Attitude Control of a Flexible Planar Space Robot

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Abstract—Rigid planar space robot is represented as Caplygin system and its base attitude can be stabilized by means of geometric phase. However, in cases when flexible arms are used for control of base attitude, inherent resonance modes are excited and instability is resulted. In this study we propose a stabilized controller is synthesized as adaptive tracking control system convined with sliding mode control to track to the arm trajectories derived from the geometric phase of the Caplygin system. Usefulness of the proposed controller is demonstrated by numerical simulations and Laboratory experiments.

I. INTRODUCTION

In ’90s, many research articles about the attitude control of the space robot have been published [1],[2],[3],[4]. However in these studies both theoretical and experimental consideration about stabilization of the planar space robot with flexible arms have not been sufficiently developed.

In this paper the attitude control problem of planar space robot with flexible dual arm is addressed. The proposed control scheme is based on the geometric phase approach [4] and the adaptive control scheme [6]. The adaptive control compensates the system uncertainties and difficulties arising from obtaining data for flexible dynamic co-ordinates. In this adaptive control, sliding mode control with Hybrid Sliding Surface (HSS) which is consisted of frequency shaped optimal sliding mode (FSOSM) and terminal sliding mode (TMS) is employed. The proposed control scheme does not directly guarantee the stability of the base satellite. However, since elastic vibration caused by the flexibility is sufficiently suppressed by the proposed control scheme, geometric phase works effectively. As a result of this control, base attitude can be regulated in a similar way to the case of rigid planar space robot.

The usefulness and validity of this control scheme can be demonstrated by hardware experiments. In our experiments free flying planar space robot with flexible dual arms floated by air on the horizontal plane has been employed.

II. MODELING OF A FLEXIBLE PLANAR SPACE ROBOT

Fig.1 shows a schematic model of a planar space robot consisting of two flexible arms connected with base satellite via revolute joints. Coordinate system \( \Sigma_0 \) is global coordinate system and \( \Sigma_1 \) is moving coordinate system fixed on the center of gravity of the base satellite. Subscripts \( \{1\} \) or \( \{r\} \) symbolizes the left arm and \( \{2\} \) or \( \{l\} \) symbolizes the right arm. \( \phi_i \) and \( \tau_i \) denote the angle of rotation of the arm and the torque supplied by the motor. Flexible arm of length \( L \) has uniform mass density \( m_l \) and uniform flexural rigidity \( EI \). Let \( J_i \) and \( J_0 \) be the moment of motor inertia and the moment of base inertia, respectively. Let \( m_L \) be the mass of the tip of arm and \( (X_i, Y_i) \) be the position coordinate of the tip. \( \theta \) denotes the base attitude. For derivation of equations of motion of the flexible planar space robot, the elastic displacement \( y_l(x, t) \) and \( y_r(x, t) \) at position \( x \) of the flexible arm is approximated by using assumed mode method as follows:

\[
y_l(x, t) = \varphi^T(x)q_{fl}(t), \quad y_r(x, t) = \varphi^T(x)q_{fr}(t) \tag{1}
\]

where \( \varphi(x) = [\varphi_1(x) \varphi_2(x) ... \varphi_m(x)]^T \) and \( q_{fl}(t) = [q_{f1}(t) q_{f2}(t) ... q_{fml}(t)]^T, \) \( \ast = l, r \), \( x \) is any point along the undeformed link. \( q_{fj}(t) \) is the jth modal displacement for the flexible arm, \( \varphi_j(x) \) is the jth assumed mode shape for the flexible arm, \( j = 1, ..., m \). By using these parameters Lagrangian \( \mathcal{L} \) can be obtained as follows[5]:

\[
\mathcal{L} = \frac{J_0}{2} \dot{\gamma}^2 + \frac{m_l}{L} \int_0^L d^2 \dot{\varphi}^2 dx + \frac{m_L}{2} \int_0^L \left( \ddot{X}_1^2 + \dot{Y}_1^2 \right) dx + \frac{m_l}{2L} \left( X_2^2 + \dot{Y}_2^2 \right) dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right)^2 dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right)^2 dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right) dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right) dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right) dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right) dx + m_l \int_0^L \left( x^2 + y_1^2 \right) \left( \ddot{\phi}_1 + \dot{\phi}_2 \right) dx
\[
\begin{align*}
+ \frac{m_l}{L} \int_0^L x \, dx \dot{\theta} + \dot{\phi}_1 \right) d \cos \phi_1 \\
+ \frac{m_l}{L} \int_0^L x \, dx \dot{\theta} + \dot{\phi}_2 \right) d \cos \phi_2 \\
- \frac{m_l}{L} \int_0^L \varphi^T(x) q_{fl} \, dx \left( \dot{\theta} + \dot{\phi}_1 \right) d \sin \phi_1 \\
- \frac{m_l}{L} \int_0^L \varphi^T(x) q_{fr} \, dx \left( \dot{\theta} + \dot{\phi}_2 \right) d \sin \phi_2 \\
- \frac{E_l}{2} \int_0^L \left( \frac{d^2 \varphi(x)}{dx^2} \right)^T \left( \frac{d^2 \varphi(x)}{dx^2} \right) dx q_{fl}^T q_{fl} \\
- \frac{E_l}{2} \int_0^L \left( \frac{d^2 \varphi(x)}{dx^2} \right)^T \left( \frac{d^2 \varphi(x)}{dx^2} \right) dx q_{fr}^T q_{fr} 
\end{align*}
\]

For the simplicity we assume \( d = 0 \), distance between center of the base and the arm joint, and truncate the flexible mode on the 2nd mode, that is \( m = 1 \). From these assumptions we can obtain the dynamic equations of motion for the controller synthesis as follows:

\[
M(q) \ddot{q} + h(q, \dot{q}) + K(q) \dot{q} = \tau
\]

where \( q = [\phi_1, \phi_2, q_{fl}, q_{fr}]^T, \tau = [\tau_1, \tau_2, 0, 0]^T \), and \( M(q) = \{M_{ij}(q)\}, h(q, \dot{q}) = \{h_{ij}(q, \dot{q})\}, K(q) = \{K_{ij}(q)\}, i, j = 1, ..., 4 \). In this equation \( q_{fl} \) and \( q_{fr} \) are used instead of \( \dot{q}_{fl} \) and \( \dot{q}_{fr} \), respectively.

Base attitude \( \theta \) does not appear in (3), though it is governed by the conservation law of angular momentum.

\[
\ddot{\theta} = \frac{1}{\lambda} \left( \dot{\alpha} \dot{\phi}_1 + \dot{\beta} \dot{\phi}_2 + \dot{\gamma} \dot{q}_{fl} + \dot{\delta} \dot{q}_{fr} \right)
\]

Where

\[
\lambda = J_h + J_1 + J_2 + 2m_l L^2 + \frac{m_l}{L} \int_0^L x^2 \, dx
\]

\[
\dot{\alpha} = -m_l L^2 - \frac{m_l}{L} \int_0^L x^2 \, dx - J_1
\]

\[
\dot{\beta} = -m_l L^2 - \frac{m_l}{L} \int_0^L x^2 \, dx - J_2
\]

\[
\dot{\gamma} = -m_l L \varphi_1(x) - \frac{m_l}{L} \int_0^L x \varphi_1(x) \, dx
\]

\[
\dot{\delta} = -m_l L \varphi_1(x) - \frac{m_l}{L} \int_0^L x \varphi_1(x) \, dx
\]

In equations (3) and (4), \( m_l/L \) can be replaced by mass density of the arms \( \rho \).

By using these approximated model we consider the following problem.

**Problem** Suppose the dynamic physical parameters written in (3) is unknown. Synthesize the stabilizing controller of the flexible dual arm space robot to stabilize the base attitude and suppress the vibration caused by the arm flexibility.

**III. CONTROLLER SYNTHESIS**

**A. Controller design for the rigid space robot**

To solve this problem we turn back to the rigid space robot. For the rigid dual arm space robot conservation law of angular momentum can be written by the following Caplygin form.

\[
\dot{\theta} = \alpha_{rigid} \dot{\phi}_1 + \beta_{rigid} \dot{\phi}_2
\]

Where \( \alpha_{rigid} \) and \( \beta_{rigid} \) are similar to the flexible case except the flexible terms. Applying the concept of the geometric phase to this Caplygin form, we can obtain the desired trajectories of the arm motions. The desired trajectories are synthesized as follows[4]:

Caplygin form (5) can be converted into

\[
\theta = \int \alpha_{rigid} \, d\phi_1 + \beta_{rigid} \, d\phi_2
\]

Suppose initial base attitude is \( \theta_0 \). Then we can design closed contour \( C \) so that

\[
-\theta_0 = \int_C \alpha_{rigid} \, d\phi_1 + \beta_{rigid} \, d\phi_2
\]

From this closed contour we can obtain the desired trajectories of \( \phi_1 \) and \( \phi_2 \) as follows:

\[
\phi_{1d}(s(t)) = -R \cos(ks(t) + \psi) + R \cos \psi
\]

\[
\phi_{2d}(s(t)) = -R \sin(ks(t) + \psi) + R \sin \psi
\]

Where \( R \) denotes a radius of the closed contour determined by relation (7), \( k \) denotes the direction of the closed contour and is assigned to 1 or -1. \( \psi \) is a constant arbitrarily assigned[4]. \( s(t) \) is monotonically increasing function and is given by

\[
s(t) = \Lambda t - \sin(\Lambda t).
\]

\( \Lambda > 0 \) indicates the angular velocity of the arm motion along the closed contour and \( T = 2\pi/\Lambda \) defines the one cycle of the contour trajectory \( C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t))) \). Then

\[
\frac{ds}{dt} = 0, \quad t = nT, \quad (n = 0, 1, 2, ...).
\]

This property shows that the desired trajectory \( C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t))) \) is at least once differentiable with respect to \( t \). Indeed it can be easily shown that \( C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t))) \) is twice continuously differentiable with respect to \( t \).

**B. Hybrid sliding surface (HSS)**

Generally speaking, conventional sliding mode control is considered to be hardly applicable to the control of the flexible system because switching control input happens to excite the inherent resonance mode of the system. To overcome this problem FSOSM has been developed [7]. Recently, Shahrvani et al [6] and Xu et al [9] proposed the new sliding mode control method with hybrid sliding surface (HSS) to apply to the control of the single arm flexible space robot.

The HSS is written as

\[
S_H = \alpha S_{FSOSM} + (1 - \alpha) S_{TSM}
\]

where subscripts FSOSM and TSM indicate frequency shaped optimal sliding mode and terminal sliding mode respectively. Weighting coefficient \( \alpha \) should be chosen properly to obtain required performance and satisfy stability.
conditions. In this subsection subscripts \{∗ = l, r\} symbolize left and right arm.

The F SOSM sliding surface is synthesized as

\[
S_{F \text{SOSM}} = e_{2*} + c_p e_{1*} + c_1 z_{1*} + c_2 z_{2*} = 0,
\]

(11)

where \( e_{1l} = \phi_1 - \phi_{1d}, e_{2l} = \phi_1 - \phi_{1d} \) and \( e_{1r} = \phi_2 - \phi_{2d}, e_{2r} = \phi_2 - \phi_{2d} \). \( z_{1*}, z_{2*} \) are filter states and are given by

\[
\dot{z}_* = F z_* + E e_{2*}
\]

(12)

where

\[
F = \begin{bmatrix} 0 & 1 \\ 0 & -2\omega_c \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]

\( \omega_c > 0 \) can be assigned arbitrarily. The main utilization of a filter would be to assure that unmodulated vibration modes in a given system are not excited in sliding mode\[7\]. \( c_p, c_1 \) and \( c_2 \) are design parameters synthesized by some optimal technique (see Appendix).

The TSM sliding surface is synthesized as

\[
S_{T \text{SM}} = e_{2*} + c_p e_{1*} = 0
\]

(13)

where \( c_p > 0, 0 < \epsilon < 1 \). Substituting (11) and (13) into (10), we can obtain the HSS as

\[
S_{H*} = e_{2*} + \alpha_c e_{1*} + (1 - \alpha) c_p e_{1*} + \alpha_c z_{1*} + \alpha_c z_{2*} = 0
\]

(14)

C. Adaptive controller

The proposed controller consists of the reference controller and adaptive controller. Desired trajectories are obtained from the reference controller synthesized for the rigid space robot. To track to these desired trajectories we synthesize the adaptive controller as follows.

First we define the HSS\(S_H = [S_{Hl}, S_{Hr}]^T\) as

\[
S_H = \begin{bmatrix} e_{2l} + \alpha_c e_{1l} + (1 - \alpha) c_p e_{1l} + \alpha_c z_{1l} + \alpha_c z_{2l} \\ e_{2r} + \alpha_c e_{1r} + (1 - \alpha) c_p e_{1r} + \alpha_c z_{1r} + \alpha_c z_{2r} \end{bmatrix}
\]

Substituting (8) and (9) into the first derivative of \( e_{2l} \) and \( e_{2r} \), we obtain

\[
\dot{e}_{2l} = \ddot{\phi}_1 - k^2 R^2 (1 - 2 \cos \Delta t + \cos^2 \Delta t) \times \cos(kt + \psi) - k R^2 \sin(kt + \psi) \sin \Delta t,
\]

\[
\dot{e}_{2r} = \ddot{\phi}_2 - k^2 R^2 (1 - 2 \cos \Delta t + \cos^2 \Delta t) \times \cos(kt + \psi) + k R^2 \sin(kt + \psi) \sin \Delta t.
\]

The \( \ddot{\phi}_1 \) and \( \ddot{\phi}_2 \) are derived from (3) and substituted into above equations. Substituting \( \ddot{e}_{2l} \) and \( \ddot{e}_{2r} \) into the first derivative of \( S_H \), we obtain

\[
\dot{S}_H = \begin{bmatrix} \dot{S}_{Hl} \\ \dot{S}_{Hr} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}
\]

\[
+ \begin{bmatrix} W_{Hl} & W_{2l} & \cdots & W_{12l} \\ W_{Hr} & W_{2r} & \cdots & W_{12r} \end{bmatrix} \begin{bmatrix} Y_{1l} \\ Y_{1r} \\ \vdots \\ Y_{12r} \end{bmatrix}
\]

\[
= \lambda_{inv} (\tau + W^T Y).
\]

(15)

Where

\[
\lambda_{inv} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix},
\]

\( \lambda_{ij} \) is obtained from \( M^{-1}(q) = (\lambda_{ij}) \). And

\[
W = \begin{bmatrix} \dot{W}_{Hl} & \dot{W}_{2l} & \cdots & \dot{W}_{12l} \\ \dot{W}_{Hr} & \dot{W}_{2r} & \cdots & \dot{W}_{12r} \end{bmatrix}^T
\]

\[
Y = [Y_{1l}, Y_{1r}, Y_{12l}, Y_{12r}]^T
\]

where \( Y_{1l} = \dot{\phi}_1, Y_{2l} = \dot{\phi}_2, Y_{3} = e_{1l}^{-1} e_{2l}, Y_{4} = e_{1r}^{-1} e_{2r}, Y_{5} = e_{2l}, Y_{6} = e_{2r}, Y_{7} = z_{1l}, Y_{8} = z_{2l}, Y_{9} = z_{1r}, Y_{10} = z_{2r}, Y_{11} = \phi_{1d}, Y_{12} = \phi_{2d} \). Since the filter states are governed by (12) and the desired trajectories are twice continuously differentiable, we can assume these parameters are bounded.

Finally from these parameterizations we synthesize the control laws and adaptive laws as follows:

\[
\begin{align*}
\tau_1 &= - (a W_{Hl}^T Y + 0) \operatorname{sat}(S_{Hl}, \delta) \\
\tau_2 &= - (b W_{Hr}^T Y + 0) \operatorname{sat}(S_{Hr}, \delta)
\end{align*}
\]

(16)

Where \( a, b, \beta_1, \beta_2, \beta_3, \beta_4 \) are synthesis parameters, \( \epsilon > 0, 0 \) is estimation of \( W^* \). \( W^* > |W_i|, i = 1, \ldots, 12 \). \( W^* \) means upper bound of \( |W_i| \) and is unknown constant. \( |Y| \) and \( \operatorname{sat}(S_{H*}, \delta) \) are

\[
|Y| = [|Y_1|, |Y_2|, \cdots, |Y_{12r}|]^T,
\]

\[
\operatorname{sat}(S_{H*}, \delta) = \left\{ \begin{array}{ll}
\operatorname{sgn}(S_{H*}) & |S_{H*}| > \delta \\
\frac{S_{H*}}{\delta} & |S_{H*}| \leq \delta, (* = l, r)
\end{array} \right.
\]

where \( \delta \) is the size of dead zone.

Remark. Since \( p < 1 \) and \( Y_3 \) may happen to be infinite.

To overcome this drawback we introduce small positive number \( \epsilon_0 \). For sufficiently small \( \epsilon_0 \) we replace \( Y_3, Y_4, S_{Hl} \) and \( S_{Hr} \) with followings.

\[
\begin{align*}
Y_3 &= e_{1l}^{-1} e_{2l}, (|e_{1l}| < \epsilon_0) \\
Y_4 &= e_{1r}^{-1} e_{2r}, (|e_{1r}| < \epsilon_0)
\end{align*}
\]

(17)

\[
S_{Hl} = e_{2l} + \alpha_c z_{1l} + \alpha_c z_{2l} + \alpha c e_{1l} + c_p (1 - \alpha) e_{1l}^{-1} e_{2l}, (|e_{1l}| < \epsilon_0)
\]

\[
S_{Hr} = e_{2r} + \alpha_c z_{1r} + \alpha_c z_{2r} + \alpha c e_{1r} + c_p (1 - \alpha) e_{1r}^{-1} e_{2r}, (|e_{1r}| < \epsilon_0)
\]

(18)

Then the following Lemmas are holded.

Lemma 1: Suppose that the control laws (16) are applied to a flexible dual arm space robot whose dynamic equation model being described by (3) and (4). Consider the following Lyapunov function.

\[
V = \frac{1}{2} S_H^T \lambda S_H + \frac{1}{2} \dot{W}^T \dot{W}
\]

(19)

Where

\[
\lambda = \begin{bmatrix} \frac{1}{\lambda_{11}} & 0 \\ 0 & \frac{1}{\lambda_{22}} \end{bmatrix}, \dot{W} = \begin{bmatrix} \dot{W}_{Hl} \\ \dot{W}_{Hr} \end{bmatrix} = \begin{bmatrix} \dot{W}_{l} - \dot{W}_{l}^* \\ \dot{W}_{r} - \dot{W}_{r}^* \end{bmatrix}
\]
Then the first derivative of $V$ is negative definite, $\dot{V} < 0$, and $S_H \to 0, \tilde{W} \to 0$ as $t \to \infty$.

**Proof:** Taking the time derivative of $V$ and using equations (15) and (16) yield

$$
\dot{V} = \frac{1}{\lambda_{11}} S_{H_1} \dot{S}_{H_1} + \frac{1}{\lambda_{22}} S_{H_2} \dot{S}_{H_2} + \dot{\tilde{W}}_r^T \tilde{W}_r + \tilde{W}_r^T \dot{\tilde{W}}_r - a|S_H|\dot{W}_r^T Y - |S_H|\dot{\epsilon} + S_H W_r^T Y - \lambda_{12} b S_{H_2} \dot{W}_r^T Y \text{sat}(S_{H_2}, \delta) - \lambda_{12} S_{H_2} \dot{\text{sat}}(S_{H_2}, \delta)
$$

$$
+ \lambda_{12} S_{H_1} W_r^T Y - \lambda_{21} a S_{H_1} W_r^T Y \text{sat}(S_{H_1}, \delta) - \lambda_{21} S_{H_1} \dot{\text{sat}}(S_{H_1}, \delta) - b|S_H|\dot{W}_r^T Y - \beta_2 W_r^T Y |S_H| - |S_H|\dot{\epsilon} + S_H W_r^T Y + \beta_1 \dot{W}_r^T Y |S_H| - \beta_1 W_r^T Y |S_H| + \beta_1 W_r^T Y |S_H| - \beta_2 \dot{W}_r^T Y |S_H|.
$$

For the simplicity we set $a = 1, b = 1, \lambda_{11} = 1, \lambda_{12} = 1, \lambda_{21} = 1, \lambda_{22} = 1$. Then $\dot{V}$ is

$$
\dot{V} < - \left( 1 - \lambda_{12} \right) |S_H| \epsilon + \lambda_{12} \dot{W}_r^T Y |S_H| - \lambda_{21} |S_H| \epsilon + \lambda_{21} \dot{W}_r^T Y |S_H|.
$$

From the definition of $\lambda_{inv}$, we can show $\lambda_{11} = \lambda_{22} > 0, \lambda_{11} > |\lambda_{12}|, \lambda_{22} > |\lambda_{21}|$. And we set $\epsilon > \max \left( \lambda_1 \dot{W}_r^T Y, \lambda_2 \dot{W}_r^T Y \right)$. Where

$$
\lambda_1^2 = \frac{2|\lambda_{12}|}{\lambda_{11} - |\lambda_{12}|}, \quad \lambda_2^2 = \frac{2|\lambda_{21}|}{\lambda_{22} - |\lambda_{21}|}.
$$

Substituting these parameters into (21), we can obtain $\dot{V} < 0$. Then, $S_H \to 0, \tilde{W} \to 0$ as $t \to \infty$.

**Lemma 2:** Suppose $S_H \to 0$ as $t \to \infty$. Then $\{e_{11}, e_{11} \} \to 0$ and $\{\dot{e}_{11}, \dot{e}_{11} \} \to 0$ as $t \to \infty$.

**Proof:** Since $S_H \to 0$, $S_{H_1} \to 0$ and $S_{H_2} \to 0$. This leads us

$$
e_{2t} = -\alpha c e_{11} - (1 - \alpha) c e_{11}^p - \alpha c z_{11} - \alpha c z_{21}$$

$$
e_{2r} = -\alpha c e_{11} - (1 - \alpha) c e_{11}^p - \alpha c z_{11} - \alpha c z_{21}.
$$

Since following discussion is satisfied in both the $l$’s case and $r$’s case, we omit the subscript $\{\}$. Substituting (22) into (12), we obtain the following state equation.

$$
x_e = A_c x_e - b_c u(y)
$$

$$
y = c_c x_e
$$

Where

$$
A_c = \begin{bmatrix}
0 & 1 & 0 & 0
\end{bmatrix}^T,
$$

$$
b_c = [0 \ 1 \ 1]^T, c_c = [0 \ 0 \ 1], x_e = [z_1 \ z_2 \ e_1]^T, u(e_1) = w(y) = c_p (1 - \alpha) e_1^p.
$$

Stability of the system (23) is shown by the Popov’s Stability Criterion[8]. If following three conditions are satisfied for an arbitrarily small $e_0$, then the origin of the system (23) is globally asymptotically stable.

1) $A_c$ is Hurwitz and $[A_c, b]_c$ is controllable.
2) The nonlinearity $u(y)$ belongs to the sector $[0, k_0]$.
3) There exists a strictly positive number $\eta$ such that

$$
\text{Re} \left[ (1 + j \eta \omega) G(j \omega) \right] \geq -\eta \left( 1 - \frac{1}{k_0} \right), \quad \forall \omega \geq 0.
$$

Where $G(s) = c_c(sI - A_c)^{-1}b_c$.

The first condition can be checked by Routh-Hurwitz method. In order to check the stability of $A_c$ we have following conditions.

$$
\omega_c = 2\omega_c + c_2 \alpha + c_3 \alpha > 0
$$

$$
\omega_c (\omega_c^2 + c_1 \alpha + 2\omega_c e_2 c_3) - \omega_c c_1 \alpha > 0
$$

$$
\omega_c^3 e_2 c_3 > 0.
$$

Since $c_c > 0$ is guaranteed by the synthesis method of the FSOSM sliding surface, if $\alpha$ is chosen as

$$
0 < \alpha < \min \left\{ 1, \frac{2\omega_c}{c_2 + c_3} \right\},
$$

then $A_c$ is Hurwitz. Furthermore transfer function $G(s) = c_c(sI - A_c)^{-1}b_c$ is given by

$$
G(s) = \frac{s^2 + 2\omega_c s + \omega_c^2}{s^3 + \omega_c s^2 + (\omega_c^2 + c_1 \alpha + 2\omega_c e_2 c_3) s + \omega_c^3 e_2 c_3}.
$$

There is no pole-zero cancellation except on the hyper plane $c_1 - (c_2 + c_3)\omega_c = 0$ in the parameter space. Therefore $[A_c, b_c]$ is controllable almost everywhere in the parameter space.

The definition of $u(y)$ the 2nd condition can be easily checked. Since $0 < \alpha < 1, c_p > 0$ and $0 < p < 1$, then $w(y) > 0$. Furthermore

$$
\frac{\partial w}{\partial y} = (1 - \alpha) p c_p c_1 (p - 1) \in (0, +\infty).
$$

Then $0 \leq w(y) \leq k y^2$. This means the 2nd condition is satisfied.

The last condition can be checked as follows. Letting $\Theta = \text{Re}(G(j \omega))$ and $\Xi = \text{Im}(G(j \omega))$, (24) can be rewritten as

$$
\Xi \leq \frac{1}{\eta} \left( \Theta + \frac{1}{k_0} \right) - \frac{e_0}{\eta} \left( \Theta + \frac{1}{k_0} \right)
$$

Where

$$
\Xi = \frac{-\omega^6 - (2\omega_c^2 + 2\omega_c e_2 c_3 - c_1 \alpha) \omega^4}{[\omega_c^2 c_3, \alpha - (2\omega_c + c_2 \alpha + c_3 \alpha) \omega^2] \omega^2}
$$

$$
- (\omega_1^2 + \omega_2^2 + 2\omega_c e_2 c_3) \omega^2.
$$

If

$$
\alpha < \frac{8\omega_c^2 c_3 - 8\omega_c^3 c_2}{4\omega_c^2 c_2^2 + c_1^2 - 4\omega_c c_1 c_2} = \alpha_{\text{bound}}.
$$
then $\Xi < 0, \forall \omega$. Therefore, if $\alpha$ is selected as $0 < \alpha < \min(1, \alpha_{\text{bound}})$, there exist $\eta$ and $k_0$ such that (24) is satisfied. From Popov’s criterion origin of (23) is globally asymptotically stable, $x_e \rightarrow 0$ as $t \rightarrow \infty$. This completes the proof.

Lemma 1 and Lemma 2 are summarized to the following theorem.

**Theorem 1:** Suppose that the control laws (16) are applied to the flexible dual arm planar space robot described by (3) and (4). Then $\{e_{1f}, e_{1r}\} \rightarrow 0$ as $t \rightarrow \infty$ and $\{\dot{e}_{1f}, \dot{e}_{1r}\} \rightarrow 0$ as $t \rightarrow \infty$.

### IV. SIMULATIONS

#### TABLE I

<table>
<thead>
<tr>
<th>Physical Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm length</td>
<td>L=10[m]</td>
</tr>
<tr>
<td>Arm rigidity</td>
<td>EI=800[Nm]</td>
</tr>
<tr>
<td>Mass density of the arms</td>
<td>$\rho=2940[kg/m^3]$</td>
</tr>
<tr>
<td>Mass of the arms</td>
<td>m=1.0[kg]</td>
</tr>
<tr>
<td>Mass of the tip of the arms</td>
<td>m=2.0[kg]</td>
</tr>
<tr>
<td>Base moment of inertia</td>
<td>$J=6.46 \times 10^{-3}[kg/m^2]$</td>
</tr>
<tr>
<td>Arm moment of inertia</td>
<td>$J=5.05 \times 10^{-3}[kg/m^2]$</td>
</tr>
<tr>
<td>Distance between the center of the base and the joint</td>
<td>$d=0.1[m]$</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>$\mu=0.0$</td>
</tr>
</tbody>
</table>

In this section, results of the numerical simulations for the closed loop system are presented. Parameters used for simulations are shown in Table 1. Conservation law of angular momentum for the rigid planar space robot is written by (5), that is

$$
\alpha_{\text{rigid}} = \frac{m_L L^2 + m_L dL \cos \phi_1}{J_{\text{arm}} + 2m_L dL (\cos \phi_1 + \cos \phi_2)} = \frac{202323 + 20000(\cos \phi_1 + \cos \phi_2)}{10000(10 + \cos \phi_2)},
$$

$$
\beta_{\text{rigid}} = \frac{m_L L^2 + m_L dL \cos \phi_1}{J_{\text{arm}} + 2m_L dL (\cos \phi_1 + \cos \phi_2)} = \frac{202323 + 20000(\cos \phi_1 + \cos \phi_2)}{10000(10 + \cos \phi_2)},
$$

where $J_{\text{arm}} = J_H + 2m_L d^2 + 2m_L L^2$. In these calculations we assume that rigid arm link does not have mass and mass $m_L$ is concentrated on the tip of the arm. For this system we can obtain the desired trajectories $\phi_{1d}(t), \phi_{2d}(t)$. Where

$$R = -1.818 k|\theta|^{\frac{1}{2}}, k = sgn(\theta).$$

Control torques are calculated from the dynamic model whose arm is rigid without mass distribution and has point mass concentrated at the tip of arm.

On the other hand control parameters of the control laws (16) are given as $a = 1, b = 1, \beta_{1f} = 1, \beta_{1r} = 1, \epsilon = 30[Nm], \delta = 0.3[rad], c_0 = 0.000001[rad], \omega_c = 2[rad/sec], a' = 10, c_e = 0.01, c_1 = -3.7879, c_2 = -3.2456, c_p = 2, p = 3/5, \alpha = 0.6$. $a'$ assigns corner frequency of the $FSOSM$.

Fig.2,3,4 and 5 show the simulation results. In these simulations parameter $\Lambda = 2$ representing the angular
velocity of the arm motion is set \(2\, [\text{rad/sec}]\). Fig.2 shows the
time histories of the base attitude \(\theta\). Solid line denotes the
performance of the conventional controller which is proposed
for the control of the rigid planar space robot[4]. Dashed line
denotes the performance of the proposed controller. Base
attitude controlled by the proposed controller converges to the
origin. On the other hand base attitude controlled by
the conventional controller cannot be converged and it is
still remained vibration. Fig.3 shows the trajectory errors
\(\{e_t, e_r\}\). Both errors are asymptotically converged to the
origin. Fig.4 and Fig.5 show the elastic deflections of the
arm as a function of two independent variables \(\{t, x\}\). In
both figures the arm deflection is enlarged at the beginning of
performance. However as shown in these figures, the elastic
deflection caused by the proposed controller is vanished
faster than that caused by the conventional controller.

V. EXPERIMENTS

In order to investigate real mechanical dynamics and also
to demonstrate the validity and effectiveness of the proposed
control laws in a practical system, the prototype of the planar
space robot floated by air on the horizontal table with air
is employed. Fig.6 shows a photograph of the prototype of
the space robot with symmetric two arms connected to the
base satellite with revolute joints. Two arms are made from
aluminum beam and physical parameters of these arms are
same. 450 [mm] length, 2 [mm] thickness and 40 [mm] width.
On the tips of two arms and center of the base air pads are
equipped separately. Through these air pads, compressed air
is exhausted on the horizontal table covered with smooth
glass to float the prototype robot. Control parameters are
same as those used for simulations except \(\Lambda = 0.1\).

Fig.7 and 8 show the experimental results. Controller
parameters are same as those in the simulations except \(\Lambda =
0.1\). Fig.7 shows the time history of the trajectory tracking
error \(e_r\). From this result it can be said that the tracking
error is sufficiently reduced. Fig.8 shows the deflection of
the tip coordinate. These experimental results demonstrate
the usefulness and validity of the proposed controller.

VI. CONCLUSION

A new method for attitude control of flexible dual arm
planar space robot has been proposed. It is based on the
geometric phase and adaptive sliding mode control with
a synthesized hybrid sliding surface. The hybrid sliding
surface makes it possible to minimize excitation of flexible
modes caused by the conventional sliding mode control. The
proposed controller has been applied to the experimental
system of the flexible planar robot floated on the flat plane by
the air. By simulation and experimental results it has been
demonstrated that the proposed control laws and adaptive
laws overcome the problems caused by the arm flexibility
and parameter uncertainty.

This work was supported in part by both the Scientific
Research Fund (B)19360110 and Hitech Research Center,
Project for Private University from the Ministry of Education,
Culture, Sports, Science and Technology.

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![Fig. 7. Trajectory errors](image)

![Fig. 8. Time histories of arm deflection](image)