta_d )</td>
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<td>Bulk modulus drill string (bar)</td>
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<td>( \rho_a )</td>
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<td>Density annulus ( (10^5 \text{Pa m}^3) )</td>
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<tr>
<td>( \rho_d )</td>
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<td>Density drill string ( (10^5 \text{Pa m}^3) )</td>
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<td>( F_d )</td>
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<td>Friction factor drill string ( (10^6 \text{Pa m}^2) )</td>
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<td>( F_u )</td>
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<td>Friction factor annulus ( (10^6 \text{Pa m}^2) )</td>
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<td>( M_u )</td>
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<td>( M_d )</td>
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<td>Friction factor annulus ( (10^6 \text{Pa m}^2) )</td>
</tr>
<tr>
<td>( v_3 )</td>
<td>1825</td>
<td>Vertical depth (m) of bit</td>
</tr>
</tbody>
</table>

**Table II**

**Parameter Values Grane Data**

The proposed observer has been tested on real data from the Grane field in the North Sea. The model (7), (12)–(13) was manually fitted to steady state data, resulting in the parameter values given in Table II. The depth of the bit was constant at \( v_3 = 1825 m \). Fig. 3 shows the measured data \( p_{\text{bit}}, p_{\text{bit}}, p_c \) and \( u_p \) (in solid lines) and the resulting fit denoted \( p_{\text{bit}}, p_{\text{fit}} \) (in dashed lines). Note that the \( p_{\text{bit}} \) measurement is lost in the time interval \( t \approx 1 hr \) to \( t \approx 1 hr 10 min \) as the flow is too low for the mud pulse telemetry system to function. The parameter values in Table II, with the exception of the ones assumed uncertain, are used in the observer test that now follows.

**Table III**

**Summary of Adaptive Observer Based on Nonlinear Model**

| Plant | \( p_0 = -a_1 q_{\text{bit}} + h_1 u_p \) |
| Observer | \( v_0 = a_2(p_p - p_c) - \hat{\theta}_1 |q_{\text{bit}}| q_{\text{bit}} + \hat{\theta}_2 v_3 + (M \hat{\theta}_1 - F_d) |q_{\text{bit}}| q_{\text{bit}} + (\rho_d g - M \hat{\theta}_2) v_3 \) |

From Table III it can be seen that the observer estimates the pressure at the bit \( p_{\text{bit}} \) well after initial transients. The
estimation error is usually less than 2 bar, although slightly higher during the transient at $t = 1\text{hr}$. The estimated parameters, $\hat{F}_a = M\hat{\theta}_1 - F_d$ and $\hat{\rho}_a = \rho_d - \frac{M}{\rho}\hat{\theta}_2$ from (10)–(11), settle at approximately 0.000068 and 0.0122 which gives a small error in the density estimate $\hat{\rho}$ while the friction factor estimate suffers as $\hat{F}_a$ is approximately 50 times larger than $F_d$, which means that $\hat{F}_a$ is very sensitive to inaccuracies in $F_d$.

V. CONCLUSIONS

Based on a newly developed nonlinear model an observer that estimates bit pressure during drilling has been presented. Through Lyapunov analysis the estimation error is shown to converge to zero. The proposed observer adapts to unknown friction and density in the annulus. Performance of the observer has been verified using data from the Grane field in the North Sea showing good results.

REFERENCES